

Machine Learning and Finance Model in Predicting Default: Merton-based Reasoning

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Abstract: When economics is combined with artificial intelligence in default prediction, difficulty arises because of the basic methodological gap, theory, and technical learning. This research empirically explores the difference between the Merton model, which calculates the probability of default by comparing the expected assets to its projection, and case-based reasoning (CBR), which learns microdata patterns. The research aims to define the characteristics of each method through a series of experiments. In addition, this study's goal is to synthesize a combined method that utilizes the advantage of each approach. Results show that the Merton model is more accurate than the CBR under a particular subjective condition. Few studies have explored and compared experimental studies between economic approaches and machine learning. The novelty of this research is how to form an optimum synthetic method of those different approaches. We recommend combining these models in the proposed algorithm to optimize the prediction performance. Furthermore, this work provides evidence supporting the fusion that yields 84% accuracy and 8% Type II error.

Keywords: the probability of default, Merton model, machine learning, case-based reasoning, bankruptcy prediction.

预测违约中的机器学习和金融模型：基于默顿的推理

摘要：当经济学与人工智能相结合进行违约预测时，由于基本的方法论差距、理论和技术学习，困难出现了。本研究通过经验探讨了默顿模型（通过将预期资产与其预测进行比较来计算违约概率）与基于案例推理（学习微观数据模式）之间的差异。该研究旨在通过一系列实验来定义每种方法的特征。此外，本研究的目标是综合利用每种方法的优势的组方法。结果表明，在特定主观条件下，默顿模型比基于案例的推理更准确。很少有研究探索和比较经济方法和机器学习之间的实验研究。这项研究的新颖之处在于如何形成这些不同方法的最佳合成方法。我们建议在提议的算法中结合这些模型来优化预测性能。此外，这项工作提供了支持产生 84% 准确度和 8% II 类错误的融合的证据。

关键词：违约概率、默顿模型、机器学习、案例推理、破产预测。

1. Introduction

A company must assess its past performance and predict its future success to make appropriate business decisions. One way to do this is to calculate the

probability of future default, for which many formulas can be used. The Altman z-score is the most popular metric and the first that predicted bankruptcy using a multivariate credit scoring model with a combination

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of financial statements and market value measures [1]. The Ohlson model is an alternative to the Altman z-score that may be more accurate because its creation involved exploring more than 2000 companies, while the Altman z-score used 66 [2]. The Merton model developed from the Black–Scholes model calculates the probability of default (PoD) by utilizing equity as a call option on an asset and is considered the first “micro-economic” model for predicting bankruptcy [3]. Merton’s perspective on the original model of Black and Scholes brought the model closer to the real corporate financial condition [4]. Many other versions were also developed from the original model, such as the extended Black and Scholes model [4], Merton jump-diffusion model [5], generalized Black–Scholes partial differential equation [6], and the frontier markets of the Merton model [7].

Economists and computer scientists have produced new perspectives in economic theory, including market design, analysis of the network accounting theory, computational complexity, and decision theory, such as the bankruptcy prediction or credit scoring model [8]. The credit scoring model is attractive to economists and financial experts and has become a popular topic in the science of artificial intelligence. Many studies have been done on bankruptcy prediction using a machine learning method. Based on Shi and Li’s bibliometric analysis [9], the number of publications on intelligence techniques, such as support vector machine (SVM) [10], decision tree and neural network [11], deep learning [12], and case-based reasoning (CBR) [13] models in bankruptcy prediction, is increasing. Many hybrid and ensemble methods are developed to increase the predictive ability [14].

Unfortunately, the implementation of various machine learning algorithms is often hampered by the issue of interpretability. Therefore, several studies highlight the importance of its nature of it in making a decision [15]. According to Altman [1], particularly on this topic, users of the bankruptcy prediction method still need a theoretical background for obtaining the prediction to take the necessary steps. That is what causes policymakers in both government and business circles to be reluctant to use machine learning algorithms unless those can be explained.

Although there is ample research in bankruptcy prediction using machine learning, the boundary between the sciences behind the two disciplines is clear. Here, we perform a comparative analysis of Merton’s finance model and a CBR machine learning model to understand how the finance and machine learning models work. We use CBR because it analyzes “case or micro pattern-based” data and maintains the interpretability of financial models. The analysis involves predicting the PoD in a given dataset, understanding each method’s means of making predictions, identifying the strengths and weaknesses

of each method, and making some recommendations to optimize each. This research shows how these two methods enhance each other’s ability to predict bankruptcy effectively.

This study consists of four parts. The first part is an introduction that explains why this research is important. The second part gives the theoretical foundations of the Merton structural model and CBR. The third part presents the results comprising an explanation of the data used in the experiments, a description of the experiments performed on the Merton and CBR models, the strategies used to expose their characteristics, and a discussion of those results. The paper’s final part presents our conclusion and recommendations for building a better bankruptcy prediction intelligent system.

2. Methods

This section describes the finance and artificial intelligence models used in this research.

2.1. Merton Structural Model

The Merton model uses the market value of the asset and its volatility. It is better than the Altman z-score model because it uses daily instead of annual accounting data. This model is widely used in economic research because it has a good performance and provides a strong theoretical analysis.

The Merton structural model is defined as follows:

$$E(t) = V_t N(d_1) - rKN(d_2) \quad (1)$$

where

$$d_1 = \frac{\ln \frac{V_t}{K} + (r + \frac{\sigma^2}{2})\Delta T}{\sigma\sqrt{\Delta T}} \quad (2)$$

$$d_2 = d_1 - \sigma\sqrt{\Delta T} \quad (3)$$

$$K = \frac{\text{Short term Debt} + \text{Total Debt}}{2} \quad (4)$$

$$= \text{Short term Debt} + \frac{\text{Long term Debt}}{2}$$

where E is the expected value of a company’s equity; V_t is the company’s asset value in period t ; K is the one-year value of a company’s debt; t is the current period; T is the future period; N is the cumulative standard normal distribution; σ is the asset volatility [16].

Using the above formulation of the Merton model, we further define a distance to default (DD), where $V_t \leq K$ is

$$DD = d_2, \quad (5)$$

and the probability of $V_t \leq K$ is equal to

$$PD = N\left(-\frac{\ln \frac{V_t}{K} + (r - \frac{\sigma^2}{2})\Delta T}{\sigma\sqrt{\Delta T}}\right) = 1 - N(d_1) \quad (6)$$

2.2. Case-Based Reasoning

The Merton model measures the distance to a company’s default, while CBR is based on a company’s data similarity. In CBR, machine learning takes historical data from a similar entity to determine a

solution to the current problem. CBR works with case-based data sources and updates by adding new cases each cycle. To reach a solution, it retrieves a set of similar cases and builds a new solution using the solutions from the retrieved cases. CBR accommodates a revision process done by the system or a human expert before the new case is added to the database [17, 18]. Fig. 1 depicts the complete cycle of CBR.

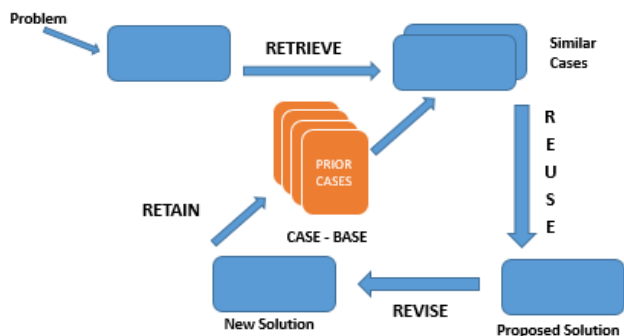


Fig 1. Case-based reasoning cycle (adopted from [18])

The distance measurement generally used is Euclidean distance, which calculates the distance from entities X to Y using each variable in $X = (x_1, x_2, .. x_n)$ and $Y = (y_1, y_2, .. y_n)$ through the following formula:

$$d_e(X, Y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2} \quad (7)$$

Many other distance measurements can be applied, such as Hamming, Minkowski, and Manhattan distance. Furthermore, other feature-based similarity metrics, such as the Jaccard index, can be used. Another developing model of similarity measurements combines local and global indices [13].

2.3. Merton-Based Reasoning

Many modifications of the CBR method were found, both in terms of the variables used and the method itself. In general, changes in variables as system inputs will affect the retrieval process. That makes CBR very easy to adapt to various disciplines, such as banking, which can be found in [13, 19] and health research [20, 21]. The retrieval process is the same, namely by applying the concept of similarity with various approaches that have been previously mentioned. The reuse and revision process will vary greatly depending on the case to be resolved and how the system built can run optimally. In addition to modifying the formula at each step, CBR can also adopt other approaches, either used as input or aligned into one of the existing 4-step processes.

We propose herein the following approach to create an optimal system. We used Merton's variables and output probability as the case representation and CBR's historical data for the solution. Fig. 2 describes the algorithm.

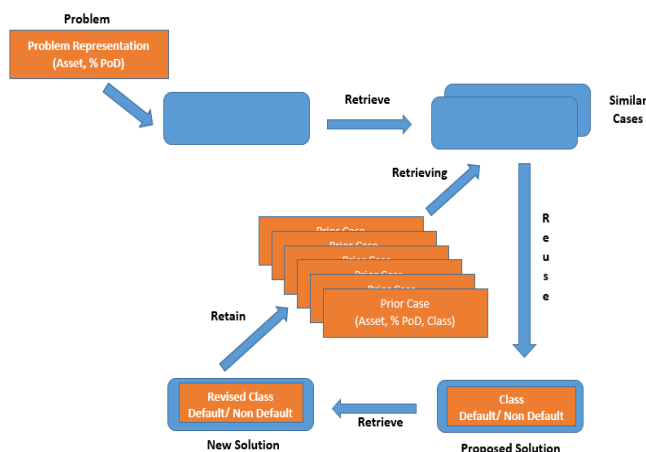


Fig. 2 Proposed Merton – CBR in determining the class

The case representation consists of the Merton model's use of asset and debt, the numeric value of PD, and default or non-default classes. The problem entered into the system has a classless case representation because it classifies it in the next step.

2.4. Data

The data used herein came from the annual financial reports from 2010 to 2016 of Indonesian public companies. We extracted important variables (i.e., net cash flow, short- and long-term liabilities, total assets, and equity) from these financial reports related to the Indonesia Stock Exchange. Before exclusions, our original data set came from over 500 companies each year.

As previously stated, the original Merton model sourced its data from the stock market daily. Our experiment used annual financial data to be implemented both on the Merton model and CBR. Thus, we assigned variable V_t as the company's asset value in the year of t , K as total liabilities per corresponding year, t as the current year, T as a future time, and σ as the volatility of the asset. That is the ratio of the previous year's assets to the current total asset. If the volatility could not be calculated, we use the mean of the value of each company. In this experiment, we forecasted the year-1 occurrences; hence, the T value in the Merton model was 1.

For determining the PoD, one theory suggests that a company is distressed if its liabilities are more than its total assets [2]. Another holds that technical insolvency occurs when a company cannot pay its current obligation on time because it does not have enough net cash flow [1]. The ground truth class in this experiment uses these technical insolvency theories defined as:

$$Tech\ Insolvency = \frac{Net\ Cash\ Flow}{Short\ term\ Liabilities}$$

A firm is categorized as being in distress if the ratio of technical insolvency is less or equal to values between 0.2 and 1. The greater the ratio, the better the company's condition because the net cash flow is sufficient to pay for these short-term liabilities. Thus, a

company is categorized as non-default if the ratio is greater than 1.

The Merton model cannot automatically specify the cut-off point; hence, we used the probability of default (PoD) greater than three specific cut-offs: 70%, 50%, and 30%. We used multiple point values better to understand the results and the most optimal cut-off. In the CBR model, the different values of this parameter are not needed because it classifies results based on historical data that already have a specific class based on the ratio of technical insolvency.

3. Results and Discussions

3.1. Experiment

Table 1 shows the results of the Merton model experiments using all data. The ratio of technical insolvency shows the effect of the data balance. The different values of the ratios in the ground truth class are described in column 1 of the table. In contrast, the

data balance ratios are given in column 2 — the greater the technical insolvency ratio, the more unbalanced the data. The ratio should be unbalanced in a real condition because the number of distressed firms is much less than the number of healthy companies. However, in this study, the number of distressed companies is more than healthy ones. It is in line with the auditors' going concern variable in each company's financial report. We categorized the evaluations into four groups, that is, true positive (TP), true negative (TN), false positive (FP), and false-negative (FN). TP indicates a healthy company that was predicted to be healthy. TN indicates a distressed company that was predicted as the real condition. FP indicates a distressed company that was predicted to be healthy. FN indicates a healthy company that was predicted to be insolvent. The inevitable thing is a high FP. It will harm the company itself because it cannot detect the bad condition and suffers its financial condition.

Table 1 Results of the Merton model experiments in terms of the true positive (TP), true negative (TN), false positive (FP), and false-negative (FN) in different ratios of technical insolvency and cut-off of the probability of default (PoD)

Technical insolvency	Healthy: distress	Cut-off PoD	TP	TN	FP	FN
0.2	1463:1510	0.7	1394	232	1278	69
		0.5	1255	851	659	208
		0.3	792	1256	254	671
0.3	1101:1872	0.7	1051	251	1621	50
		0.5	980	938	934	121
		0.3	692	1518	354	409
0.4	849:2124	0.7	811	263	1861	38
		0.5	771	981	1143	78
		<u>0.3</u>	<u>594</u>	<u>1672</u>	<u>452</u>	<u>255</u>
0.5	666:2307	0.7	632	267	2040	34
		0.5	606	999	1308	60
		0.3	501	1762	545	165
0.6	549:2424	0.7	519	271	2153	30
		0.5	499	1009	1415	50
		0.3	429	1807	617	120
0.7	471:2502	0.7	443	273	2229	28
		0.5	424	1012	1490	47
		0.3	369	1825	677	102
0.8	408:2565	0.7	382	275	2290	26
		0.5	371	1022	1543	37
		0.3	328	1847	718	80
0.9	376:2597	0.7	353	278	2319	23
		0.5	343	1026	1571	33
		0.3	306	1857	740	70
1	343:2630	0.7	321	279	2351	22
		0.5	312	1028	1602	31
		0.3	277	1861	769	66

The results showed that the maximum true positive and true negative predictions were achieved when the technical insolvency ratio was 0.4 and the cut-off point was 30% PoD. It means that the company is classified as insolvent if the number of their Net Cash Flow is less than the Short-term Liabilities and that the ratio does not exceed 40%.

The total number of TP and TN was 2.266, showing 76.21% accuracy. The number of FPs indicates the false prediction of distressed companies being healthy.

We determined that 30% PoD is the best cut-off option in every step of technical insolvency, which is confirmed by the chart in Fig. 3 showing the grey line has the best average of accuracy. Thus, a company is

classified as insolvent if the PoD is greater than or equal 30%.

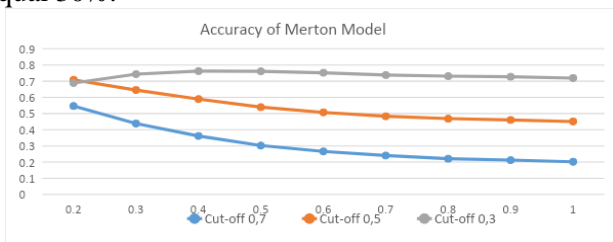


Fig. 3 Accuracy of Merton

Table 2 presents the results of the five experiments. Scenarios 1 and 3 were done to compare Merton and CBR using the data set from the 2015 and 2016

Table 2 Prediction performance in different experiments and using different variables

No	Scenario	TP	TN	FP	FN	Accuracy (%)	Type I error	Type II error
1	Merton cut-off: 0.3	126	445	152	70	72.00	19.17	8.83
2	Merton cut-off: 0.5	161	257	340	35	52.71	42.88	4.41
3	CBR: four variables	128	470	127	68	75.40	16.01	8.57
4	CBR: 23 low-correlated variables	17	582	58	112	77.89	7.54	14.56
5	CBR model: 13 highest importance variables	22	600	40	112	80.36	5.17	14.47

CBR showed a greater accuracy of 75% compared to Merton with 72%. The Merton model had the additional weaknesses of requiring the selection of the best cut point with which to classify the condition of companies. Using the intersection point value of 50% probability, the Merton model only achieves 52.17% accuracy. Scenario 2 describes the experiment result. The false-positive number was high at 340 out of 793. It can be dangerous because the insolvent company predicted to be in good condition would make wrong decisions based on that prediction.

Fig. 4 shows the total assets in billions of rupiah and 10 times the PoD value to scale to plot in the same canvas. An increase in the assets makes the PoD decrease. Conversely, a decrease in assets makes the PoD increase. This variable seems to have significantly affected the final result. The final concern is how the liability variable affects the prediction.

Scenarios 4 and 5 illustrated the CBR results when more variables were used in conjunction with an uncorrelated variable selection algorithm and the highest feature importance of a set of accounting variables.

The experiments yielded better accuracies of 75.53% and 78.43% compared to the previous Merton model and CBR experiments. We cannot provide a theoretical explanation for this because of the technical feature selection algorithm. An expert opinion is needed to determine an appropriate financial decision at this stage.

financial reports of companies with similar variables. The important variables for CBR are the total assets, total short-term liability, and total long-term liability. Scenario 1 describes the Merton model's result with a cut-off of 30% and a 40% ratio of technical insolvency. Using the Merton model, this value was used based on the best ratio and cut-off in the previous experiment.

The Merton model results in Scenario 1 showed almost the same number of true positives and a different number of true negatives than Scenario 3. The number of true positives for the Merton model was 126, while that for CBR was 128. The Merton model had 445 true negatives, while the CBR had 470.

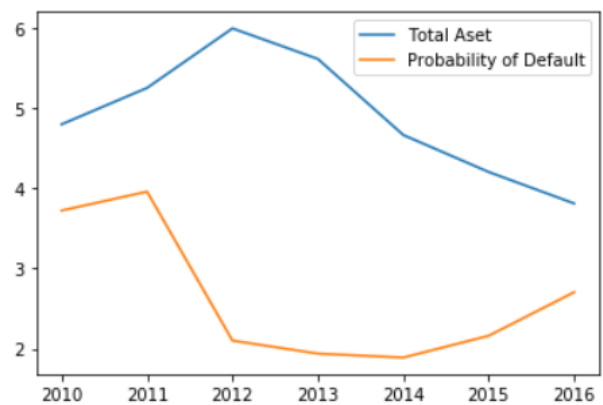


Fig. 4 The total asset and probability of default company XYZ

3.2. Discussion

Table 3 presents the differences between the Merton model and CBR in terms of the base works, output, measurements, variables, explanation, weaknesses, and strengths. The Merton model is based on asset volatility; hence, it depends on this measurement. In comparison, CBR works by using historical cases; therefore, it depends on the choice of variables. A random variable retrieval negatively affects the analysis quality and cannot be used as an explanatory variable, even though it has a better result. Explanatory variables mean that the variable can explain how it can affect the final decision and what recommendation can be made. The metrics of the default probability measurement should be explained to form a sound financial policy.

Table 3 Characteristics of Merton and CBR

Term	Merton model	CBR
Base works	Asset volatility	Historical case
Output	Percentage of probability	Class of default
Measurements	Normal distribution	Similarity metrics
Variables	Defined by theory	Defined as subjective or based on a data-selection algorithm

Continuation of Table 3

Explanatory	Based on variable and theory	Based on a rule or pattern of data
Weakness	Subjective to cut-off decision	Variable decision
Strength	Clearer analysis of variable use; Better classification of the positive condition	Objective in data classification; Better classification of the negative condition

The output of the Merton model is a number representing the PoD. This numerical value makes further analysis easier and makes the default or non-default classification subjective. However, the CBR output is default or non-default, which means further analysis to produce a recommendation on a financial issue should be based on the variable, although its classification is very objective. The experiments concluded that CBR works better in classifying the negative class and minimizing the number of false positives. The better performance may be caused by using similar measurements from historical cases. The good qualities of CBR could be used to optimize the Merton model's subjective ability to decide on the final class. Even though the number of true positives predicted by the Merton model does not differ much from that of CBR, a bankruptcy prediction system aims to predict the likelihood that a company will experience insolvency, which means predicting a negative condition.

Implementing these proposed algorithms will improve the accuracy and lower the number of FP errors. We experimented with the asset because it is the Merton model's dominant variable and the output (PoD). The system measures the similarity of the data on these variables using a particular variable weight. Table 4 presents the results. The approach accuracy is 84.36%, decreasing the Type II errors to 8%. The weight in this experiment was 0.2, 0.2, 0.2, and 0.4 for assets, short-term debt, long-term debt, and PoD, respectively. Fig. 5 depicts the comparison of five different scenarios and confirms the superiority of the proposed model.

Table 4 Optimized Merton–CBR model's result

	Predicted as healthy	Predicted as distressed
Healthy in reality	136	60
Distressed in reality	64	533

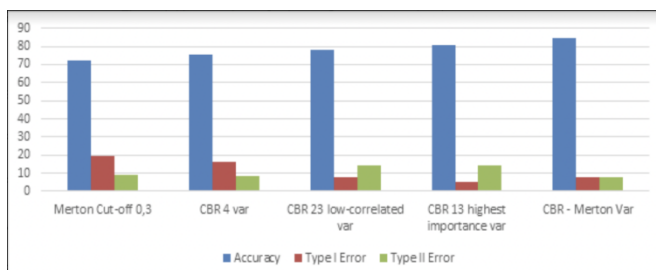


Fig 5 Accuracy and types I and II error percentage of various scenarios

This approach can also explain the CBR by strengthening the theoretical foundation.

Understanding the theoretical basis for decision-making in economics makes the variables more meaningful. The variable characteristics can help policymakers prevent the predicted bankruptcy. The Merton model explains the assets and PoD in terms of the projection and the expected value of the asset used to build the PoD and why the PoD can reflect the bankruptcy potential. Thus, further analysis can be based on these variables to form the financial recommendation action.

4. Conclusion

Although financial distress prediction has both economic and computational scopes, its application is typically for economic, strategic decision making in macro and microeconomics, banking, investments, or a company's ranking. Then, the computational algorithm supports the enhancement of economic application. The novelty of our research is that we can synthesize the advantages of each characteristic of the Merton model and CBR approach into a fusion method that optimizes performance to predict financial distress in companies. It is formed by conducting experiments using those two approaches and various variables implementing some experimental constraint. Next, we analyze the results, identify the strengths and weaknesses, and then build a combined algorithm to make better predictions.

The huge differences between the Merton and CBR models are their base theories and classify conditions. Merton uses normal distribution while CBR is a data-driven algorithm. It is not easy to determine the cut-off of the Merton model's PoD to separate the classes and vice versa easy for CBR. In CBR, it is difficult to define the variable that contributes based on the data only. Otherwise, we found it easier in Merton because the theory-based formula is clear.

Based on our experiment, the best cut-off point for the Merton model was 30% PoD using 40% for the technical insolvency ratio percentage and exhibited 72% accuracy. The single CBR algorithm implemented the Euclidean distance measurement and achieved 75% accuracy, slightly better than the Merton model. In addition to being more accurate, CBR did not require certain cutting points to determine its class because it was calculated using similarity levels. Even though the CBR can achieve high accuracy when implementing randomized variables, it cannot explain how to form a sound policy recommendation.

We can use the algorithm to obtain the best combination of the technical insolvency ratio and the cut-off point of PoD and achieve an optimal result with

the Merton model. The cut-off point may be based on the firm's characteristics, as proposed in the previous section or based on the form of its generated curve, which can be engineered by machine learning. Better performance of CBR can be achieved by analyzing the theoretical reason for the variable use. Many variables cannot be ensured to produce better results. However, we can learn from the previous research on the financial model of bankruptcy prediction and combine it with the feature selection algorithm to choose the optimum set. Novelty is reflected in the proposed approach of incorporating the variables and outputs of the Merton model into the CBR. It improved performance with 84% accuracy and a lower Type II error of 8%. The ability of this combined method is better than the single Merton. Although other machine learning methods do it better, as described in the Rahayu and Suhartanto research [18], we highlight the explainable variables supported by the theory than on the performance only. By implementing a CBR approach that is data-driven and high-explainable and using some Merton variables as input that has a strong theoretical foundation, the result is more accurate. It can be used to set a coherent policy because it can be found out for cause and effect from the point of view of economic theory.

This research has some limitations due to some constraints. First, the data is sourced from one country. It did not implement other benchmark data. Second, the prediction time frame was 1-year before the distress occurred. So, further analysis should include more benchmarks data set and make predictions for a time beyond one year. This study is carried out from the perspective of a computing expert who intends to create a method that can be explained and a more functional approach to be applied in economics. That is because many methods have been developed in computer science but are deemed less effective for adoption in the field of economics due to the lack of a theoretical basis for analyzing the results and concocting them into a decision-making application.

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