

The Intuitive Thinking Process of High Ability Students in Mathematical Problem Solving Based on Gender

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Abstract: Intuitive thinking is one approach that can be done in solving mathematical problems. This study aims to analyze the process of the emergence of intuitive thinking of high-ability students in solving mathematical problems related to number patterns in terms of gender. This research is a descriptive study using a qualitative approach with four class X students with high mathematical abilities, each of two male and female students used as research subjects. Research data was obtained through written answers and interviews with subjects. Credible data was obtained through time triangulation and was analyzed by interactive methods. The results show that the emergence of intuitive thinking in male subjects uses an extension of intuitive thinking flow with a construction strategy. This extension of thinking involves two components of intuitive thinking, feeling, and intervention. In contrast, the emergence of intuitive thinking on female subjects uses intuitive thinking with the strategy of guess and test. Intention thinking flow involves two components of intuitive thinking, feeling and intrinsic. The novelty of our studies demonstrated that gender differences in students determine the method of solving the problem, especially more pronounced in the guessing and re-checking stage, involving the intention thinking flow in female and construction strategy in the male participants.

Keywords: high ability, intuitive thinking, mathematics problem, problem-solving, gender.

基于性别的数学问题解决中高能力学生的直觉思维过程

摘要：直觉思维是解决数学问题的一种方法。本研究旨在从性别角度分析高能力学生在解决与数字模式相关的数学问题时直觉思维的出现过程。这项研究是一项使用定性方法的描述性研究，有四名具有高数学能力的X班学生，每名两名男性和女性学生作为研究对象。研究数据是通过书面回答和对受试者的采访获得的。通过时间三角测量获得可靠的数据，并通过交互方法进行分析。结果表明，男性受试者直觉思维的出现使用了直觉思维流程的扩展和构建策略。这种思维的延伸涉及直觉思维、感觉和干预两个组成部分。而女性被试直觉思维的出现，则是运用直觉思维的意图和猜测与测试的策略。意图思维流涉及直觉思维、感觉和内在思维两个组成部分。我们的研究的新颖性表明，学生的性别差异决定了解决问题的方法，尤其是在猜测和复核阶段更为明显，涉及女性的意图思维流和男性参与者的构建策略。

关键词：高能力、直觉思维、数学问题、解决问题、性别。

1. Introduction

Mathematics is a fundamental part of human knowledge that informs our decisions in life and is one of the main foundations of the technological revolution [1]. Learning mathematics aims to understand the mathematical concepts and ideas applied in problem-

solving, investigating, reasoning, communication, and making connections to everyday life [1, 2]. Regarding the importance of mathematics, the vision of mathematics learning must develop high-order thinking abilities, namely mathematical creativity through problem-solving, social empowerment through

mathematics that contributes to critical citizenship, and broader mathematical appreciation.

Mathematical problem solving is one of the most important components in any mathematics program or curriculum [3, 4]. It helps students improve and develop standard abilities to solve real-life problems, develop critical thinking and reasoning skills, gain a deeper understanding of concepts, work in groups, work together, and interact with each other [3]. Successful mathematical problem solving depends on many factors and skills with different characteristics. One of the main difficulties in learning problem solving is that many of the skills students need to acquire the skill of effective problem solvers [3]. Skills, well-connected knowledge, rich schemes, and self-confidence are critical for becoming successful problem solvers [1, 3].

Mathematical problem solving is a complex cognitive activity to find solutions using the stock of mathematical knowledge and experience already possessed. The problem-solving process can be done through two approaches, namely rational or intuitive. A rational approach occurs through formal/analytical step-by-step such as using formulas and rules of logic. While the intuitive approach can be described when someone can solve it directly, spontaneously, quickly, and do not have regular completion steps, not trying to solve problems systematically and analytically [5]. Mathematical problem solving is still dominated by the development of formal analytic thinking. If students stop before getting the final results in problem-solving, they cannot continue their work, so students are considered to fail in solving problems.

Solving problems involves a student's skills and creativity because, in general, problem-solving problems cannot be directly resolved through daily or routine procedures. Polya [6] compiled some procedures that help to solve mathematical problems. They often become a research reference relating to problem-solving: understanding the problem, making plans, carrying out plans, and re-checking. In addition, Polya also wrote seven mathematical problem-solving strategies: making drawings or diagrams, finding patterns, making tables, systematically paying attention to all possibilities, guessing and testing, backward work strategies, and using open sentences [6]. The guess and test strategy, which means guessing and checking again, is Polya's strategy that can be used to support the emergence of intuitive thinking.

Intuitive thinking is identified with spontaneous, fast, and easy thinking processes based on past experiences, including knowledge, skills, perceptions, and feelings [7]. Intuitive thinking is a process of thinking through feelings different from analytic thinking, where the explanation of the truth of a statement will always be proven. Intuitive thinking means explaining the truth of a statement that emerges subjectively and is accepted directly without any proof [5] and through feelings and interventions [8]. Intuitive thinking is a cognitive

process that leads to ideas to solve problems. Intuitive students answer written and oral problems spontaneously, without analytical thinking [9]. Intuition is a decision center where information has been stored in long-term memory. In its execution, intuition thinking is done spontaneously in problem-solving [10] and is very important in learning mathematics [11]. Intuitive understanding is needed as a bridge of thinking when someone tries to solve a problem because, with intuition, students have creative ideas in solving mathematical problems [8].

Seeing the reality of the importance of intuitive thinking in learning and problem solving, more specifically mathematical problem solving, several studies that examine the characteristics of intuitive thinking in solving mathematical problems for secondary school students [8, 12, 13], for students [10, 14] by paying attention to the mathematical abilities and cognitive styles of research subjects, and the development of intuitive thinking in mathematics learning [15]. Besides, intuitive thinking is also studied to solve physics problems [16].

Based on the results of previous studies, most of the focus is on the exploration and characterization of intuitive thinking in problem-solving, not on the emergence of problem-solving intuitive thinking and has not been viewed from gender. Research on gender differences in classroom learning and mathematical problem solving has been widely studied [17]; gender (sex) has a significant effect on learning settings and mathematical problem solving [18, 19]. Therefore, it is essential to study observing how the role of gender in using intuitive cognition in solving mathematical problems. This research will analyze the process of the emergence of intuition (intuitive thinking) of students in solving mathematical problems based on gender. The selection of students with high mathematical abilities is expected to be a reference in developing intuitive thinking for students whose abilities are mediocre or low.

2. Literature Review

2.1. Intuitive Thinking in Solving Mathematical Problems

The thought process or approach to solving problems is an intellectual activity classified into an analytic or intuitive process [20]. Analytic thinking is thinking to describe, detail, and analyze information used to understand knowledge using logical reasoning and mind, not based on guesswork [20]. At the same time, intuitive thinking is a cognitive process that can produce hypotheses to develop further knowledge. The characteristic of intuitive thinking is the process carried out or occurs subconsciously individual, the involvement of personal feelings and emotions in it, the process occurs quickly seems like automatic, and is

holistic or comprehensive and not detailed or partial [21].

Mathematicians define intuition as a way of understanding evidence and conceptualizing problems. At the same time, psychologists state that intuitive thinking is useful in various domains, including clinical diagnosis, creativity, decision making, reasoning, and problem-solving [22]. Problem-solving is a set of actions to identify the known and asked elements and combine the knowledge possessed to arrive at the correct solution [23, 24]. There needs to be staged for problem-solving activities to run smoothly, namely understanding the problem, making plans, resolutions, and re-checking [6]. Many students who are good at solving math problems often use intelligent ways beyond expectations and habits to give short and accurate answers. However, medium and low-ability students do not do this. That shows the relationship between mathematical abilities possessed by students with intuition used by students in solving mathematical problems.

Regarding problem-solving, intuition is grouped into affirmatory, conjecture, anticipatory and conclusive intuition [25]. Affirmatory is intuition to understand problems while anticipating, planning, and solving problems. These intuitions must solve problems so that maximum results will be obtained. Intuitive thinking processes that can be passed in understanding the problem are direct cognition (self-evident), intrinsic certainty, coerciveness, prediction (extrapolativeness), and overall (globally). [26] explains that intuitive thinking is a cognitive process through instincts and perceptions involving global intervention. Instinct is used in understanding the problem at hand, while perception, intervention, and global are related to the overall problem-solving process.

Intuitive thinking in solving mathematical problems has two parts: intention and extension. The intention is a spontaneous answer that appears suddenly and is associated with a guess and test strategy based on feeling and intrinsic in solving problems. In contrast, extension is a spontaneous answer that links to previous knowledge based on feeling and intervention. Feeling occurs when students guess an answer by looking and reading the problem. At the same time, intrinsic occurs when students suddenly answer directly from ideas that arise in their minds with characteristics that do not need external supporters, answer directly without knowing the reason for the answer, and feel confident about what is done. Feeling happens when students understand a problem and try to find a solution, while intrinsically done by students in generating ideas to solve problems with the help of the strategies used. In addition to feeling and intrinsic, intuitive thinking also arises because it is supported by feeling and intervention. In contrast to

intrinsic explanation, the interventions referred to associating ideas that arise in students' minds with knowledge that they have previously learned [27].

Based on the above stages of problem-solving, the components of intuitive thinking in problem-solving that will be examined in this study are seen from the step of understanding the problem, planning/strategy, and problem-solving as formulated through feelings with intrinsic and intervention.

2.2. Gender and Student Abilities in Mathematics

Gender and academic performance in mathematics found in the literature have many different views [28]. Gender differs significantly in the perseverance factor and self-concept towards mathematics. Females tend to last longer but have lower mathematical self-concepts than males [29]. Gender differences in mathematics show that males are dominant in mathematics and science [17]. However, there are no gender differences in attitudes to study mathematics [30]. Gender differences in mathematical abilities are so small that men and women do the same in math [28]. Male students were more successful than female students in solving mathematical problems, but there were no statistically significant differences [18]. Male students are better than women in counting, while women are superior to men in reading, writing, spelling, and grammar [31].

Cognitive ability factors, information processing speed, and learning styles contribute to gender differences in mathematical problem-solving. The influence of gender factors in mathematics is due to biological differences in the brains of males and females, that women are generally superior in language and writing, while men are superior in mathematics because of their better spatial abilities. The difference between spatial and verbal abilities also influences women's and men's strategies. Male tend to be more flexible using more abstract strategies and retrieval strategies, while females use manipulative and more concrete strategies.

Many mathematical problems can be solved either by spatial or verbal approaches or by both; the difference between spatial and verbal abilities will affect how students approach mathematical solutions. Therefore, a hypothetical conclusion can be drawn that males and females have different patterns (strategies) in problem-solving. There are differences in the emergence of intuitive thinking in problem-solving in terms of gender. Thus, this conceptual framework paper deals with students' intuitive thinking in understanding problems, making plans/strategies, and solving problems based on intrinsic feelings and interventions for male and female students. This conceptual framework paper is illustrated in Fig. 1.

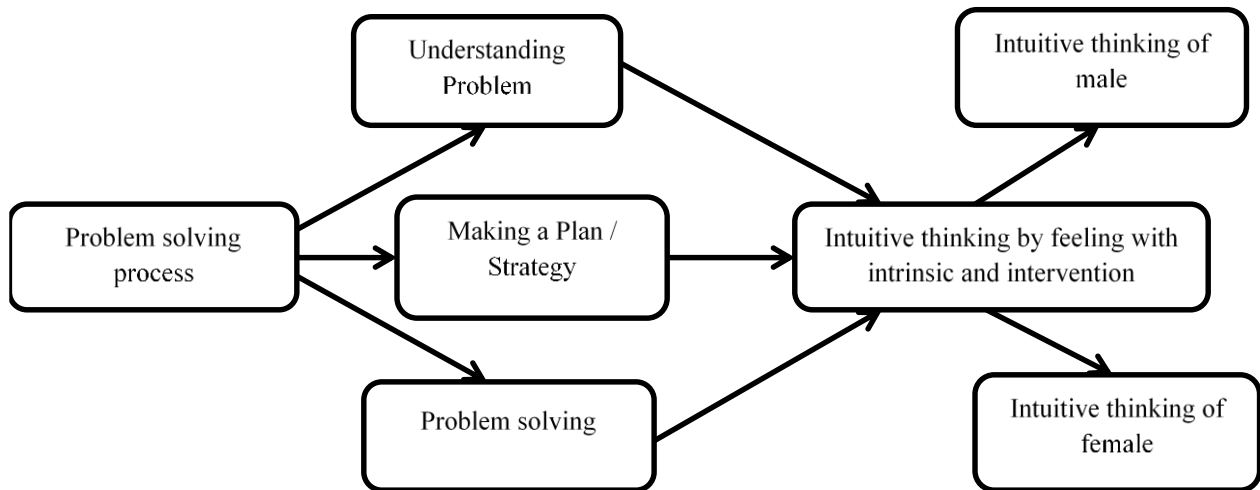


Fig. 1 Conceptual research framework

3. The Research Methods

3.1. Research Design

This research is a descriptive qualitative study with a qualitative approach, research that describes the process of the emergence of intuitive thinking in solving mathematical problems in terms of gender without any treatment of the subject [32].

3.2. Subjects

Four high school class X students with two men and two women who have high mathematical abilities and can communicate well were made the research subject. Gender differences were chosen because of differences in biological factors in the brains of people who are male and female sexually, resulting in someone who is female gender better in terms of language and writing. In contrast, a sexed male is better in mathematics and logical thinking [31]. Choosing subjects with high mathematical ability is important because intuitive thinking will appear in students with high mathematical abilities. While good oral communication because the solution written by the subject requires direct confirmation to see how the emergence of intuitive thinking when the subject is doing mathematical problem-solving. The selected research subjects represented each gender, namely two male gender students, L1 and L2, while two female gender students were P1 and P2.

3.3. Data Collection

The data in this study are in the form of the emergence of intuitive thinking of research subjects in solving problems of mathematical number patterns. The process of the emergence of the subject's intuitive thinking is obtained through written answers (documents) when the subject solves a mathematical problem, direct observation when the subject resolves the problem, and records of semi-structured interviews

conducted to dig deeper about the emergence of ideas in solving the given problem.

The instruments in this study area consisted of the main instruments and supporting instruments. The main instrument in this study is the researcher himself. At the same time, the supporting instruments are a sheet of problem-solving tests of mathematical numbers, interview guidelines, and recording tools. The instrument of problem-solving tests related to number patterns consists of a single item in the description. Each problem is expected to bring up the intuitive thought flow indicator intention and extension thinking flow. The instrument has been validated by experts who are competent in mathematics related to the construct and language of the questions and tested for readability by high school students. The interview guide is designed following the stage of problem-solving. It includes questions about understanding the questions given and leading to questions that will answer the category of intuitive thinking flow. The interview is carried out after the subject has finished writing and is semi-structured. The interview guidelines used in this study are as follows: 1) what do you understand about this problem; 2) how did you find this answer; 3) have you ever gotten a question like this; 4) are you sure about this answer? A recording device is used to document the interview activities.

The Head of Research Centre and Community Service University of Muhammadiyah Malang confirms the data collection process to comply with all regulations, and no ethical violation is observed. Data credibility can be seen from the validity of the data in the study by using several types of source triangulation and triangulation methods [33]. Source triangulation is done by comparing subjects 1 and 2 of each sex. Triangulation method by comparing the interview data and the contents of documents on the results of tests of solving mathematical number patterns.

3.4. Data Analysis

Credible data are analyzed by interactive methods, namely data reduction, data presentation, and conclusions [32]. After data reduction, there is a transcript of interview data. After the subject has solved the problem of number patterns, there is re-checking the transcript results by playing back the recording to hear the answers of the subjects during the interview. There is also a transcript of the results of documents from the test of solving the number pattern problem and reducing the transcript results by selecting and disposing of data that is not needed. Then it is focused on the main things relating to indicators of students' intuitive thinking in solving numbers problem patterns.

Presentation of the data is done in a descriptive manner intended to dig up the information obtained in-depth and is described as following the results obtained. Data is described in the form of information coherently and clearly so that it is possible to conclude. The next step is concluding. Conclusions are intended to explain the data meaning of what has been presented and adjusted to the research objectives of the emergence of intuition of male and female students in solving problems.

4. Results

4.1. The Process of the Emergence of Intuitive Thinking in Female Subjects

The results of interviews conducted on female subjects showed that they still had difficulty understanding the problem when they first read the questions. Their understanding of the problem occurs when both subjects read the problem for a second time. With understanding the problem, spontaneous ideas arise to solve the problem. Further demonstrating that female subjects understand the problem and solve it is when mentioning that to get 15 and 43 must operate three other numbers, namely 138, 3, and 9, by using two different number operations, spontaneous answers. The answer is an idea that appears suddenly and then will be processed to answer the problems given. The feeling that occurs with three given numbers and two different number operations. This feeling appears by not linking new information obtained with prior knowledge.

The answer written by the female subject is the correct answer that to get 15 is by way of $(138 - 3) : 9$ and to get 43 is the way $(138 - 9) : 3$. It shows that for getting the results 15 and 43 of the other three known numbers, the subject uses two number operations: the subtraction and division operation of the three given numbers as in Fig. 2.

Penyelesaian:

$$(138 - 3) : 9 = 15$$

$$(138 - 9) : 3 = 43$$

Fig. 2 The process of female subjects solving problems

If seen from the operations and numbers used to get results 15 and 43, then the method or strategy used by the subject is to try to arrange the three numbers and guess the calculated operation used. It is in line with the interview results, the idea that arises from the subject that three numbers, namely 3, 9, and 138, will be operated with two different number operations resulting in 15 and 43. From the ideas that emerge, then the subject can do problem-solving. Interview excerpts are presented below.

Researcher: how do you arrange the three numbers and determine the two number operations you then use?

Subject: I did it by experimenting with numbers 9, 3, and 138 and experimenting with several number operations. I remember that the first try was bigger than what was asked. Then I tried again until I got 15 and 43 results.

Researcher: When experimenting, do you use prior knowledge?

Subject: No, I only use what is in the problem.

In solving problems that occur, first, the subject makes feelings towards three different numbers given and two different number operations. Furthermore, with intrinsic assistance, the subject gave rise to obtaining 15 and 43 using the subtraction operation and the division operation against these three numbers. From the feeling and intrinsic that happened, the two subjects used the guess and test strategy to get results 15 and 43. The guess and test strategy were carried out when the subject tried to answer the question by guessing and dabbling with the right number operations for the other three numbers, thus getting the results 15 and 43. The subject's problem-solving is a decision that is believed to be correct and executed correctly through ideas that suddenly appear in his mind and are not based on prior knowledge.

Intuitive thinking arises when the subject places feelings and intrinsic on three numbers 3, 9 and 138 with the corresponding number operations so that the numbers are sought, $(138 - 3) : 9 = 15$ and $(138 - 9) : 3 = 43$. It shows that for getting 15 and 43, subjects use the numbers 138, 3, and 9 and use subtraction and division. So the subject's thought process in problem-solving can be called intended intuitive thinking. Overall, the emergence of intuitive thinking from the subject is shown in Fig. 3.

Problem

The process of the emergence of intuition in problem solving

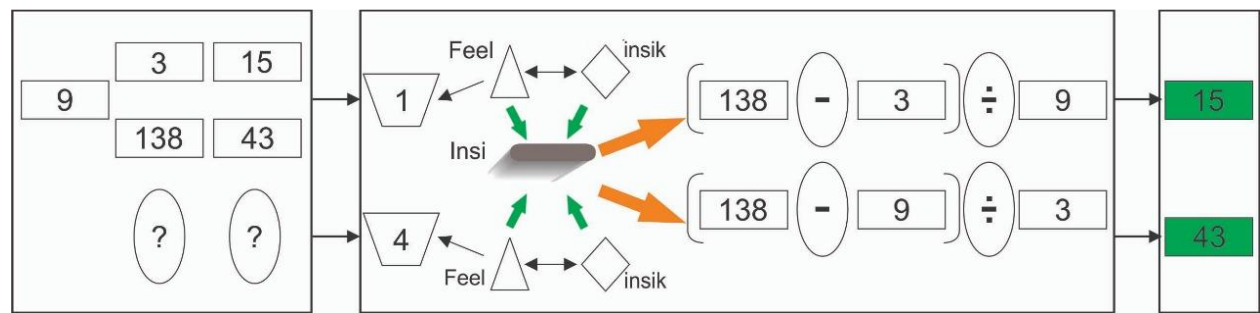


Diagram 1. The process of the emergence of female subject intuition in problem solving

Information:

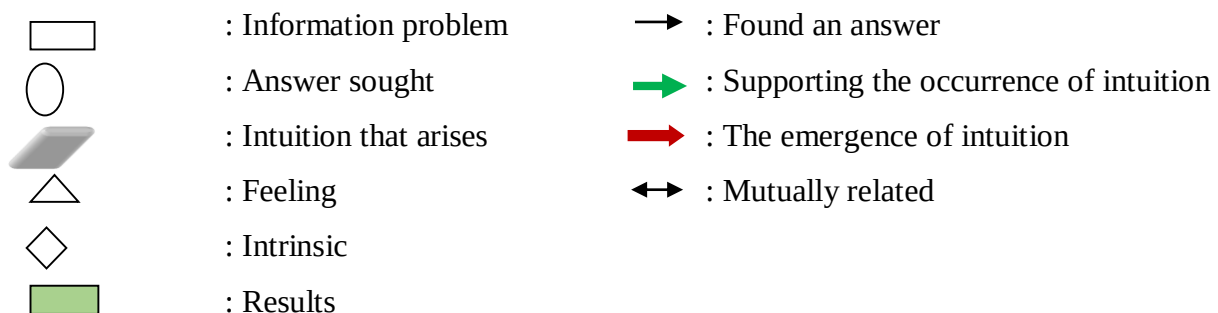


Fig. 3 Emergence of female intuitive thinking from the subject

4.2 The Process of the Emergence of Intuitive Thinking in Male Subjects

The results of interviews related to the process of male subjects in understanding the problem showed that the subjects obtained can immediately understand the problem after reading the questions by thinking about obtaining results 15 and 43, which must use three other numbers with two arithmetic operations. For getting this result, the subject thinks first and takes the largest number, 138. It means that intuition and feeling occur when selecting the largest number happens, understanding this problem as an idea that arises spontaneously to determine the answer to the problem.

The answer written by male subjects and women is the correct answer, in which to get 15 is by way of $(138 - 3) : 9$ and to get 43 is by the way $(138 - 9) : 3$. It shows that to get the results 15 and 43 of the other three known numbers and use two number operations, the subtraction and division operation for the three numbers given is following the subject's understanding and interview. The subject's written answers are presented in Fig. 4.

Penyelesaian:

a. $(138 - 3) : 9 = 15$
 b. $(138 - 9) : 3 = 43$

Fig. 4 The process of problem-solving by male subjects

Interview quotes relating to the problem-solving process are presented as follows.

Researcher: How do you arrange these three numbers and determine what number operations to use to get the results in question?

Subject: I first thought I had to take the largest number, 138

Researcher: Why 138?

Subject: Because 138 minus 3 results as 135, and 135 divided by 9 is 15

Researcher: How do you get 43?

Subject: That means the opposite of 15. Because the numbers and operations used are the same. But the numbers just need to be arranged again to produce 43

In the problem-solving process, the subject first makes a feeling of a completed number that is 138. Then, by using prior knowledge about the operation of the subject numbers using a construction strategy to produce the desired answer by performing a count reduction operation (division) of 138 numbers followed by division operations (reduction) so that results in 15 and 43 are obtained. The subject's problem-solving is a decision that is believed to be right and executed correctly through ideas that suddenly appear in his mind and are based on prior knowledge.

Intuitive thinking arises when subjects place feelings and interventions on numbers by the corresponding number operations involving numbers 138, 3, and 9 so that they get the number sought, $(138 - 3) : 9 = 15$ and $(138 - 9) : 3 = 43$. So the subject's thought process in problem-solving can be called an intuitive thinking

extension. Overall, the emergence of intuitive thinking from the subject is shown in Fig. 5.

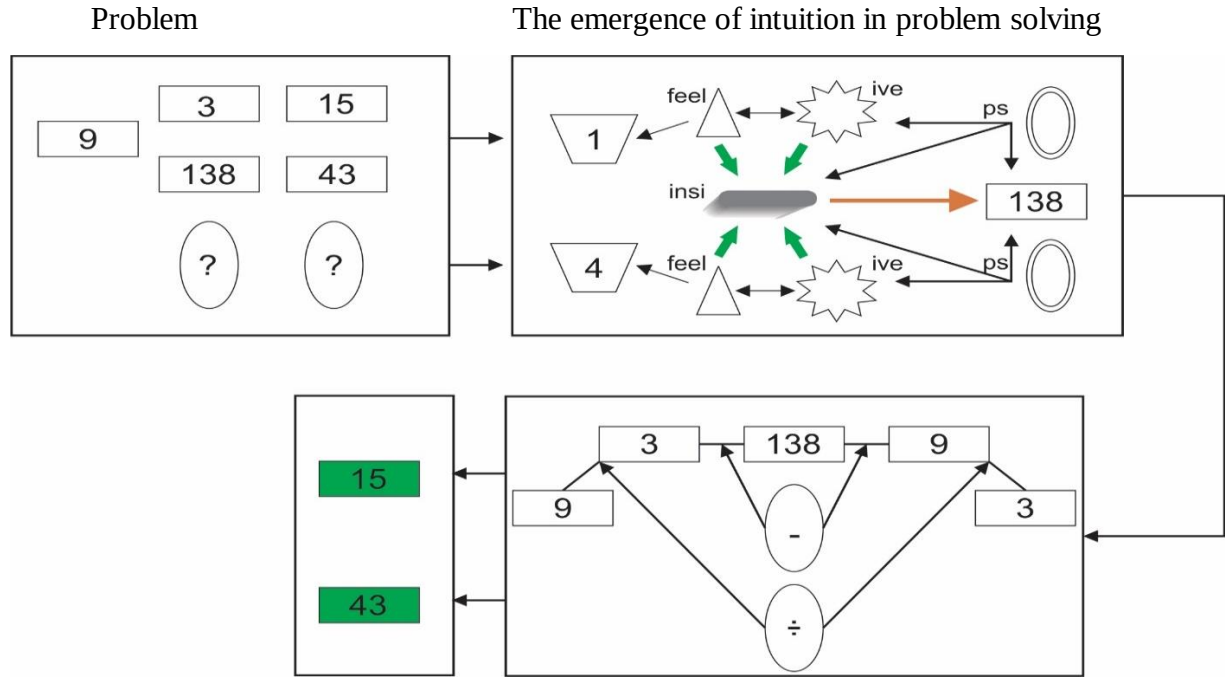


Diagram 2. Intuition Process of Male Subjects in Problem Solving

Information:



: Information problem

: Answer sought

: Intuition that arises

: Feeling

: Intervention

: Results

: Previous knowledge

→ : Found an answer

→ : Supporting the occurrence of intuition

→ : The emergence of intuition

↔ : Mutually related

Fig. 5 Emergence of male intuitive thinking from the subject

5. Discussion

The problem-solving process carried out by male subjects uses an intuitive thinking flow extension with components supporting feelings and interventions and construction strategies. The feeling means the emergence of ideas in students' minds to solve existing problems and decide to produce spontaneous answers [34, 35]. While intervention is an attempt by students to associate ideas that arise in their minds with the knowledge they have previously learned related to selecting two arithmetical operations. The process of intervention that links the problem with already possessed knowledge starts when the subject understands the problem resulting in building a construction strategy in problem-solving. The subject's response to such problems involves instinct, a component of intuitive thinking.

The emergence process of intuitive thinking is a male subject when understanding problems start when it spontaneously compiles numbers other than 15 and 43 using two number operations. In this context, the subject performs relations between given numbers to answer the problem called intuition affirmation [25]. Developing a problem-solving strategy with a construction strategy starts with intuitive thinking when the subject spontaneously (feeling) focuses on the largest number, called intuition conjecture [25]. Based on his knowledge, the subject chooses several arithmetic operations at the largest number to get the solution to the problem. When the subject develops a strategy and solves the problem, he relates it to the intervention component. When a male subject spontaneously uses several arithmetic operations in the greatest number, his intuition emerges as an awareness that is accepted as a feeling without further checking and verification because empirical data and prior knowledge support it.

That is the one by [25] as self-evident and extrapolativeness in intuitive thinking, so the subject tends to reject alternative interpretations that contradict his intuition (coerciveness). The problem-solving process produced by female subjects begins with understanding the problem, choosing a strategy, and implementing the problem-solving. When understanding a problem, the subject is only based on the information given to the problem and, by reading it repeatedly, does not relate to previous experience. In understanding the problem, female subjects immediately used the feeling of three other numbers other than those asked. Furthermore, spontaneously the female subject dabbled in several arithmetic operations to get the desired answer. It means that an idea that comes to mind suddenly as a strategy for making a decision produces spontaneous answers in solving intrinsic problems. In solving problems, female subjects do not use a particular process such as linking with prior knowledge, illustrating, or making diagrams. Thus, the subject follows intuitive thinking with two supporting components: the feeling and intrinsic components and the guess and test strategy. Understanding the subject matter of women was obtained shortly after reading the questions. That means the subject understands the problem using accepted direct cognition without further checking and proof of the problem at hand. Such thought processes are referred to as intuitive self-evident [36]. Furthermore, understanding the subject matter does not use the external support needed to obtain an immediate certainty, both formally and empirically. Intuitive thinking is called an intrinsic certainty component [13, 36]. Furthermore, if associated with opinions, there is an instinctual component that means understanding the problem with the emergence of response in thinking of the problem being faced. The similarities occurring on men and women when making plans and solving problems based on the explanation above can be related to opinions of [25] that the anticipatory component is the resolution of the problem contrary to the allegations in general. It means that students claim an unusual statement, method, or procedure, and the claim feels right. Meanwhile, suppose it is associated with the opinion of [19]. In that case, the method used by male and female subjects is included in the perception and global components because students perceive the solution to the answers generated and then resolve to obtain results. At the same time, the difference between males and females lies in subjects' strategy in understanding and solving the problem of number forms [17].

6. Conclusion

6.1. Scope of Results

Based on the data gathered and the findings of this study, it demonstrates how students' intuitive thinking emerges when solving problems. The emergence of

intuitive thinking on male subjects begins when understanding the problem using intuition of relational affirmation followed by building a construction strategy to solve the problem as a result of feeling and intervention. Such thought processes are referred to as intuitive thought flow extension. While the subject of women uses self-evident intuition to understand the problem, the confidence obtained from the problem is provided by the strategy of guessing and testing due to feeling and intrinsic. The process of thinking in solving problems like this uses the intuitive thinking flow of intention.

6.2 Novelty and Limitation

The novelty in our study revealed that intuitive thinking in Poya's stage of guessing and re-checking occurred, where the gender factor characterizes the difference of intuitive thinking nature. We observed that the intention thinking according to feeling and intrinsic in intuitive thinking was more pronounced in females while solving the problem. On the contrary, the construction strategy was carried out in male students through feeling and intervention. These differences should be further elaborated using an inter-disciplinary approach to explain the phenomenon. Further research may employ the factor of patriarchy dominance as a social construct that supports the idea and how its development in Indonesia fast-changing society, thus making the differences trait of intuitive thinking and problem solving between male and female students in Indonesia.

The study conducted in this article has identified in a practical sense the cultural values of the education system in Indonesia, which appears to have broader challenges, which affect many aspects, including the mathematic ability between gender differences. Notably, one interaction/lecturing is still the most accepted method in the day-to-day basis of teaching, where the student is a passive-received participant in the class setting. Thus, the prior intelligence profile and student experiences were determined the intuitive thinking process between gender in Indonesia.

This research is limited to examining the emergence of intuitive thinking in solving the problem of number patterns with highly capable subjects viewed from gender. Therefore, further research is still on intuitive thinking with various subject characteristics, such as moderate or low ability, learning style, and cognitive style with problems other than number patterns. Since problem-solving can use analytic and intuitive thinking, mathematics teachers should allow students to use their intuitive thinking to solve mathematical problems. Thus, they are not always focused on solving problems by analytic thinking alone. One way that teachers can use is to train students by giving practice questions that can lead to intuitive thinking.

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