第 48 卷 第 11 期 2021 年 11 月
湖南大学学报（自然科学版）
Journal of Hunan University（Natural Sciences）
Vol. 48 No. 11 November 2021

Open Access Article

Model of Discharge Coefficient for Rubber Weir Construction Steel Protected (Obermeyer Crest Gate)

Andri Rachmanto Wibowo[1*], M. Bisri[2], Sumiadi[2], Very Dermawan[2]

[1] Doctoral Program in Water Resources Engineering and Water Engineering, Faculty of Engineering, Universitas Brawijaya, Malang, Indonesia

[2] Department of Water Resources, Faculty of Engineering, University of Brawijaya, Jl. MT Haryono No 167 Malang, Indonesia

Abstract: Crest Gate Rubber Weir is a modification of rubber weir with the addition of a crest gate from metal on the front side. The function of elevating water is enacted by the crest gate, which gets support from rubber weir behind it. The crest gate also protects the rubber weir from debris and has a curvature to give room for rubber when in a deflated condition. This weir can be operated from an angle of 0º to an angle of 53º from the riverbed based on the desired water elevation. However, discharge on downstream of this weir cannot be determined due to the absence of a discharge coefficient representing this type of rubber weir. This research aims to find the discharge coefficient equation on the Crest Gate Rubber Weir. Experiments with a hydraulic model in the laboratory resulted in an equation of discharge coefficient for Crest Gate Rubber Weir, which applies to all angles of weir opening. The discharge coefficient is a function of the head over the weir (H), the vertical height of the weir (P), and the slope of the weir (tan **θ**).

Keywords: crest gate weir, discharge coefficient, hydraulic model, rubber weir, sloped upstream.

橡胶堰结构钢保护（奥伯迈耶冠门）流量系数模型

摘要：顶门橡胶堰是橡胶堰的改进型，在正面增加了一个由金属制成的顶门。提升水位的功能是由闸门实现的，闸门由后面的橡胶堰支撑。顶部闸门还可以保护橡胶堰免受碎屑的影响，并具有弯曲度，以便在放气情况下为橡胶留出空间。根据所需的水位高度，该堰可以从与河床成0度到53度的角度进行操作。然而，由于缺乏代表这种橡胶堰的流量系数，无法确定该堰下游的流量。本研究的目的是寻找顶板式橡胶堰上的流量系数方程。实验室中的水力模型实验得出了一个适用于所有堰口角度的冠门橡胶堰流量系数方程。流量系数是堰上水头(H)、堰的垂直高度（磷）和堰的斜率（棕褐色 θ）的函数。

关键词：闸门堰，流量系数，水力模型，橡胶堰，向上游倾斜。

1. Introduction

Rubber Weir Construction Steel Protected (Obermeyer Crest Gate) is a product of Obermeyer Hydro Inc. under the brand of Spillway Crest Gate System and Inflatable Bladder. This kind of weir has been a registered patent and installed in the United States since 1998. It has also been installed in some other countries such as t Portugal, Germany, United Kingdom, Indonesia, and Malaysia. This weir has a crest gate from a curved metallic plate at the upstream face of the rubber weir. The crest gate functions as a water barrier and also serves as rubber weir protection. On the other hand, the rubber only serves as a support for the crest gate as shown in Fig. 1.

Received: July 24, 2021 / Revised: September 2, 2021 / Accepted: October 4, 2021 / Published: November 30, 2021

About the authors: Andri Rachmanto Wibowo, Doctoral Program in Water Resources Engineering and Water Engineering, Faculty of Engineering, Universitas Brawijaya, Malang, Indonesia; M. Bisri, Sumiadi, Very Dermawan, Department of Water Resources, Faculty of Engineering, University of Brawijaya, Malang, Indonesia

Corresponding author Andri Rachmanto Wibowo, andriwibowo22@pu.go.id

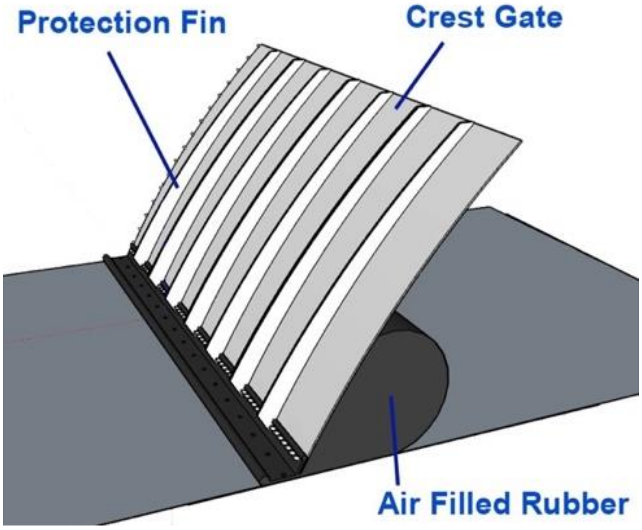

Fig. 1 Components of spillway crest gate and inflatable bladder

This weir has several unique characteristics compared to other rubber weirs. The first is that its height can be adjusted like other rubber weirs but the water flowing over it does not follow the curvature of the rubber. It is because the water only comes into contact with the crest gate and has no contact with the rubber. Since the crest gate is formed from thin metal, the flow behavior over this weir can be studied by approaching the sharp crested weir.

The second uniqueness is that the shape of the crest gate is not flat but curvy due to the circular arc. This curvature of the crest gate will affect the water flowing through the weir. Considering this uniqueness, the performance of the weir in the form of a discharge coefficient needs to be specifically studied because its discharge coefficient cannot be equated with other weirs.

The analysis of the discharge coefficient (Cd) for this type of weir is performed by taking Tirtonadi Rubber Weir in Surakarta City, Indonesia as a model. Tirtonadi weir was constructed in 2018, has a height of 3.035 m, a width of 30 m at the middle span, and a curvature with a radius of 10.98 m as shown in Fig. 2.


Fig. 2 Tirtonadi Weir in Surakarta, Indonesia

2. Literature Review

Chow [1] showed that discharge overrunning the sharp-crested weir could be expressed as [2, 3]:

$$Q = \frac{2}{3} C_d \sqrt{2g} B H^{1,5} \quad (1)$$

where:
B – the length of the weir, [L];
Cd - discharge coefficient, [-];
g - gravitational acceleration, [L/T2];
H - depth of water over the crest without $v^2/2g$, [L].

Equation 1 shows that the discharge amount overrunning the sharp-crested weir is influenced by the weir width (B) and the height of the water above the weir (H). A disparity between the observed flow rate and the calculated discharge is expressed as the discharge coefficient (Cd). Several studies have been conducted to determine the factors that affect the value of the Cd.

Rehbock [16] discovered that the ratio of water head over weir breadth and weir height (H/P) had a significant effect on the magnitude of the Cd [3]. Kinsvater and Carter [4] stated that the influence of weir width over channel width (b/B) alongside the H/P influenced the value of the Cd. Swamee [5] attempted to develop a Cd equation that would take into account the geometry of the weir, be it a sharp-crested weir, a narrow-crested weir, a broad-crested weir, or a long-crested weir. Bagheri and Heidampour [6], [7] proposed a Cd equation after researching water velocity on a nappe that overran a weir. Some proposed Cd equations are listed in Table 1.

Table 1 Discharge coefficient equations by different researchers

Researcher	Equation
Rehbock [16]	$C_d = 0.611 + 0.08\, H/P$
Kinsvater and Carter [4]	$C_d = 0.602 + 0.075\, H/P$
Bagheri and Heidampour [6, 7]	$C_d = 0.829 \ln(1 + \frac{0.73\, H/P + 3.64}{6.11})$

In addition to being influenced by the H/P ratio, United States Bureau of Reclamation (USBR) [17] discovered that the upstream face of the sloping weir influenced the weir's performance. The Ogee Weir, which is derived from a sharp-crested weir, has upstream facing angles of 1:3, 2:3, and 3:3. USBR provides a graph of the Cd for the Ogee Weir.

Shaker and Sarhan [8] conducted a study that compared weirs of different slopes and heights. They found that the slope of the weir with an upstream face influences the Cd and that, generally, milder sloping weirs generate a higher Cd than sharper sloping weirs.

3. Laboratory Experiment

The laboratory experiment was conducted using a physical hydraulic model based on the Tirtonadi weir prototype. As indicated in Fig. 3, a hydraulic model was constructed as a simple form of prototype by eliminating the elevated base and removing the pillars and the downstream apron slopes.

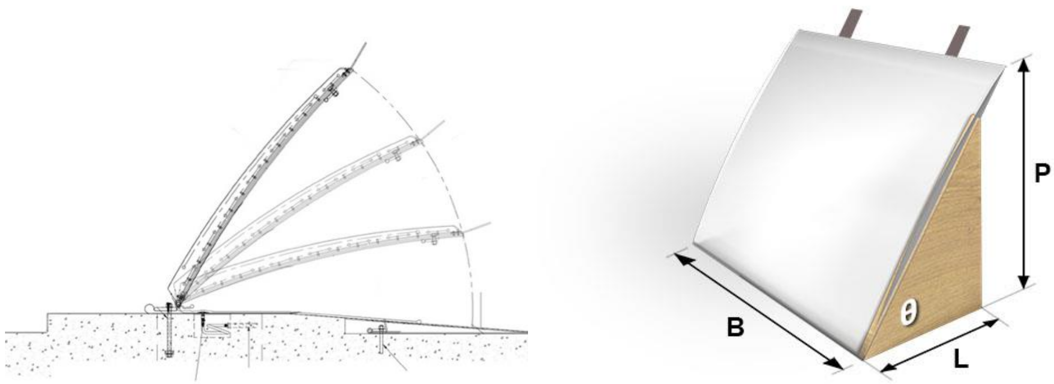

Fig. 3 Cross of weir prototype (left) and isometric view of hydraulic weir model (right)

The research was conducted in the River Engineering Laboratory of Universitas Brawijaya. The acrylic weir model has a curvature and a geometric scale of 1:10, a width 0.4 m, or as wide as the flume. The weir model curvature is a circular arc with a radius of 109.8 cm, a chord length of 38 cm, and a sagitta of 1.6 cm. Two nappe breakers with a width of 1 cm and a length of 4.6 cm are located at the weir's end tip. B_{eff} will be used to determine B, which will account for the width reduction caused by the nappe breakers.

The acrylic models are angled at 53°, 45°, 35°, and 18° to simulate the prototype weir's crest gate opening. The prototype weir has a 53° angle that represents a fully closed condition, a 35° angle that represents a two-thirds opening, and an 18° angle that represents a one-third opening. To test the Cd model produced from the other three weir models, a model with a 45° angle is built monitored. The weir model is placed 5 meters downstream of the pump and stilling system to ensure that the water flow on the weir model is stable.

A rectangular flume with a pump simulates water flow over hydraulic models. Flume has a dimension of 0.55 m height, 0.4 m width, and 9 m length and has a Rechbock weir to regulate required discharges being simulated. This research was conducted with discharges of 2, 5, 8, 11, 14, 17, and 20 l/s.

Measurement of water level and velocity are conducted by point gauge and pitot after water flow is stable. Measurements are conducted on the upstream, above the model, and on the downstream of the model. Measurements on the upstream are arranged at a distance of 20 cm, 40 cm, 60 cm, and 110 from the weir model. Measurements above the model are carried out at the weir base point, 1/3 of the weir length, 2/3 of the weir length, and at the weir's end tip or brink point. Measurements on the downstream of the model are conducted at a distance of 10 cm, 20 cm, 40 cm, 60 cm until 250 cm downstream of the weir.

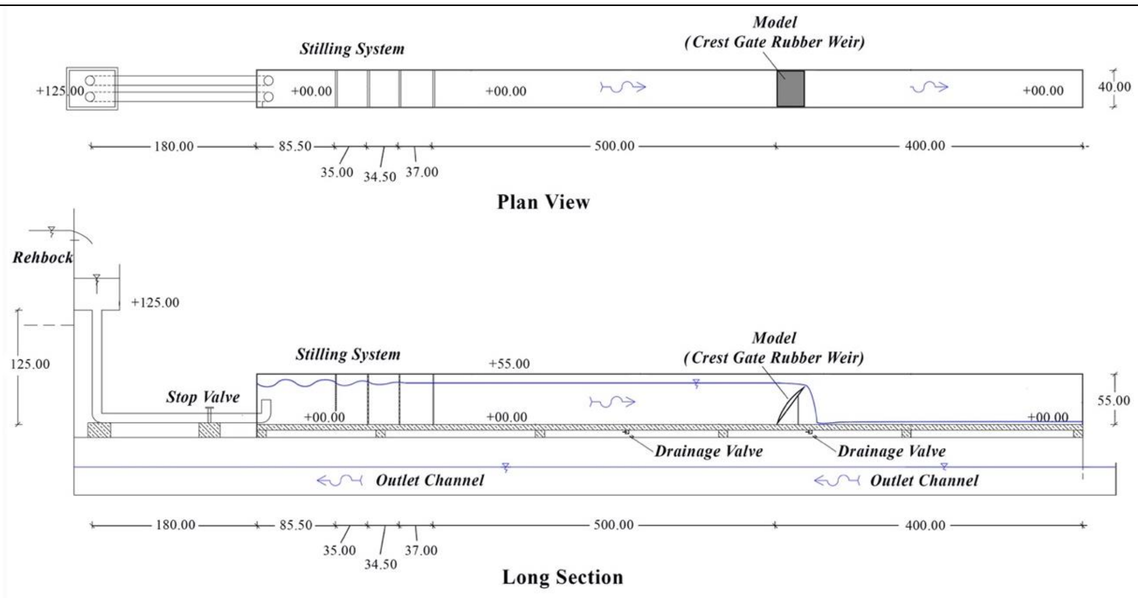

Fig. 4 Flume and weir hydraulic model (unscaled)

4. Results and Discussion

After the weir model was drained, it was measured and documented from the flume side, from the top, and from the upstream and downstream sides. Some photos of the side view are in shown in Fig. 5-7.

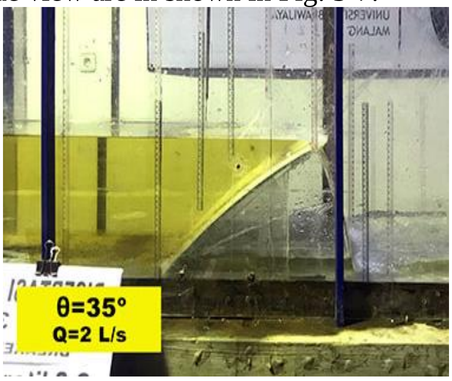

Fig. 5 Weir angle of 35^o and discharge of 2 L/s

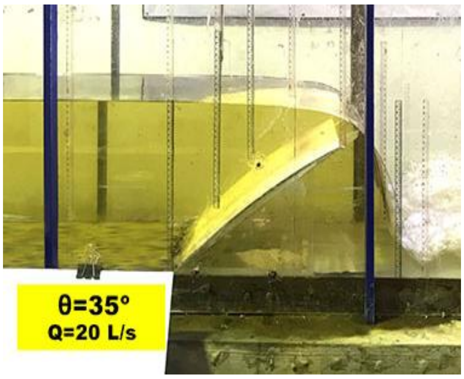

Fig. 6 Weir angle of 35^o and discharge of 20 L/s

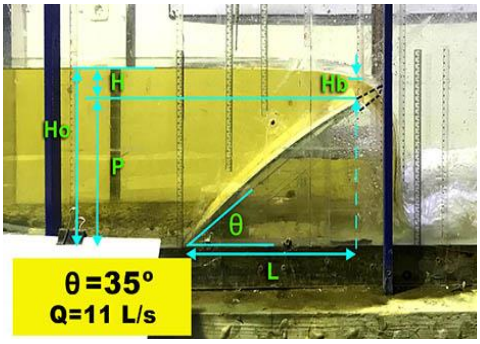

Fig. 7 Weir angle of 35^o at discharge of 11 L/s and measurements

The findings for the measurements of the water head (Ho), the water head over the weir (H), and the brink head (Hb) are listed in Table 2. The flows upstream of the weir are subcritical, with an Fr range from 0.01 to 0.18. Flows at the end of the weir are supercritical and range from 1.04 to 1.28. These values have been entered into equation (1) to calculate the values of Cd for each weir angle at different discharges; the results are shown in Table 2.

Table 2 Measurement at various weir model angles

Angle	Discharge	Water Head	Head over Weir	Brink Head	Discharge Coefficient
	Q	Ho	H	Hb	Cd
(o)	(L/s)	(cm)	(cm)	(cm)	
18	2	13.60	1.60	1.28	0.881
	5	15.30	3.30	2.27	0.743
	8	16.40	4.40	3.10	0.772
	11	17.50	5.50	3.79	0.760
	14	18.50	6.50	4.52	0.753
	17	19.40	7.40	5.10	0.753
	20	20.30	8.30	5.56	0.745
35	2	23.6	1.64	1.32	0.849
	5	25.1	3.14	2.31	0.801
	8	26.4	4.44	3.18	0.762
	11	27.5	5.54	3.95	0.752
	14	28.5	6.54	4.62	0.746
	17	29.3	7.34	5.24	0.762
	20	30	8.04	5.74	0.782
45 (for Model Verification)	2	28.50	1.63	1.34	0.856
	5	30.10	3.23	2.37	0.768
	8	31.20	4.33	3.26	0.791
	11	32.40	5.53	4.05	0.754
	14	33.40	6.53	4.80	0.748
	17	34.20	7.33	5.38	0.763
	20	35.00	8.13	5.80	0.769
53	2	32	1.65	1.38	0.841
	5	33.6	3.25	2.49	0.761
	8	34.8	4.45	3.35	0.759
	11	35.9	5.55	4.16	0.750
	14	36.9	6.55	4.91	0.744
	17	37.7	7.35	5.58	0.760
	20	38.6	8.25	5.93	0.752

From the experiment, it can be seen that an increase in Ho is related to an increase in Q. The value of Cd in equation (1) is influenced by the value of H; the relationship between H and Q is plotted in Fig. 8. As the figure shows, the relationship between Q and H resembles a rating curve; H increases more at low discharges than at large discharges. This shows that the relationship between H and Q is not linear but exponential.

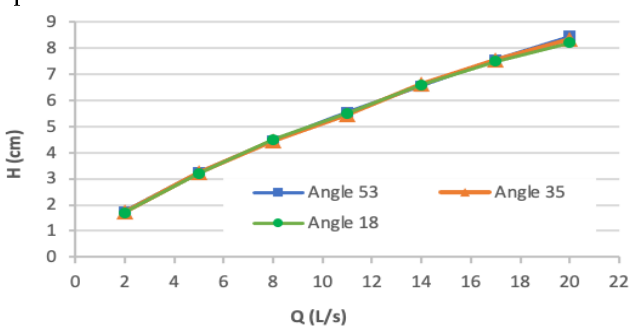

Fig. 8 Relation between the discharge and the head over the weir

Using the values of H and Q in equation (1) yields the value of Cd for each discharge, as shown in Table 2. The plot comparing the discharge coefficient to the ratio of H/P for each weir angle is shown in Fig. 9. As the figure shows, when the H/P value is constant, a weir with a gentler angle results in a higher Cd value than one with a sharper angle. For example, for an H/P value of 0.14, the Cd value for a weir angle of 18° is 0.881, while the Cd value is 0.801 for a weir angle of 35° and 0.759 for a weir angle of 53°. This phenomenon is explained by Shaker and Sarhan [8], who find that weir performance is influenced by the slope of the upstream face because of the difference in impact from the collision of the flow with the weir surface.

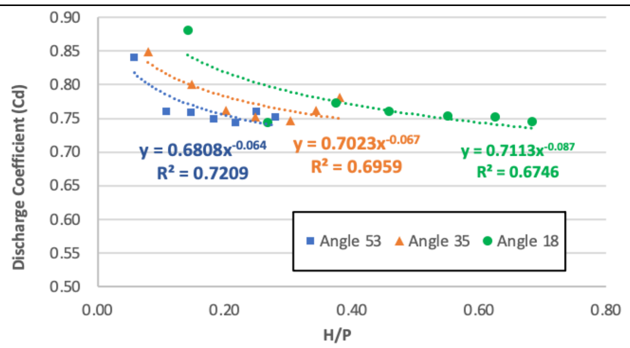

Fig. 9 Discharge coefficients and trendlines for different weir angles

Fig. 9 shows that the trendline has a similar exponential curve shape but also has distinct curves for different angles. This is probably caused by the slope factor of the weir angle. This slope can be expressed as a tangential function or as the ratio between the height (P) and length (L) of the weir. Based on the graph above, the proposed discharge coefficient model is:

$$Cd = a\left(\frac{H}{P}\right)^{b} + c\left(\frac{P}{L}\right)^{d} \tag{2}$$

Equation (2) is a general form of previous research shown in Table 1, which shows that Cd is a function of the H/P ratio. The P/L term has been added to account for the impact of the weir's slope factor on weir performance. This form was chosen for the power equation because the trendline formed by constituent of three angles is a power trendline rather than a straight line [9, 10].

A non-linear regression analysis was conducted using IBM Statistical Product and Service Solution (SPSS), version 25. The initial values of a, b, c, and d were set to 1. After 44 iterations of the model, the result was as follows:

$$Cd = 0.116\left(\frac{H}{P}\right)^{-0.302} + 0.573\left(\frac{P}{L}\right)^{-0.068} \quad \text{or} \tag{3}$$

$$Cd = 0.116\left(\frac{H}{P}\right)^{-0.302} + 0.573(\tan\theta)^{-0.068}$$

Predicted Cd values were synthesized with the observed Cd values using equation 3; the results are shown in Fig. 10. The figure shows that the Cd values in the model are quite close to the observed values, indicating that the model accurately accounts for the slope of the weir.

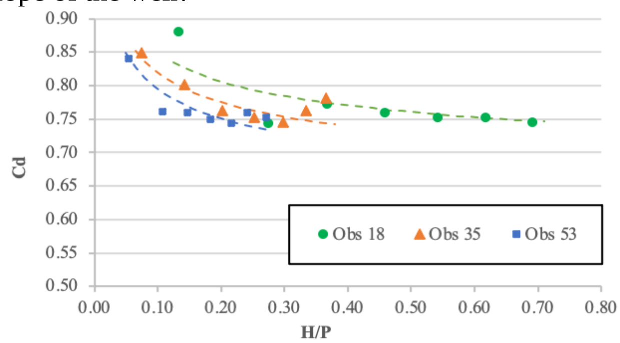

Fig. 10 Discharge coefficients for different weir angles

To further examine the accuracy of the proposed model, statistical tests were conducted to measure the root mean squared error (RMSE), mean absolute error (MAE) and Nash-Scutliffe efficiency (NSE) [11], as shown in equations 4, 5, and 6.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Cd_{obs} - Cd_{model})^2} \tag{4}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|Cd_{model} - Cd_{obs}| \tag{5}$$

$$NSE = 1 - \frac{\sum_{i=1}^{n}(Cd_{model} - Cd_{obs})^2}{\sum_{i=1}^{n}\left(Cd_{obs} - \overline{Cd}_{obs}\right)^2} \tag{6}$$

These calculations yielded an RMSE value of 0.027 and an MAE value of 0.014. Both of these values are quite small, indicating that the Cd values predicted by the model and the observed Cd values did not differ significantly [12]. The result of the NSE test was 0.712, indicating good performance [13]. All three statistical tests for the proposed discharge coefficient model yield acceptable results.

The discharge coefficient model was then verified using the measurements of the 45° weir model. Table 3 shows the observed Cd values for a weir with an angle of 45° and the Cd values predicted by the model.

Table 3 Comparison of the Observed Cd and Cd model

Discharge	Observed Head Over Weir	Discharge Coefficient (Cd)		Difference
Q	H	Cd Obs	Cd Model	
L/s	cm			%
2	1.63	0.856	0.843	-1.5%
5	3.23	0.768	0.793	3.3%
8	4.33	0.791	0.774	-2.1%
11	5.53	0.754	0.760	0.8%
14	6.53	0.748	0.751	0.4%
17	7.33	0.763	0.745	-2.4%
20	8.13	0.769	0.739	-3.8%

As Table 3 shows, the Cd values predicted by the discharge coefficient model differ from the observed values only slightly; this difference ranges from -3.8% to 3.3%. The average absolute difference is 2.1%. Therefore, the model equation may be applied to other angles in this type of weir.

The prototype was then validated [14, 15] to determine the model's accuracy for the Tirtonadi Weir. Discharge was measured by measuring the flow velocity and depth of the cross section of the Pepe River at a weir height (P) of 1.72 m and a weir length (L) of 3.39 m with an angle of 26.9° from the horizontal line. Measurements of the water elevation 100 m upstream of the weir showed an H value of 0.386 m. The effective width of the weir, after the width of the left and right pillars was subtracted, is 29.99 m.

Using the data from the prototype in equation (1) results in a discharge coefficient of 0.913 for the

Tirtonadi Weir. When the measured values of H, P, and L are entered into the discharge coefficient model shown in equation (3), the Cd for the model is 0.782. The difference between the observed Cd and the modelled Cd is 14.32%, which is acceptable. This difference may be affected by disparities between the weir model and the prototype, such as an elevation of 0.4 m to the base of the prototype, the absence of pillars in the weir model, and the sloped side walls of the prototype. Those differences impact head over weir and flow velocity, resulting in different Cd values for the model and the prototype.

5. Conclusion

This research found that the slope of the crest gate will affect the performance of Crest Gate Rubber Weir. Discharge from a gentler slope will be higher than discharge from a steeper slope because of lesser impact in flow collision with weir.

The discharge coefficient value will also change according to the flow magnitude, where Cd will be high at a small discharge and gradually decrease for a larger discharge. This phenomenon is seen in the power of the H/P ratio, which is the number of negative fractions.

The discharge coefficient model for Crest Gate Rubber Weir in any angle (θ) is:

$$Cd = 0.116\left(\frac{H}{P}\right)^{-0.302} + 0.573(\tan\theta)^{-0.068}$$

The equation above applies to any Crest Gate Rubber Weir with the same curvature and flow with Fr less than 0.18 at the upstream of the weir and Fr > 1.0 at the end tip of the weir.

Understanding Cd values at different flows and angles of the weir is important in this weir operation. It can lead to optimal operation between flood control and maintaining water elevation for water usage.

References

[1] CHOW V. T. *Open Channel Hydraulics*. Tata McGraw-Hill Publication, 2009.

[2] SUBRAMANYA K. *Flow in open channels.* 3rd ed. Tata McGraw-Hill, New Delhi, 2009.

[3] AYAZ M.D., & MANSOOR T. Discharge coefficient of oblique sharp crested weir for free and submerged flow using trained ANN model. *Water Science*, 2018, 32(2): 192-212. https://doi.org/10.1016/j.wsj.2018.10.002

[4] PANDEY R., MITTAL S. K., and CHOUDHARY M. K. Study of flow characteristics of contracted sharp crested rectangular weir. *International Journal of Innovative Science, Engineering & Technology*, 2016, 3(2): 97-103. http://ijiset.com/vol3/v3s2/IJISET_V3_I2_13.pdf

[5] MAHTABI G., & ARVANAGHI H. Experimental and numerical analysis of flow over a rectangular full-width sharp-crested weir. *Water Science and Engineering*, 2018, 11(1): 75-80. https://doi.org/10.1016/j.wse.2018.03.004

[6] AKHBARI A., ZAJI A. H., AZIMI H., and VAFAEIFARD M. Predicting the discharge coefficient of triangular plan form weirs using radian basis function and M5' methods. *Journal of Applied Research in Water and Wastewater*, 2017, 4(1): 281-289. https://doi.org/10.22126/ARWW.2017.767

[7] KHANPOUR M., ZARRATI, KOLAHDOOZAN M., SHAKIBAEINIA A., and JAFARINIK S. Numerical modeling of free surface flow in hydraulic structures using smoothed particle hydrodynamics. *Applied Mathematical Modelling*, 2016, 40(23-24): 9821-9834. https://doi.org/10.1016/j.apm.2016.06.032

[8] SHAKER S. A., & SARHAN S. A. Performance of flow over a weir with sloped upstream face. *ZANCO Journal of Pure and Applied Sciences*, 2017, 29(3): 43-54. http://zancojournals.su.edu.krd/index.php/JPAS/article/view/681

[9] ZHANG Y., CHIEW F. H. S., LI M., and POST D. Predicting runoff signatures using regression and hydrological modeling approaches. *Water Resources Research*, 2018, 54: 7859–7878. https://doi.org/10.1029/2018WR023325

[10] MAHMOUDI M. On Comparing Two Dependent Linear and Nonlinear Regression Models. *Journal of Testing and Evaluation*, 2019, 47(1): 449-458. https://doi.org/10.1520/JTE20170461

[11] ADNAN R. M., KHOSRAVINIA P., KARIMI B., and KISI O. Prediction of hydraulics performance in drain envelopes using K-means based multivariate adaptive regression spline. *Applied Soft Computing*, 2021, 100: 107008. https://doi.org/10.1016/j.asoc.2020.107008

[12] ADNAN R. M., PARMAR K. S., HEDDAM S., SHAHID S., and KISI O. Suspended Sediment Modeling Using a Heuristic Regression Method Hybridized with Kmeans Clustering. *Sustainability*, 2021, 13(9): 4648. https://doi.org/10.3390/su13094648

[13] KNOBEN W. J. M., FREER J. E., and WOODS R. A. Technical note: Inherent benchmark or not? Comparing Nash–Sutcliffe and Kling–Gupta efficiency scores. *Hydrology and Earth System Sciences*, 2019, 23: 4323–4331. https://doi.org/10.5194/hess-23-4323-2019

[14] MOHAMED A. A., AMIN K. A., ABU-ZEID T. S., and HASSAN R. Effect of the free over fall weirs Top corners curvatures on the discharge coefficient. *International Journal of Constructive Research in Civil Engineering*, 2016, 2(4): 1-11. http://dx.doi.org/10.20431/2454-8693.0204001

[15] HASAN H. H., MOHD RAZALI S. F., AHMAD ZAKI A. Z. I., and MOHAMAD HAMZAH F. Integrated Hydrological-Hydraulic Model for Flood Simulation in Tropical Urban Catchment. *Sustainability*, 2019, 11(23): 6700. https://doi.org/10.3390/su11236700

[16] REHBOCK T. Wassermessung mit sharfkantigen uberfallwehren. *Zeitschrift des Vereins Deutscher Ingenieure*, 1929, 73: 343.

[17] UNITED STATES BUREAU OF RECLAMATION. *Embankment dams.* 1987.

参考文:

[1] CHOW V. T. 明渠液压。塔塔麦格劳-希尔出版社，2009。

[2] SUBRAMANYA K. 明渠中的流动。第3版。塔塔麦格劳希尔，新德里，2009。
[3] AYAZ M.D., & MANSOOR T. 使用训练有素的人工神经网络模型的自由和淹没流的斜尖顶堰的排放系数。水科学, 2018, 32(2): 192-212. https://doi.org/10.1016/j.wsj.2018.10.002
[4] PANDEY R.、MITTAL S. K. 和 CHOUDHARY M. K. 收缩尖顶矩形堰的流动特性研究。国际创新科学、工程与技术学报, 2016, 3(2): 97-103. http://ijiset.com/vol3/v3s2/IJISET_V3_I2_13.pdf
[5] MAHTABI G., & ARVARAGHI H. 矩形全宽尖顶堰上流动的实验和数值分析。水科学与工程, 2018, 11(1): 75-80. https://doi.org/10.1016/j.wse.2018.03.004
[6] AKHBARI A.、ZAJI A. H.、AZIMI H. 和 VAFAEIFARD M. 使用弧度基函数和米5'方法预测三角形平面堰的流量系数。水与废水应用研究杂志, 2017, 4(1): 281-289. https://doi.org/10.22126/ARWW.2017.767
[7] KHANPOUR M.、ZARRATI、KOLAHDOOZAN M.、SHAKIBAEINIA A. 和 JAFARINIK S. 使用平滑粒子流体动力学对水力结构中的自由表面流动进行数值模拟。应用数学建模, 2016, 40(23-24): 9821-9834。 https://doi.org/10.1016/j.apm.2016.06.032
[8] SHAKER S. A., & SARHAN S. A. 上游倾斜面堰上的流动性能。赞科纯粹与应用科学杂志, 2017, 29(3): 43-54。 http://zancojournals.su.edu.krd/index.php/JPAS/article/view/681
[9] 张 Y.、CHIEW F. H. S.、LI M. 和 POST D. 使用回归和水文建模方法预测径流特征。水资源研究，2018，54：7859-7878。https://doi.org/10.1029/2018WR023325
[10] MAHMOUDI M. 关于比较两个相关线性和非线性回归模型。测试与评估杂志, 2019, 47(1): 449-458。 https://doi.org/10.1520/JTE20170461
[11] ADNAN R. M.、KHOSRAVINIA P.、KARIMI B. 和 KISI O. 使用基于钾均值的多元自适应回归样条预测排水包络线中的水力性能。应用软计算，2021，100：107008。https://doi.org/10.1016/j.asoc.2020.107008
[12] ADNAN R. M.、PARMAR K. S.、HEDDAM S.、SHAHID S. 和 KISI O. 使用启发式回归方法与克米恩斯聚类混合的悬浮沉积物建模。可持续性，2021，13(9)：4648。https://doi.org/10.3390/su13094648
[13] KNOBEN W. J. M.、FREER J. E. 和 WOODS R. A. 技术说明：固有基准与否？比较纳什-萨特克利夫和克林-古普塔效率得分。水文与地球系统科学，2019，23：4323–4331。https://doi.org/10.5194/hess-23-4323-2019
[14] MOHAMED A. A.、AMIN K. A.、ABU-ZEID T. S. 和 HASSAN R. 自由落体堰的影响顶角曲率对流量系数的影响。国际土木工程建设研究杂志, 2016, 2(4): 1-11. http://dx.doi.org/10.20431/2454-8693.0204001
[15] HASAN H. H.、MOHD RAZALI S. F.、AHMAD ZAKI A. Z. I. 和 MOHAMAD HAMZAH F. 热带城市集水区洪水模拟的综合水文水力模型。可持续性，2019，11(23)：6700。https://doi.org/10.3390/su11236700
[16] REHBOCK T. 使用锋利的挡水堰进行水测量。德国工程师杂志, 1929, 73: 343。
[17] 美国垦务局。堤坝。1987。