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Financial Performance of Selected Portfolio Schemes in India: Using Model of Markowitz

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Abstract: The paper's objective is to analyze the performance of portfolios of selected mutual funds in India. This study focused on the Harry Markowitz formula applied to the selected ten mutual fund schemes. Five years of monthly and yearly data were collected from the BSE India from 2013 to 2018. There are the highest returns in selected schemes' first, third, and fifth years. ICICI prudential blue-chip fund and Mirae Asset Emerging blue-chip fund have high returns in third and fifth year continuously. The researcher presents the correlation matrix among the selected schemes of the portfolio. Through the modern portfolio theory, it is observed that the individual portfolio has more risk as compared to the group portfolio standard deviation. This paper will be helpful for investors, academic portfolio managers, jobbers, and brokers. The major concern for the paper is to maximize the high expected rate of return and mitigate the risk along with capturing the different types of securities in the portfolio.

Keywords: mutual fund schemes, risk and return, correlation, Markowitz portfolio theory.

印度选定投资组合计划的财务表现：使用马科维茨模型

摘要：该论文的目的是分析选定的共同基金在印度的投资组合的表现。本研究侧重于应用于选定的10个共同基金计划的哈里· 马科维茨公式。从印度疯牛病收集了2013年至2018年的5年月度和年度数据，其中选定计划的第一年、第三年和第五年的回报最高。国际商会审慎蓝筹基金和未来资产新兴蓝筹基金连续第三年和第五年获得高额回报。研究人员提出了投资组合选定方案之间的相关矩阵。通过现代投资组合理论，可以观察到，个人投资组合比群体投资组合标准差具有更大的风险。本文将对投资者、学术投资组合经理、求职者和经纪人有所帮助。该论文的主要关注点是最大化高预期回报率并降低风险，同时捕获投资组合中的不同类型的证券。

关键词： 共同基金计划，风险和回报，相关性，马科维茨投资组合理论。

1. Introduction

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Value shared assets (equity mutual funds) are the best venture alternative to accomplish long-term financial-related objectives. Innumerable investigations have demonstrated that stocks can offer better returns than other resources over an extensive period. Equity mutual fund schemes are the best available option for investors with a long-term budgetary objective, that is, no less than five to seven years [2]. Be that as it may, it is similarly imperative to pick the correct value for a common equity mutual fund scheme. The decision to invest in a mutual fund scheme should be coordinated with the objectives, speculation skyline, and risk contour. For instance, a traditional financial specialist contributing to a five-year objective may consider putting resources into an extensive large-cap mutual fund scheme [3]. In the event that the financial specialist needs to rid the corpus of most instability, he may likewise think about putting resources in aggressive hybrid schemes. Aggressive hybrid schemes put resources into a blended arrangement of equity (65 to 80 percent) and debt (obligation) (20 to 35 percent). The debt part of the plan offers solidity to the portfolio during volatile times. Financial specialists (investors) with moderate risk profiles can consider putting resources into multiple schemes that contribute crosswise over market capitalizations. These schemes are riskier than large-cap schemes; however, they may be able to offer prevalent returns [4]. Aggressive financial specialists (investors) with high risk-resistance and a longer speculation horizon can consider putting resources into midcap and small-cap schemes. Generally, these plans put resources into fair-sized organizations and can turn profoundly unstable amid a volatile stage in the market. Nonetheless, these organizations develop to become expansive and reward investors with additional profits for the additional risk they are bearing. Sector schemes or topical plans are intended for refined financial specialists or those with high total assets. These plans can assist financial specialists in mastering a segment or subject that they need to wager. In any case, mutual fund consultants trust that investors ought to put resources into these schemes just if they can time their entrance to and exit of these schemes and have close learning about the sector [5]. High total assets speculators can likewise see worldwide plans to expand their portfolio crosswise coverage. In any case, mutual fund guides trust that small investors should give these schemes a chance as it would weaken the total return from their portfolio [2].

Table 1 Selected mutual fund schemes to invest [1]

S.N.	Schemes	NAV
1	Motilal Oswal Multicap 35 Fund	22.73
2	SBI Bluechip Fund	34.43
3	SBI Magnum MultiCap Fund	42.34
4	Mirae Asset Emerging Bluechip Fund	27.36
5	L&T Tax Advantage Fund	50.29
6	DSP BlackRock Equity Opportunities Fund	193.88
7	Aditya Birla Sun Life Tax Relief 96	28.56
8	ICICI Prudential Bluechip Fund	38.12
9	ICICI Prudential Equity & Debt Fund	21.94
10	L&T India Value Fund	32.46

1.1. Net Asset Value

Net asset value (NAV) is calculated as the value of a fund's asset, less the value of its liabilities per unit. Net asset value is normally determined on a per-share basis. A fund’s net asset value fluctuates with the value of its underlying investments. The formula for calculating the net asset value is:

Net asset value = (Market Value of All Securities Held by Fund + Cash and Equivalent Holdings - Fund Liabilities) / Total Fund Shares Outstanding

A fund’s net asset value will change every day as the value of a fund’s securities, cash held, liabilities, and the number of shares outstanding fluctuates [6].

1.2. Harry Markowitz’s Modern Portfolio Theory (MPT)

Harry Markowitz is Nobel Prize winner in economics. In 1950, Markowitz developed the modern portfolio theory (PMT), purporting that the expected returns can be maximized and investment risk minimized by applying efficient frontier in the portfolio selection, as this would also result in the highest elasticity of returns for the specified dimension of risk. These ideas have given rise to the Capital Asset Pricing Model [7].

1.3. Objectives of the Study

1. To minimize the risk and get the maximum return dynamic of portfolios;
2. To apply the modern portfolio theory for the investment.

2. Literature Review

Ample body of empirical evidence indicates that most measurable exchange methodologies, such as sets exchanging and its speculations, depend on the development of mean reverting range with a specific level of consistency. However, co-integration tests are rarely applied to identify stocks to be utilized in sets of exchanging methodologies, which can be applied to create a set exchanging portfolio dependent on appropriate profitability markers. These gaps are addressed in the present study, whereby system profitability is determined using information obtained from the São Paulo Stock Exchange pertaining to the period from January 2005 to October 2012. Observational investigation demonstrates that the planned procedure yields 16.38% annual overabundance returns, with a 1.34 Sharpe Ratio and low relationship with the market [8].

Mangram [9] focused on the value of Microsoft Excel in performing MPT calculations and validating them against alternative numerical models. His findings indicate that MPT is valuable in practice, especially when combined with Microsoft Excel and other technological tools [9]. Besides, it also demonstrates that the more often the property information is accessible, the more significant the advantage. This has significant ramifications for the SEC's ongoing decision on the recurrence of property exposure and the data plan supporters should gather from portfolio administrators. Again, it additionally investigates the impact of molding betas on full-scale factors as recommended by Ferson and Schadt in 1996 to recognize unrivaled performing shared assets and the elective method for utilizing possessions information proposed by Grinblatt and Titman in 1993 [10].

This paper determines the ideal system for arbitrageurs with a finite skyline and non-nearsighted inclinations when engaging in mean-returning exchange opportunities (e.g., value sets exchanges). The research finds that intertemporal choice plays an essential role in arbitrageur decisions on how forcefully to exchange against mispricing to ensure an expansive division of the aggregate distribution to the exchange opportunity. While arbitrageurs typically wager against mispricing, this paper systematically demonstrates that, in the past, basic levels of mispricing have occurred, which encourage differences and accelerate distribution decreases. Our ideal methodology conveys a significant change in the acknowledged Sharpe proportion and welfare concerning a basic edge running the show [11].

This research empirically studies and proposes a technique for the ideal portfolio choice. It utilizes the Bayesian choice theoretical system and notes two weaknesses of the conventional Markowitz approach involving the capacity to address higher methods and parameter vulnerability. The author utilizes the skew-normal distribution, which has an excellent ability for demonstrating multivariate returns. The study’s outcomes indicate the importance of consolidating higher request methods in portfolio determination. Further, the study’s correlation with different techniques, where parameter vulnerability is either disregarded or employed in an impromptu way, demonstrates that its methodology prompts higher expected utility than contending strategies, such as the resampling strategies basic to finance activities [12].

This paper’s basic-level hypothetical establishment of conduct portfolio determination varies from the idea of objective portfolio choice under vulnerability. Social portfolio choices regarding given benchmark portfolio abuses have established objective truths. This study proposes a conduct model of portfolio choice, which fuses balanced portfolio choice and benchmarking and determines its investigative arrangement. In this model, the primary utility capacity depends on Tukey’s lament hypothesis and has two target variables rather than one. Comparing the EVC standard clarifies disputable issues in portfolio determination and considers testable ramifications [13].

The authors have empirically studied and built up a group of completely inferred estimators of the covariance matrix. Here, the essential thought is to acquire appraisals of covariance grid exclusively from current costs of plain vanilla alternatives. These costs are reflected to advertise assumptions regarding the arrival disseminations of the hidden resources. Along these lines, the methodology is forward-looking and differs on a very basic level from the regressive-looking methodology that utilizes the time arrangement of profits. The individuals from the family differ concerning the snapshot of the inferred appropriation they use to acquire the covariance Metrix. The authors execute the estimation dependent on the second and fourth moments. The nature of the new estimators is tested by dissecting the out-of-test execution of related global minimum variance (GMVP) techniques for the 30 stocks incorporated into the Dow Jones Industrial Average (DJIA). The study found that they work similarly. Their execution is compared with a few benchmark methodologies and demonstrates a major difference in the procedures used, but only in times of high data flow. At that point, the speculator can be improved by utilizing current market information as in our completely suggested methodology [14].

Meucci [15] empirically studies and presents "factors on interest," which measures a multi-resource class return deterioration structure that surpasses both the standard orderly and particular methodology. This structure rests on the contingent connection between the flexible, base-up estimation factor model and the flexible, best-down attribution factor model, which together achieve higher logical power, exactness and hypothetical consistency when compared to standard methodologies [15].

The objective of this paper is to explore whether one can utilize alternative suggested data to enhance the determination of mean-difference portfolios with an expansive number of stocks, while reporting which parts of choice inferred data are most valuable for enhancing their out-of-test execution. Experimental proof demonstrates that utilizing choice suggested unpredictability decreases portfolio instability. However, utilizing alternative suggested connection does not enhance any of the measurements. Yet, utilizing alternative suggested unpredictability, chance premium, and skewness to modify expected return does prompt generous enhancement in the Sharpe proportion, even after precluding short deals and representing exchange costs. The authors have examined how data suggested in cost of investment opportunities can be utilized to enhance evaluations of these three methods to enhance the out-of-test execution of portfolios with a substantial number of stocks. Execution is estimated by evaluating portfolio instability, Sharpe proportion, and turnover. The benchmark portfolios include the following: the 1/N portfolio, four kinds of least fluctuation portfolios, four sorts of mean-difference portfolios (dependent on recorded returns), and the parametric arrangement of Brandt, Santa-Clara, and Valkanov [16]. Benchmarks are measured by chronicled returns and size, esteem, and force qualities [17].

The authors propose a unified, completely broad system to analyze diversification in any condition, incorporating long-short exchanges that profoundly correspond with markets characterized by complex subsidiaries. Initially, diversification circulation is considered, corresponding with the appropriation of the uncorrelated wagers in the portfolio that are consistent with the portfolio requirements. The authors present the mean-diversification efficient boondocks, a diversification tool for portfolio improvement. Finally, we propose a practical approach to mean diversification advancement considering exchange and market affect costs by simply exchanging a couple of ideally picked securities [18].

The authors have contemplated the more than one hundred measures that have been proposed in related literature to assess the execution of a store, including the ideas of return and hazard. Each performance indicator has its positive qualities as well as its shortcomings or breaking points. They encompass a variety of measurements that bode well for a large portion of investment strategies. However, it is reckless to assume that "one size fits all." The authors endeavor to distinguish between the established methodologies in order to identify whether one may be considered a superior rule for clarifying portfolio execution while maintaining constancy. Additionally, the measurement systems will be evaluated for pertinence under different financial settings (unpredictable or not, shoulder or bull), with respect to various kinds of assets (simple stocks, including bonds) and lengths (present moment, medium term, long haul). Overall, investigations of the constancy in execution and the location of the best portfolio supervisors ought to sufficiently envelop the significant components of execution [19].

A long-short portfolio determination display is dependent on the following three measurements of arrival circulation: mean, fluctuation, and CVaR. Further, it must be within the sight of discrete resource decision limitations of (i) purchase and move edges and (ii) cardinality confinement on the quantity of stocks to be held in the given portfolio. In any case, the long-short portfolios involve a significantly larger number of stocks than the long-only ones. By presenting cardinality imperatives, the authors demonstrate that the better execution of long-short portfolios is not simply due to the "broadening impact." Notwithstanding forcing indistinguishable cardinality limitations from the long-only portfolios, the long-short portfolios have better returns for appropriations with much lower chances for exposure and higher estimations of execution measures (Sharpe and Sortino proportions). Regarding the long-short models, it is vital to note that by forcing solid cardinality limitations, the relating increment in CVaR is low when contrasted with the instance of no cardinality confinements. Likewise, our investigations demonstrate significantly better out-of-test execution of long-short portfolios than long-only portfolios [20].

A differentiated benchmark is called the Critical Implied Reference Portfolio. This benchmark suggests that while portfolios on the productive outskirts might be ideal in the scientific sense, they are regularly very focused and not all-around broadened. Productive portfolios are ineffectively differentiated and thus unsuitable for reference purposes. Therefore, the CIR portfolio is proposed as a predominant reference portfolio. Additionally, the CIR portfolio will not generally be ruled by, when measured against weighted reference portfolios in mean change space. Such guileless reference portfolios are introduced in the idea of the Diversified Opportunity Set examination [21].

This paper envisages a dynamic model of portfolio management procedure identified by outflanking a benchmark. It hinges on the expectation that supervisors will embrace a benchmarking system for speculation. The ideal portfolio approach is state-subordinate, including a particular time period to reaching the speculation skyline, requires meeting the given benchmark, and features the return on the speculation portfolio itself. Thus, an unequivocal equation for ascertaining the ideal portfolio loads is acquired in a ceaseless time setting. Additionally, regarding hazard sensitivities, the authors examined a model for portfolio execution assessment. Hazard sensitivities are evaluated utilizing a dynamic coordinating methodology that limits the margin of error between the real returns and the profits suggested by the model. Experimental examination demonstrates that there is a solid factual connection between the dimension of performance and the managers’ risk sensitivities [22].

This paper has joined financial specialist inclinations for advanced methods into a PGP advancement work. This enables us to explain different contending (and regularly conflicting) hedge investment portion targets inside a 4-moment framework. Flexible investments exchange against one another similarly as in the fundamental security markets. Nonpartisan and worldwide large-scale reserves advertising equity have critical tasks to carry out in ideal fence investment portfolios because of their appealing co-fluctuation, co-skewness, and co-kurtosis properties. Neutral funds advertising value act as unpredictability and kurtosis reducers, while worldwide large-scale funds act as skewness enhancers [23].

The authors have experimentally contemplated that a standout amongst the utmost influential thoughts in the previous 30 years of the Journal of Portfolio Management is the efficient market hypothesis stating that showcase costs fuse all data normally and immediately. In any case, the rising control of finance and financial aspects of conduct has tested this theory, contending that business sectors are not objective but determined by dread and eagerness. Later on, research in the psychological neurosciences recommends that these two points of view are inverse sides of an almost similar coin. In this paper, the authors also put forward another system that accommodates showcase efficiency with conduct choices by applying the standards of advancement—competition, adaptation, and natural selection to financial corporations [24].

This current study examines the aggregate return of speculation portfolios made of common assets and the breakdown of the commitments of vital resource distribution, strategic planning, and security determination. Using information from five model common reserve portfolios covering a wide variety of benefit class blends over a multi-year timeframe exhibits that vital resource allotment strategy determines more than 90 percent of the variety in complete portfolio return and that strategic planning choice and security determination may likewise contribute essentially to the variety in all out-return. The study extends the current knowledge by reproducing a pooled relapse, which merges all portfolios. It also involves analysis to incorporate a few types of risk measures, inspecting the information in terms of absolute and relative variance, and an assessment of the portfolios regarding balancing risk and return [25].

This paper is maybe the first to assess the out-of-test execution of an enhanced estimation of the co-difference structure of fence stock investment file returns, concentrating on its utilization for ideal portfolio determination. Due to the nearness of substantial estimation chance in the assessed anticipated returns, we center on the base fluctuation arrangement of fence investments records. Optimal inclusion of speculative stock investments in a speculator portfolio can create a sensational decline in the portfolio instability on an out-of-test premise. The difference in mean returns, then again, is not measurably significant, showing that the hazard control enhancement does not occur because of lower anticipated returns [26].

Portfolio execution estimation has been a point of enthusiasm among practitioners and in academic networks for quite a while. Notwithstanding, a sensible worry among those using a specific measure raises the question of whether the director assessed may respond by endeavoring to control it. A few articles have noticed that even if the evaluator knows the moments of the return distribution, it is yet conceivable to utilize informationless exchanges to support the expected Sharpe ratio. This paper has demonstrated that, without a doubt, such a measure exists. Manipulation proof measures can be portrayed (up to a monotonic change) as the weighted average of a utility-like capacity. Not only is it manipulation proof; it is not harder to compute than the Sharpe ratio and is impressively less demanding than huge numbers of the other execution estimates that have been proposed in the past to fix the Sharpe ratio's inadequacies [27].

The present study has investigated the impact of estimation chance on the portfolio choice issue in the lower incomplete method framework. The assessed outskirts give off an impression of a situation dependent on the information and the evaluated parameters; however, the genuine wilderness truly depends on the genuine parameters. Three different techniques for enhancing portfolio optimizations have been utilized in this section. They contrasted with the established methodology in which the example gauges are utilized to contribute to the portfolio improvements. The three different techniques were two different shrinkage estimators, James-Stein and Bayes-Stein, and the third strategy was a bootstrap approach. The outcomes demonstrated that the blunders in the upgraded portfolios, the difference between the actual and genuine portfolios, could be shockingly substantial in the average incomplete method structure [28].

3. Research Methodology

Research is a roadmap and systematic process of analyzing information about the content of the study. In the current study, the function of a researcher is to understand the phenomenon and communicate the understanding to others regarding the financial performance of selected mutual fund schemes in India. The statement declared the portfolio performance of the schemes in India, and the researchers focused on the top

10 schemes working on the Bombay Stock Exchange (BSE) cited by Economic times. This report is based on secondary data depending on descriptive mode. The data were collected from BSE India and the top 10 securities working in BSE. Monthly and yearly data were collected from BSE India.

3.1. Tools and Techniques

3.1.1. Mean

Mean is the average of all numbers, the arithmetic mean. To calculate the mean, divide the sum of all the numbers in a set by the total count of numbers given.

$$\bar{\mathbf{X}} = \frac{\mathbf{\Sigma x}}{\mathbf{n}}$$

where n - number of the items in the sample; Σx - sum of all data value; $\bar{X}$ = mean of the sample.

3.1.2. Variance

Variance is defined as the expectation of the squared deviation of a random variable from its mean. Essentially, it measures how far a set of (random) numbers are from their average given value.

$$\boldsymbol{\sigma}^2 = \frac{\mathbf{N\Sigma R^2 - (\Sigma R)^2}}{\mathbf{N^2}}$$

where ΣR^2 - the sum of square of the return; ΣR - the sum of the return; N - number of items.

3.1.3. Standard Deviation

Standard deviation is defined as a measure of dispersant in statistics. “Dispersant” refers to how the data is spread out. Specifically, it shows the extent to which the data is spread out around the given mean or given average.

$$\boldsymbol{\sigma} = \sqrt{\frac{\mathbf{N\Sigma R^2 - (\Sigma R)^2}}{\mathbf{N^2}}}$$

3.1.4. Modern Portfolio Theory

Harry Markowitz, a scholar in portfolio construction, propounded the Modern Portfolio Theory. Hence, the authors have selected this theory to evaluate and construct the portfolio.

There are two main concepts under the Modern Portfolio Theory:

- Any investor's goal is to maximize Return for any level of Risk.
- Risk can be reduced by creating a diversified portfolio of unrelated assets.

3.1.5. Formula of Variance of Portfolio

$$\boldsymbol{\sigma}^2\boldsymbol{\rho} = \mathbf{W}_1^2\boldsymbol{\sigma}_1^2 + \mathbf{W}_2^2\boldsymbol{\sigma}_2^2 + 2\mathbf{W}_1\boldsymbol{\sigma}_1\mathbf{W}_2\boldsymbol{\sigma}_2\mathbf{r}_{(1,2)}$$

where:

$\sigma^2\rho$ = Portfolio variance.

W_1 = Proportion of funds invested in the first security.

W_2 = Proportion of funds invested in the second security.

σ_1^2 = Variance of first security.

σ_2^2 = Variance of second security.

σ_1 = Standard deviation of first security.

σ_2 = standard deviation of second security.

$r_{(1,2)}$ = correlation coefficient between the returns of first and second security.

3.1.6. The Variance of Selected Portfolio of 10 Securities

$$\begin{aligned}
\boldsymbol{\sigma}^2\boldsymbol{\rho} = {} & \mathbf{W}_1^2\boldsymbol{\sigma}_1^2 + \mathbf{W}_2^2\boldsymbol{\sigma}_2^2 + \mathbf{W}_3^2\boldsymbol{\sigma}_3^2 + \mathbf{W}_4^2\boldsymbol{\sigma}_4^2 + \mathbf{W}_5^2\boldsymbol{\sigma}_5^2 + \mathbf{W}_6^2\boldsymbol{\sigma}_6^2 + \mathbf{W}_7^2\boldsymbol{\sigma}_7^2 + \mathbf{W}_8^2\boldsymbol{\sigma}_8^2 + \mathbf{W}_9^2\boldsymbol{\sigma}_9^2 + \mathbf{W}_{10}^2\boldsymbol{\sigma}_{10}^2 \\
& + 2W_1\sigma_1W_2\sigma_2r_{(1,2)} + 2W_1\sigma_1W_3\sigma_3r_{(1,3)} + 2W_1\sigma_1W_4\sigma_4r_{(1,4)} + 2W_1\sigma_1W_5\sigma_5r_{(1,5)} \\
& + 2W_1\sigma_1W_6\sigma_6r_{(1,6)} + 2W_1\sigma_1W_7\sigma_7r_{(1,7)} + 2W_1\sigma_1W_2\sigma_8r_{(1,8)} + 2W_1\sigma_1W_9\sigma_9r_{(1,9)} \\
& + 2W_1\sigma_1W_{10}\sigma_{10}r_{(1,10)} + 2W_2\sigma_2W_3\sigma_3r_{(2,3)} + 2W_2\sigma_2W_4\sigma_4r_{(2,4)} + 2W_2\sigma_2W_5\sigma_5r_{(2,5)} \\
& + 2W_2\sigma_2W_6\sigma_6r_{(2,6)} + 2W_2\sigma_2W_7\sigma_7r_{(2,7)} + 2W_2\sigma_2W_8\sigma_8r_{(2,8)} + 2W_2\sigma_2W_9\sigma_9r_{(2,9)} \\
& + 2W_2\sigma_2W_{10}\sigma_{10}r_{(2,10)} + 2W_3\sigma_3W_4\sigma_4r_{(3,4)} + 2W_3\sigma_3W_5\sigma_5r_{(3,5)} + 2W_3\sigma_3W_6\sigma_6r_{(3,6)} \\
& + 2W_3\sigma_3W_7\sigma_7r_{(3,7)} + 2W_3\sigma_3W_8\sigma_8r_{(3,8)} + 2W_3\sigma_3W_9\sigma_9r_{(3,9)} + 2W_3\sigma_3W_{10}\sigma_{10}r_{(3,10)} \\
& + 2W_4\sigma_4W_5\sigma_5r_{(4,5)} + 2W_4\sigma_4W_6\sigma_6r_{(4,6)} + 2W_4\sigma_4W_7\sigma_7r_{(4,7)} + 2W_4\sigma_4W_8\sigma_8r_{(4,8)} \\
& + 2W_4\sigma_4W_9\sigma_9r_{(4,9)} + 2W_4\sigma_4W_{10}\sigma_{10}r_{(4,10)} + 2W_5\sigma_5W_6\sigma_6r_{(5,6)} + 2W_5\sigma_5W_7\sigma_7r_{(5,7)} \\
& + 2W_5\sigma_5W_8\sigma_8r_{(5,8)} + 2W_5\sigma_5W_9\sigma_9r_{(5,9)} + 2W_5\sigma_5W_{10}\sigma_{10}r_{(5,10)} + 2W_6\sigma_6W_7\sigma_7r_{(6,7)} \\
& + 2W_6\sigma_6W_8\sigma_8r_{(6,8)} + 2W_6\sigma_6W_9\sigma_9r_{(6,9)} + 2W_6\sigma_6W_{10}\sigma_{10}r_{(6,10)} + 2W_7\sigma_7W_8\sigma_8r_{(7,8)} \\
& + 2W_7\sigma_7W_9\sigma_9r_{(7,9)} + 2W_7\sigma_7W_{10}\sigma_{10}r_{(7,10)} + 2W_8\sigma_8W_9\sigma_9r_{(8,9)} + 2W_8\sigma_8W_{10}\sigma_{10}r_{(8,10)} \\
& + 2W_9\sigma_9W_{10}\sigma_{10}r_{(9,10)}
\end{aligned}$$

3.2. Empirical Data Analysis

Table 2 presents all the selected schemes of portfolio indicators reflecting R_1-R_{10}.

Table 2 Indicators of Selected Portfolio Schemes

S.N.	Schemes	Indicators
1	MotilalOswalMulticap 35 Fund	R1
2	SBI Bluechip Fund	R2
3	SBI Magnum MultiCap Fund	R3
4	Mirae Asset Emerging Bluechip Fund	R4
5	L&T Tax Advantage Fund	R5
6	DSP BlackRock Equity Opportunities Fund	R6
7	Aditya Birla Sun Life Tax Relief 96	R7
8	ICICI Prudential Bluechip Fund	R8
9	ICICI Prudential Equity & Debt Fund	R9
10	L&T India Value Fund	R10

Table 3 presents the returns of selected portfolios from different time frames, which indicate the monthly and yearly values. The current table of selected returns shows negative values for the months 1, 3, and 6. However, after being converted to yearly values, it becomes a part of the positive returns series. ICICI Prudential Bluechip fund recorded the highest return out of the selected schemes. In the second and third years, the highest returns were recorded by Mirae Asset Emerging blue-chip fund and SBI Magnum MultiCap Fund, respectively. In the last five years, the highest returns were recorded by Mirae Asset Emerging blue-chip fund. Selected returns of portfolio were used for further calculation using Markowitz's Modern Portfolio Theory.

Table 3 Returns of selected mutual fund schemes

S.N.	Schemes	1 Month	3 Months	6 Months	1 Year	2 Years	3 Years	5 Years
1	MotilalOswalMulticap 35 Fund	-0.1	-3.1	-5.1	8.1	19	13.3	19.0
2	SBI Bluechip Fund	-0.7	-1.6	-3.8	6	11.1	9.2	18.1
3	SBI Magnum MultiCap Fund	-0.5	-2.7	-6.5	7.6	13.2	11.9	20.9
4	Mirae Asset Emerging Bluechip Fund	-0.9	-3	-11.6	2.9	18.6	16.1	30.2
5	L&T Tax Advantage Fund	1.1	-1.3	-7.3	8.1	17	12.6	19.4
6	DSP BlackRock Equity Opportunities Fund	-1.5	-4.2	-10.7	4.2	14.1	11.6	18.9
7	Aditya Birla Sun Life Tax Relief 96	0.8	-0.4	-3.5	13	17.3	12.9	22.6
8	ICICI Prudential Bluechip Fund	-0.3	0.5	-3.6	9.8	14	10.2	17
9	ICICI Prudential Equity & Debt Fund	-0.9	-2.5	-6.1	4	12	10.1	17.3
10	L&T India Value Fund	-1	-5.2	-12.1	1.3	14.9	12.5	24.5

Table 4.1 summarizes the mean variance analysis of mathematical framework for assembling a portfolio of selected schemes. Consequently, the portfolio size increases with the reduction in the risk of the portfolio as a whole. In the selected top five portfolios, Mirae Asset Emerging blue-chip fund (R4) is highly volatile with seeking high return, and SBI Bluechip Fund (R2) is less volatile with an average return, i.e., 18 percent.

Table 4.1 Volatility of 5 selected mutual fund schemes

Periods	R1	$[R1]^2$	R2	$[R2]^2$	R3	$[R3]^2$	R4	$[R4]^2$	R5	$[R5]^2$
1 Month	-0.10	0.01	-0.70	0.49	-0.50	0.25	-0.90	0.81	1.10	1.21
3 Months	-3.10	9.61	-1.60	2.56	-2.70	7.29	-3.00	9.00	-1.30	1.69
6 Months	-5.10	26.01	-3.80	14.44	-6.50	42.25	-11.60	134.56	-7.30	53.29
1 Year	8.10	65.61	6.00	36.00	7.60	57.76	2.90	8.41	8.10	65.61
2 Years	19.00	361.00	11.10	123.21	13.20	174.24	18.60	345.96	17.00	289.00
3 Years	13.30	176.89	9.20	84.64	11.90	141.61	16.10	259.21	12.60	158.76
5 Years	0		18.10	327.61	20.90	436.81	30.20	912.04	19.40	376.36
Total		639.13		588.95		860.21		1669.99		945.92
Mean	5.35		5.47		6.27		7.47		7.09	
Variance	77.90		54.20		83.56		182.75		84.92	
Std. Dev.	8.83		7.36		9.14		13.52		9.22	

Table 4.2 summarizes the mean-variance analysis of the mathematical framework for assembling a portfolio of the selected schemes. Consequently, the size of the portfolio increases with the reduction in the risk of the portfolio as a whole. In the second top five schemes, the highest volatility is 11.84%, from L&T India Value Fund R(10). The lowest is 7.32%, from ICICI Prudential Blue-chip Fund (R8).

Table 4.2 Volatility of 5 selected mutual fund schemes

Periods	R6	$[R6]^2$	R7	$[R7]^2$	R8	$[R8]^2$	R9	$[R9]^2$	R10	$[R10]^2$
1 Month	-1.50	2.25	0.80	0.64	-0.30	0.09	-0.90	0.81	-1.00	1.00
3 Months	-4.20	17.64	-0.40	0.16	0.50	0.25	-2.50	6.25	-5.20	27.04
6 Months	-10.70	114.49	-3.50	12.25	-3.60	12.96	-6.10	37.21	-12.10	146.41
1 Year	4.20	17.64	13.00	169.00	9.80	96.04	4.00	16.00	1.30	1.69
2 Years	14.10	198.81	17.30	299.29	14.00	196.00	12.00	144.00	14.90	222.01
Continuation of Table 4.2										
3 Years	11.60	134.56	12.90	166.41	10.20	104.04	10.10	102.01	12.50	156.25

5 Years	18.90	357.21	22.60	510.76	17.00	289.00	17.30	299.29	24.50	600.25
Total		842.60		1158.51		698.38		605.57		1154.65
Mean	4.63		8.96		6.80		4.84		4.99	
Variance	98.95		85.27		53.53		63.06		140.09	
Std. Dev.	9.95		9.23		7.32		7.94		11.84	

Table 5 represents the Net Assets Value (NAV) of the selected portfolio, which extracts the weights of the individual schemes, which will be further associated to calculate the set of variances of portfolio.

Table 5 S.D & weight (NAV) of the selected mutual fund schemes

S.N.	**Schemes**	**NAV**	**Weight**	**S.D (σ)**
1	Motilal Oswal Multicap 35 Fund	22.73	0.05	8.83
2	SBI Blue-chip Fund	34.43	0.07	7.36
3	SBI Magnum MultiCap Fund	42.34	0.09	9.14
4	Mirae Asset Emerging Blue-chip Fund	27.36	0.06	13.52
5	L&T Tax Advantage Fund	50.29	0.10	9.22
6	DSP Black Rock Equity Opportunities Fund	193.88	0.39	9.95
7	Aditya Birla Sun Life Tax Relief 96	28.56	0.06	9.23
8	ICICI Prudential Blue-chip Fund	38.12	0.08	7.32
9	ICICI Prudential Equity & Debt Fund	21.94	0.04	7.94
10	L&T India Value Fund	32.46	0.07	11.84
Total		492.11	1.00	

Table 6 shows that the correlation coefficient range is from 0 to 1. The frequency of the value 1 is 4 times, hence it indicates a perfect positive correlation. Portfolio standard deviation can be obtained by taking the square root of portfolio variance. Here, the variance of the portfolio is not simply a weighted average of the variances of the individual securities in the portfolio. The relationship between each security in the portfolio and every other security as measured by the covariance of return must also be considered.

Table 6 Correlation and standard deviation of the selected mutual funds

Schemes	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**	**R9**	**R10**
R1	-	0.992	0.983	0.963	0.986	0.980	0.973	0.971	0.989	0.963
R2	0.992	-	0.996	0.979	0.973	0.979	0.984	0.977	0.991	0.977
R3	0.983	0.996	-	0.978	0.985	0.989	0.987	0.982	0.994	0.979
R4	0.963	0.979	0.978	-	0.969	0.986	0.941	0.941	0.992	0.998
R5	0.986	0.973	0.985	0.969	-	0.996	0.979	0.983	0.988	0.974
R6	0.980	0.979	0.989	0.986	0.996	-	0.969	0.971	0.995	0.989
R7	0.973	0.984	0.987	0.941	0.979	0.969	-	0.996	0.972	0.942
R8	0.971	0.977	0.982	0.941	0.983	0.971	0.996	-	0.971	0.939
R9	0.989	0.991	0.994	0.992	0.988	0.995	0.972	0.971	-	0.992
R10	0.963	0.977	0.979	0.998	0.974	0.989	0.942	0.939	0.992	-

Table 7 is a correlation matrix showing correlation coefficients for combinations of 10 variables. The diagonal of the table is always a set of ones, because the correlation between a variable and itself is always 1.

Table 7 Matrix of the correlation of the selected mutual fund schemes

S.N.	**r (x, y)**	**CORREL**	**$\sigma^2\rho$**	**S.N.**	**r (x, y)**	**CORREL**	**$\sigma^2\rho$**	**S.N.**	**r (x, y)**	**CORREL**	**$\sigma^2\rho$**
1	r (1,2)	0.992	0.912	16	r (2,9)	0.991	0.691	31	r (5,6)	0.996	22.169
2	r (1,3)	0.983	1.585	17	r (2,10)	0.977	1.786	32	r (5,7)	0.979	2.156
3	r (1,4)	0.963	1.542	18	r (3,4)	0.978	2.640	33	r (5,8)	0.983	2.252
4	r (1,5)	0.986	1.846	19	r (3,5)	0.985	3.019	34	r (5,9)	0.988	1.528
5	r (1,6)	0.980	18.599	20	r (3,6)	0.989	22.040	35	r (5,10)	0.974	3.022
6	r (1,7)	0.973	0.341	21	r (3,7)	0.987	1.884	36	r (6,7)	0.969	19.522
7	r (1,8)	0.971	1.039	22	r (3,8)	0.982	1.965	37	r (6,8)	0.971	19.803
8	r (1,9)	0.989	0.573	23	r (3,9)	0.994	1.297	38	r (6,9)	0.995	17.604
9	r (1,10)	0.963	1.585	24	r (3,10)	0.979	2.698	39	r (6,10)	0.989	22.092
10	r (2,3)	0.996	1.787	25	r (4,5)	0.969	2.956	40	r (7,8)	0.996	1.296
11	r (2,4)	0.979	1.742	26	r (4,6)	0.986	21.912	41	r (7,9)	0.972	0.750
12	r (2,5)	0.973	2.039	27	r (4,7)	0.941	1.811	42	r (7,10)	0.942	1.858
13	r (2,6)	0.979	19.229	28	r (4,8)	0.941	1.894	43	r (8,9)	0.971	0.804
14	r (2,7)	0.984	1.134	29	r (4,9)	0.992	1.270	44	r (8,10)	0.939	1.940
15	r (2,8)	0.977	1.198	30	r (4,10)	0.998	2.686	45	r (9,10)	0.992	1.310

Up to this point, the researchers have considered portfolios with only two securities. Table 8 shows the benefits from the divergence as the size of the portfolio increases. The return given is less than perfectly positively correlated with the increase in the number of securities. As the number of securities added to a portfolio increases, the smaller the standard deviation of the portfolio becomes. Therefore, an investor should make the portfolio risk arbitrarily small by creating a portfolio composed of a large number of securities with negative and positive correlation in the given portfolio.

Table 8 Variance and S.D of the selected portfolios

S.N.	Particular	Statistics
1	Portfolio Variance	89.34
2	Portfolio Std. Dev.	0.95

4. Conclusion

This study focuses on the performance of the portfolios of selected companies. These are the top 10 companies from the BSE, based on five years of data. The data were collected from the Economic Times (ET). Harry Markowitz theory was applied to the selected portfolios using the approach Harry Markowitz suggested for reducing the risk of the portfolio. Harry Markowitz used a formula based on risk and expected return along with weighting. Weightings were extracted from the NAV and are also given on ET's website. Harry Markowitz formulated the portfolio evaluation based on risk, return, and the correlation. The researchers calculated all the individual standard deviations and weights to determine the group-based portfolio standard deviation. These were used to construct the correlation matrix showing the correlations among the selected portfolio schemes. The individual portfolios were found to have more risk than the group portfolios, as the standard deviation is less than to individual ones. This is the aim of modern portfolio theory: to fulfill the expected returns and to mitigate risk. This study will be helpful for investors who want to include all three categories of stocks in their portfolio: small cap, mid cap and large cap. This study has a perfect mix of securities from the different groups. It is not limited to one type of investment only.

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