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Modelling Autism Spectrum Disorders: An Optimization Approach with Application

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Abstract: Autism spectrum disorders (ASDs) refer to a group of neurodevelopmental disorders that can cause significant behavioral, communication, and social challenges. Since the biological causes are genetic, the genes responsible for the causes of autism are still yet to be identified. In Malaysia, approximately 9,000 children are born with autism yearly. However, to date, no study has been done to model this disorder through an optimization approach. Hence, modeling autism spectrum disorders using an optimization method may lead to new research in this area. Therefore, this paper proposes a new hybrid conjugate gradient method for solving unconstrained optimization problems and an autistic regression model. The authors parameterized cases of autistic children to construct an autistic optimization model and apply the Hybrid Conjugate Gradient (HCG) method to seek the solution. The hybrid HCG method is an efficient optimization algorithm. It helps to solve large-scale unconstrained optimization problems due to its low memory requirement and excellent convergence results. Additionally, it features efficient numerical performance. The authors established the convergence analysis of the proposed method under some suitable conditions. The authors further extended the proposed method to solve the problem of portfolio selection. The authors prove that the hybrid HCG optimization model efficiently solves large-scale unconstrained optimization problems based on the preliminary results.

Keywords: autism spectrum disorders, Malaysia, unconstrained optimization, convergence analysis, line search.

自闭症谱系障碍建模： 种应用优化方法

摘要: 自闭症谱系障碍或自闭症是指一组可导致重大行为、沟通和社会挑战的神经发育障碍。由于生物学原因是遗传的，导致自闭症的基因仍有待确定。在马来西亚，每年约有 9000 名儿童患有自闭症。然而，迄今为止；没有研究使用优化方法对这种紊乱进行建模。因此，使用优化方法对自闭症谱系障碍进行建模将导致该领域的新研究。因此，在本文中；我们将自闭症儿童的案例参数化以构建自闭症优化模型，并提出了一种混合共轭梯度方法来求解该模型。混合方法由于其内存要求低，收敛效果好，是解决大规模无约束优化问题的有效优化算法；除了高效的数值性能。接下来，在一些合适的条件下建立了所提出方法的收敛性分析，我们进一步扩展了所提出的方法来解决投资组合选择问题。基于初步结果，证明了混合优化模型在解决本研究中提出的大规模无约束优化问题方面是有效的

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关键词： 自闭症谱系障碍， 马来西亚， 无约束优化，收敛分析，线搜索。

1. Introduction

Autism spectrum disorder (ASD) is a complex developmental condition characterized by repetitive behaviors, nonverbal communication, speech, and social skills [1–3]. It usually becomes apparent during the first three years of life since it affects brain functions. Autism spectrum disorder has no social, racial, or ethnic boundaries. It cuts across family educational levels, income, and lifestyle [4]. This disorder is four times more prevalent for boys than girls. A study by the Centers for Disease Control [5] states that one in every 54 children born in the United States today has an ASD diagnosis. Generally, ASD and its related behaviors occur in approximately one of every 68 newborns [6, 7]. According to several works, ASD possesses a broad multi-factorial etiology. The following factors predispose the condition: genetic susceptibility, nutrient deficiencies, and exposure to toxic chemicals. One of the most efficient methods to determine the metabolites in any biological samples of children with ASD is the analytical method [8].

In the ASD-related literature, the research trend only focuses on underlying biology or behavior. Little attention is paid to mathematical aspects. For example, the work [9] studied brain bridges and the behavior of children with autism, and proposed a mathematical model, which illustrates how the neurons in the brain's circuits are stuck in the overdrive. This model provides a means of interpreting the fast-growing body of different studies, which predicts the neural disorder basis.

The autism system-wide prevalent recommends that autism is likely to alter neural computation broadly instead of narrowly impacting people's systems, including vision or affection. Based on this, [10] suggests that nonlinear alterations, canonical experimentation that occur all around the brain may be fundamental in the characteristics behavior of people with autism.

One such computation, called divisive normalization, balances a neuron's net excitation with inhibition, reflecting the overall activity of the neuronal population. The researchers investigated the level of alterations through neural network simulations. Their work studied the divisive normalization capable of influencing autism symptomatology. Results show that reducing the number of inhibitions that occur via divisive normalization can account for the emotional effects of autism.

The work [11] presented some models to examine the possible roles of oxytocin receptors and oxytocin in the development of autism. The researchers used normalized data from the Stanford study and employed mathematical operations to establish a correlation between the oxytocin blood levels of children with ASD and the severity of the condition. The results illustrated the significance of oxytocin receptors. The work [12] studied the impact of autism in gastrointestinal, immunological, mitochondrial, and metabolic systems. The authors developed models suggesting various systems capable of replacing those affected by autism.

However, the study on the optimization approach for solving a parameterized autism regression model is yet to be explored by researchers. This study considered the cases of autism among children in Malaysia to parameterize an autism regression model [13, 14].

Table 1 Statistics of children with autistic disorder in Malaysia

Age (x)	**Statistics (y)**
1	1
2	3
3	8
4	10
5	5
6	7
7	5
8	5
9	4
10	2

In addition, the authors developed a hybrid conjugate gradient method to obtain the solution of the parameterized autism regression model and some benchmarked optimization models. The authors established convergence of the proposed method under an exact line search. The authors further extended the proposed HCG method to solve problems of portfolio selection. Finally, the authors analyzed the performance of the proposed parameter with the help of tables and graphs based on the number of iterations and CPU processing time.

2. New Method and Parameterized Model

Let us consider the following model:

$$\min f(x)\,,\ \ x \in \mathbb{R}^n \qquad (1)$$

where $f: \mathbb{R}^n \to \mathbb{R}$ is a smooth function, whose gradient is denoted by $g(x) = \nabla f(x)$ [15]. Model (1) is an unconstrained optimization model solved using different numerical methods [16]. One of the most widely used numerical methods is the conjugate gradient (CG) method because of its simplicity, convergence properties, and low memory requirements [17-19]. The CG algorithm generates its iterative sequence $\{x_k\}$ via the following formula:

$$x_{k+1} = x_k + \alpha_k d_k, \quad (2)$$

where $k = 0,1,2,\ldots$ and $\alpha_k > 0$ is the step size computed using a line search method along the search direction d_k [20,21]. For $k = 1$, then $d_{k-1} = -g_{k-1}$which is known as the steepest descent direction [22]. However, subsequent iterations are computed as follows:

$$d_k = -g_k + \beta_k d_{k-1}. \quad (3)$$

One of the most important properties for a CG method to be considered is the process of choosing the right step size, which could be an exact line search or an inexact line search [23]. The exact line search is computed such that α_k satisfy:

$$f(x_k + \alpha_\kappa) = \lim_{\alpha_k \geq 0} f(x_\kappa + \alpha d_k). \quad (4)$$

On the other hand, the most commonly used inexact line search is the Strong Wolfe's (SWP) line search with the formula defined as:

$$f(x_k + \alpha_k d_k) - f(x_\kappa) \leq \delta \alpha_\kappa g_k^T d_k \quad (5)$$

$$|g_{\kappa+1}^T d_\kappa| \leq -\sigma g_k^T d_k. \quad (6)$$

where $0 < \delta \leq \sigma < 1$ [24].

To derive the unconstrained optimization regression model for the cases of autism in Malaysia, the authors need to parameterize the data from Table 1. Given the regression analysis, the function is defined as:

$$y = h(x_1, x_2, \ldots, x_p + \varepsilon) \quad (7)$$

where ε is the value of error, $x_i, i = 1,2,\ldots,p, p > 0$, y represent the predictor and response variable, respectively. Regression analysis is a statistical process proposed for estimating relationships between independent and dependent variables [25]. The linear regression function can be obtained by computing for y such that:

$$y = a_0 + a_1x_1 + a_2x_2 + \cdots + a_px_p + \varepsilon. \quad (8)$$

where $a_0, \ldots, a_p$ denotes the estimated regression parameters such that the ε is minimized. In certain cases, a straight line is often used to approximate situations where the linear regression method is the relationship between y and x. However, the above cases are rare; hence, the nonlinear regression process is usually considered. This study also considers the nonlinear regression procedure.

From the autism data given in Table 1, the authors derive the approximate function. The authors considered data for children aged 1 to 10 with the ages denoted by x-variable and the corresponding number of children in the age group denoted as y-variable. Only the data for children aged from 1 to 9 would be considered for the data fitting while reserving the data for children of age 10 for error analysis.

$$f(x) = 29.52381 - 13.45390x + 1.36039x^2. \quad (9)$$

Now, the approximate function for the nonlinear least square method is given as follows:

$$\min_{x \in R^n} f(x) = \sum_{j=1}^{n} \left((u_0 + u_1x_j + u_2x_j^2) - y_j \right)^2. \quad (10)$$

Based on Table 1, the authors derive the nonlinear quadratic function for the least square method. This function is further applied to develop the corresponding unconstrained optimization function. From data x_j and the value of y_j, it is obvious that they possess some parabolic relations with the regression parameters $u_0, u_1$and u_2 and the regression function (1).

$$\min_{x \in R^2} \sum_{j=1}^{n} E_j^2 = \sum_{j=1}^{n} ((u_0 + u_1x + u_2x^2) - y_j)^2. \quad (11)$$

Equation (11) can be transformed using the data from Table 1 to derive the following nonlinear quadratic unconstrained minimization function:

$$9u\_1\text{^}2 + 90u\_1\,u\_2 + 570u\_1\,u\_3 - 96u\_1 + 285u\_2\text{^}2 + 4050u\_2\,u\_3 - 498u\_2 + 15333u\_3\text{^}2 - 3022u\_3 + 314. \quad (12)$$

The well-known CG coefficient for solving (12) is the PRP method proposed by [25,26] with THE formula as follows:

$$\beta_k^{PRP} = \frac{g_k^T y_{k-1}}{\|g_{k-1}\|^2}, \quad (13)$$

where $y_{k-1} = g_k - g_{k-1}$. This method is among the common classical methods used to compare the efficiency of a new conjugate gradient method. Recently, [27] modified the denominator of the PRP method by replacing $\|g_{k-1}\|^2$ with $\|d_{k-1}\|^2$ as follows:

$$\beta_k^{RMIL} = \frac{g_k^T y_{k-1}}{\|d_{k-1}\|^2}, \quad (14)$$

Furthermore, it established the convergence of the method under exact line search. Numerical results obtained show that the RMIL method is efficient and more promising compared to other existing methods. The work [28] further extended the results to RMIL to propose RMIL+ with the coefficients defined as follows:

$$\beta_k^{RMIL+} = \frac{g_k^T (y_{k-1} - d_{k-1})}{\|d_{k-1}\|^2}. \quad (15)$$

The authors proved the global convergence of the method under exact and inexact line searches. The computation shows that the proposed method is competitive. For an excellent review on advances of the CG method, the authors refer the readers to a survey by [29].

Motivated by efficient numerical results and nice convergence properties, the authors present a hybrid CG method that combines RMIL and RMIL+. The formula of the new method is defined as follows:

$$\beta_k^{hSNM} = \max\left\{ \frac{g_k^T y_{k-1}}{\|d_{k-1}\|^2}, \frac{g_k^T (y_{k-1} - d_{k-1})}{\|d_{k-1}\|^2} \right\}. \quad (16)$$

Next, the authors present the algorithm of the proposed method as follows:

Algorithm 1. (hSNM method)

Step 1: Starting with $x_0 \in \mathbb{R}^n$ as the initial point, set $d_0 = -g_0$, set $k - 1 = 0$.

Step 2: Terminate the computation if $\|g_k\| < \epsilon$, where $\epsilon = 10^{-6}$. Else, go to Step 3.

Step 3: Compute search direction d_k using (3).

Step 4: Compute the step-size α_k using an exact line search (4).

Step 5: Update the iteration using (2).

Step 6: Set $k - 1 = k$ and return to Step 2.

3. Convergence Analysis

This section presents the convergence analysis of the proposed method under the exact minimization condition. An efficient algorithm must satisfy some basic properties, including the sufficient descent condition and global convergence properties.

3.1. Sufficient Descent Condition

This section shows that proposed CG coefficient possess sufficient descent condition. For this condition to hold,

$$g_k^T d_k = -\|g_k\|^2. \tag{17}$$

Theorem 1. Let the sequence $\{x_k\}$ be generated by Algorithm 1, where the search direction d_k is defined by (3) and $\beta_k = \beta_k^{hSNM}$. Then the sufficient descent condition (17) holds for all $k \geq 0$.

Proof. The proof of this theorem is induction. Let us assume $k = 0$, then, $g_0^T d_0 = -\|g_0\|^2$. This implies that (17) is true. Next is to show that the condition also is true for $k \geq 1$. Multiplying (3) by g_k results in

$$g_k^T d_k = -\|g_k\|^2 + \beta_k g_k^T d_{k-1}.$$

However, for the exact line search, then $g_k^T d_{k-1} = 0$. Hence,

$$g_k^T d_k = -\|g_k\|^2,$$

which implies that d_k satisfies the descent condition (17) and thus, complete the proof.

3.2. Global Convergence Properties

To proceed with the convergence analysis of the proposed method, simply β_k^{hSNM}. From (16), (14), and (15), then

$$\beta_k^{hSNM} = \max\{\beta_k^{RMIL}, \beta_k^{RMIL+}\}.$$

However, from [28], it follows that:

$$0 \leq \beta_k^{RMIL} \leq \frac{\|g_k\|^2}{\|d_{k-1}\|^2}. \tag{18}$$

In addition, from [29], it follows that

$$\beta_k^{RMIL+} < \beta_k^{RMIL} \leq \frac{\|g_k\|^2}{\|d_{k-1}\|^2}. \tag{19}$$

Now from (16), (18), and (19), it follows that,

$$\beta_k^{hSNM} \leq \frac{\|g_k\|^2}{\|d_{k-1}\|^2}. \tag{20}$$

The above simplification would ease the theoretical proof.

For the global convergence analysis of the CG method, the following important assumptions are always required.

Assumption A

i. The level set $\Omega = \{x \in \mathbb{R}^n | f(x) \leq f(x_0)\}$ is bounded from below, such that there exists a positive constant c where $\|x\| \leq c, \forall x \in \Omega$.

ii. The objective function f is continuously differentiable in some neighborhood N of Ω and $g(x)$ is Lipchitz continuous, namely, there exists a constant $L > 0$ satisfying:

$$\|g(x) - g(y)\| \leq L\|x - y\|, \forall x, y \in N. \tag{21}$$

Based on the above assumption, the following lemma Zoutendijk [30] holds for the exact minimization rule [31].

Lemma 1. Let us suppose that *Assumption A* is true. Let the sequence $\{x_k\}$ be defined by Algorithm 1, where d_k is a descent direction, and α_k satisfies the exact minimization condition (4). Then, the Zoutendijk condition below holds.

$$\sum_{k=1}^{\infty} \frac{(g_{k-1}^T d_{k-1})^2}{\|d_{k-1}\|^2} < \infty. \tag{22}$$

The proof of this lemma follows from Zoutendijk [30]. From Lemma 1 and (20), the authors obtain the following convergence theorem.

Theorem 2. Let us suppose that $\{x_k\}$ is generated by *Algorithm 1* and *Assumption A* is true, with d_k is a descent direction, and α_k is computed via exact minimization condition (4). In addition, the sufficient descent condition (17) is true. Then either

$$\lim_{k\to\infty} \|g_{k-1}\| = 0 \text{ or } \sum_{k=1}^{\infty} \frac{(g_{k-1}^T d_{k-1})^2}{\|d_{k-1}\|^2} < \infty. \tag{23}$$

Proof. The proof of this theorem will be by contradiction. That is, let us suppose that *Theorem 2* is not true, then, there exists a constant $\lambda > 0$ such that,

$$\|g_k\|^2 \geq \lambda. \tag{24}$$

However, from (3), then

$$g_k^T + d_k = \beta_k d_{k-1}.$$

By squaring both sides of the above equation, then dividing by $\left(g_k^T d_k\right)^2$, then

$$\frac{\|d_k\|^2}{(g_k^T d_k)^2} = \beta_k^2 \frac{\|d_{k-1}\|^2}{(g_k^T d_k)^2} - \frac{2}{(g_k^T d_k)^2} - \frac{\|g_k\|^2}{(g_k^T d_k)^2}$$

$$= \beta_k^2 \frac{\|d_{k-1}\|^2}{(g_k^T d_k)^2} - \left(\frac{1}{\|g_k\|} - \frac{\|g_k\|}{g_k^T d_k}\right)^2 + \frac{1}{\|g_k\|^2}$$

$$\leq \beta_k^2 \frac{\|d_{k-1}\|^2}{(g_k^T d_k)^2} + \frac{1}{\|g_k\|^2}.$$

Substituting the value of β_k from (20) and using the sufficient descent condition $g_k^T d_k = -\|g_k\|^2$, then

$$\frac{\|d_k\|^2}{(g_k^T d_k)^2} \leq \frac{\|g_k\|^4}{\|d_{k-1}\|^4} \frac{\|d_{k-1}\|^2}{\|g_k\|^4} + \frac{1}{\|g_k\|^2}$$

$$= \frac{1}{\|d_{k-1}\|^2} + \frac{1}{\|g_k\|^2}$$

Based on Lemma 4 from [18], then there is a relation $\frac{1}{\|d_k\|^2} \leq \frac{1}{\|g_k\|^2}, \forall\, k \geq 0$, then

$$\frac{\|d_k\|^2}{(g_k^T d_k)^2} \leq \frac{1}{\|g_{k-1}\|^2} + \frac{1}{\|g_k\|^2}.$$

By using (24), then

$$\frac{\|d_k\|^2}{(g_k^T d_k)^2} \leq \frac{1}{\lambda} + \frac{1}{\lambda} = \frac{2}{\lambda}.$$

Thus,

$$\frac{\left(g_k^T d_k\right)^2}{\|d_k\|^2} \geq \frac{2}{\lambda}.$$

This implies that,

$$\sum_{k=0}^{n} \frac{\left(g_k^T d_k\right)^2}{\|d_k\|^2} \geq \sum_{k=0}^{n} \frac{2}{\lambda} = \frac{2(n+1)}{\lambda}.$$

Furthermore, if $n \to \infty$, then

$$\sum_{k=0}^{\infty} \frac{\left(g_k^T d_k\right)^2}{\|d_k\|^2} \geq \lim_{n\to\infty} \frac{2(n+1)}{\lambda} = \infty.$$

Hence,

$$\sum_{k=0}^{\infty} \frac{\left(g_k^T d_k\right)^2}{\|d_k\|^2} \geq \infty.$$

which contradicts (22) and thus, complete the proof.

4. Numerical Experiments

This section presents the performance analysis of the proposed method on the unconstrained minimization model (1) derived from Table 1. The performance of the proposed method was compared with that of other CG methods. Applying the new hybrid hSNM and other existing algorithms on the model (1), the authors obtain Table 2.

Computing u_0, u_1, u_2 values via matrix inversion has always been a difficult task for researchers. In this study, the proposed hSNM method was presented with nine different initial points. This idea was proposed in order to overcome the difficulties to compute u_0, u_1, u_2. In the process of the iterations, if any of the following conditions hold:

1. The algorithm fails to solve the model.
2. The number of iterations exceeds 1000.

The iteration would stop, and that point will be denoted as Fail.

Table 2 Numerical results for optimization of quadratic model

No	Initial Points	RMIL		RMIL+		hSNM	
		NOI	CPU	NOI	CPU	NOI	CPU
N1	$(-3,-3,-3)$	4	0.00308884	7	0.00546788	5	0.00287208
N2	(0,0,0)	5	0.00483278	9	0.01257624	4	0.00237247
N3	(0.5,0.5,0.5)	6	0.00303671	5	0.00285361	4	0.00265409
N4	(3,3,3)	34	0.16383639	5	0.00475270	4	0.00336021
N5	(5,5,5)	34	0.14033876	6	0.00352113	5	0.00463532
N6	(7,7,7)	5	0.00445304	29	0.09175968	4	0.00270746
N7	(13,13,13)	5	0.00281995	6	0.00522525	4	0.00243856
N8	(15,15,15)	5	0.00455940	34	0.11680952	4	0.00344436
N9	(27,27,27)	5	0.00383437	5	0.00303671	5	0.00381590

The next step is to employ the proposed hSNM and least-squares methods to estimate Table 1. The Microsoft Excel software was used to plot the trend line where the equation would be in a nonlinear quadratic equation. In the trend line graph, x and y represent *the* x-axis and y-axis, respectively, as shown in Figure 1.

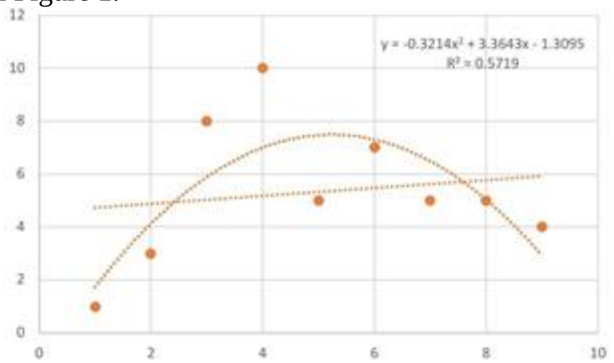

Fig. 1 Quadratic trend line for children with autism spectrum disorder

The approximate functions of the hSNM algorithm are compared with that of the trend line and least square methods, as presented in Table 3.

Table 3 Numerical results for optimization of a quadratic model

Models	Estimation Point	Relative Error
hSNM	19256.790790	0.23786793800
Least Square	18186.200000	0.280239046000
Trend Line	18186.200000	0.280239046000

Based on the results from Table 3, it is obvious that the hSNM algorithm presents the least relative error compared to the Trend line and Least-square methods. Since the regression analysis aims to estimate the parameters $a_0, a_1, \dots, a_p$ such that the error ε is minimized, the authors can deduce that the proposed method can be used as an alternative to both Trend line and Least-square methods.

Furthermore, the authors present numerical experiments of the proposed hSNM method based on 56 test-benchmarked functions (see Table 4) with dimensions ranging from 2 to 50,000. The performance was compared with the RMIL method proposed by [27] and the RMIL+ method presented by [28]. The name set of test functions used for the experiments can be found in Table 4.

The authors use MATLAB R2019a on a personal computer with specifications; Intel Core i7 processor, 16 GB RAM, 64bit Windows 10 Pro operating system to write all codes. The authors marked failure denoted by "-" if the number of iterations exceeded 10,000. In addition, the authors used the stopping criteria for each method is either $\|g_k\| < 10^{-6}$ or never reach the optimal point. Information containing the initial points, dimensions, and numerical results of numerical experiments based on the number of iterations and CPU time is available in Table 5.

Table 4 List of test functions

No	**Function**	**No**	**Function**
F1	Extended White and Holst	F29	Extended Quadratic Penalty QP1
F2	Extended Rosenbrock	F30	Quartic
F3	Extended Freudenstein and Roth	F31	Matyas
F4	Extended Beale	F32	Colville
F5	Raydan 1	F33	Dixon and Price
F6	Extended Tridiagonal 1	F34	Sphere
F7	Diagonal 4	F35	Sum Squares
F8	Extended Himmelblau	F36	DENSCHNA
F9	FLETCHCR	F37	DENSCHNC
F10	NONSCOMP	F38	DENSCHNF
F11	Extended DENSCHNB	F39	Staircase S1
F12	Extended Penalty	F40	Staircase S2
F13	Hager	F41	Staircase S3
F14	Extended Maratos	F42	Extended Block-Diagonal BD1
F15	Six Hump Camel	F43	HIMMELBH
F16	Three Hump Camel	F44	Extended Hiebert
F17	Booth	F45	Tridiagonal White and Holst
F18	Trecanni	F46	ENGVAL1
F19	Zettl	F47	ENGVAL8
F20	Shallow	F48	Linear Perturbed
F21	Generalized Quartic	F49	QUARTICM
F22	Quadratic QF2	F50	Brent
F23	Leon	F51	Deckkers-Aarts
F24	Generalized Tridiagonal 1	F52	El-Attar-Vidyasagar-Dutta
F25	Generalized Tridiagonal 2	F53	Price 4
F26	POWER	F54	Rotated Ellipse 2
F27	Quadratic QF1	F55	Wayburn Seader 1

F28	Extended Quadratic Penalty QP2	F56	Zirilli or Aluffi-Pentini's

Table 5 Numerical results

No	**Dimension**	**Initial Points**	**RMIL**		**RMIL+**		**hSNM**	
			NOI	**CPU**	**NOI**	**CPU**	**NOI**	**CPU**
F1	1,000	(-1.2, 1, …)	28	0.8948	16	0.4244	29	0.7561
F1	10,000	(-1.2, 1, …)	28	7.282	19	4.83	29	7.4158
F2	1,000	(-1.2, 1, …)	28	0.1369	22	0.0806	28	0.1002
F2	10,000	(-1.2, 1, …)	28	0.5125	23	0.4503	28	0.5276
F3	1,000	(0.5, -2, ...)	22	0.0851	12	0.1135	9	0.0404
F3	10,000	(0.5, -2, ...)	24	0.5016	12	0.2484	10	0.2183
F4	1,000	(1, 0.8, …)	52	1.5775	52	1.469	52	1.4401
F4	10,000	(1, 0.8, …)	54	14.792	54	14.8493	54	14.9358
F5	10	(1,…,1)	19	0.0575	19	0.0532	19	0.053
F5	100	(1,…,1)	98	0.3003	99	0.3074	98	0.3046
F6	1,000	(1,…,1)	181	4.9043	165	4.5566	197	5.5173
F6	10,000	(1,…,1)	271	74.0323	283	77.744	281	77.8244
F7	1,000	(1,…,1)	3	0.0207	3	0.0193	3	0.017
F7	10,000	(1,…,1)	3	0.0725	4	0.0769	3	0.0583
F8	1,000	(1,…,1)	11	0.0496	11	0.0425	11	0.0424
F8	10,000	(1,…,1)	12	0.2626	12	0.2396	12	0.2384
F9	1,000	(2,…,2)	21	0.0832	21	0.085	21	0.0885
F9	10,000	(2,…,2)	21	0.4764	21	0.4757	21	0.4735
F10	1,000	(2,…,2)	29	0.1128	29	0.1116	29	0.1098
F10	10,000	(2,…,2)	29	0.625	29	0.6542	29	0.6325
F11	1,000	(1,…,1)	6	0.0298	6	0.0283	6	0.0277
F11	10,000	(1,…,1)	6	0.1286	6	0.1225	6	0.1329
F12	10	(1, 2,…,10)	20	0.0512	13	0.0448	13	0.0431
F12	100	(1, 2,…,100)	25	0.0756	-	-	18	0.0504
F13	10	(1,…,1)	12	0.04	12	0.035	12	0.0342
F13	100	(1,…,1)	25	0.0878	25	0.087	25	0.0871
F14	4	(-1,…,-1)	18	0.0479	18	0.048	21	0.0527
F15	2	(5,5)	6	0.0295	6	0.0308	6	0.0286
F15	2	(10,10)	8	0.0344	8	0.0387	8	0.0385
F16	2	(5,5)	-	-	16	0.0629	16	0.0616
F16	2	(10, 10)	-	-	-	-	25	0.0954
F17	2	(5, 5)	3	0.0182	3	0.0181	3	0.0178
F17	2	(10, 10)	3	0.0186	3	0.0215	3	0.015
F18	2	(-1, 0.5)	1	0.0053	1	0.0067	1	0.0056
F18	2	(-5, 10)	6	0.0344	6	0.0354	6	0.0285
F19	2	(-1, 2)	21	0.0813	21	0.0749	21	0.0739
F19	2	(10, 10)	19	0.076	19	0.0662	19	0.0641
F20	1,000	(0,…,0)	26	0.1391	26	0.1189	26	0.1188
F20	10,000	(0,…,0)	37	0.7005	29	0.5514	29	0.5437
F21	1,000	(1,…,1)	6	0.0456	5	0.0375	5	0.0331
F21	10,000	(1,…,1)	6	0.1873	5	0.1308	5	0.1303
F22	10	(0.5,…,0.5)	27	0.0992	27	0.0893	27	0.0913
F22	100	(0.5,…,0.5)	135	0.4058	135	0.4002	135	0.4017
F23	2	(2,2)	15	0.0523	22	0.075	18	0.0702
F23	2	(8,8)	30	0.118	36	0.1235	27	0.0881
F24	10	(2,…,2)	22	0.089	22	0.0858	22	0.0865
F24	100	(2,…,2)	-	-	31	0.3121	23	0.124
F25	4	(1,…,1)	4	0.0233	4	0.0255	4	0.0233
F25	10	(1,…,1)	138	0.3549	138	0.3592	138	0.3655
F26	10	(1,…,1)	123	0.3356	123	0.3215	123	0.3285
F26	50	(1,…,1)	2470	7.4066	2495	5.9509	2502	6.0822
F27	500	(1,…,1)	422	3.2311	469	4.2617	430	3.2147
F27	1,000	(1,…,1)	991	13.1075	949	9.8479	944	9.9397
F28	500	(10,…,10)	58	0.6273	89	0.8869	57	0.6496
F28	1,000	(10,…,10)	250	3.9973	-	-	502	5.8223
F29	5	(1,…,1)	15	0.0552	15	0.0571	15	0.059
F29	10	(1,…,1)	18	0.0646	18	0.0662	18	0.0669
F30	4	(10,10,10,10)	807	5.7637	802	2.3553	802	2.2966
F30	4	(15,15,15,15)	807	2.9649	793	2.2904	793	2.2707
F31	2	(1, 1)	1	0.0051	1	0.0064	1	0.0056
F31	2	(20, 20)	1	0.0103	1	0.0103	1	0.0062
F32	4	(-1,…,-1)	413	1.0662	407	1.719	433	1.0594
F32	4	(10,10,10,10)	392	1.0984	332	0.8546	373	0.9009
F33	4	(1,…,1)	41	0.1327	41	0.1332	41	0.1269
F33	10	(1,…,1)	90	0.2495	90	0.2559	90	0.2414

No	Dimension	Initial Points	RMIL		RMIL+		hSNM	
			NOI	CPU	NOI	CPU	NOI	CPU
F34	5,000	(1,…,1)	1	0.0173	1	0.0191	1	0.0173
F34	10,000	(1,…,1)	1	0.0509	1	0.0342	1	0.0211
F35	10	(10,…,10)	28	0.0901	28	0.0917	28	0.0919
F35	100	(10,…,10)	146	0.3989	146	0.418	146	0.3836
F36	10,000	(7,…,7)	13	3.9376	13	4.0727	13	3.9429
F36	50,000	(7,…,7)	13	17.9999	13	26.937	13	30.7808
F37	10,000	(-1,…,-1)	13	3.6091	10	2.907	12	3.3582
F37	50,000	(-1,…,-1)	14	17.9479	11	14.9031	13	16.8131
F38	5,000	(-1,…,-1)	9	0.1155	9	0.1172	9	0.1168
F38	10,000	(-1,…,-1)	10	0.2377	10	0.2336	10	0.2316
F39	2	(1,1)	1	0.0055	1	0.0074	1	0.0061
F39	2	(-1, -1)	1	0.0097	1	0.0063	1	0.0061
F40	2	(-1, -1)	1	0.0054	1	0.0052	1	0.0053
F40	2	(7,7)	1	0.0116	1	0.0074	1	0.0062
F41	2	(2,2)	1	0.0063	1	0.0067	1	0.005
F41	2	(7,7)	1	0.0119	1	0.007	1	0.0065
F42	1,000	(1,…,1)	9	0.0789	12	0.0991	12	0.0877
F42	10,000	(1,…,1)	9	0.3475	13	0.4566	13	0.4565
F43	200	(0.8,…,0.8)	5	0.0561	5	0.0526	5	0.0489
F43	900	(0.8,…,0.8)	6	0.7623	6	0.6413	5	0.4221
F44	10	(5.001,…)	36	0.0946	35	0.0857	30	0.0757
F44	10,000	(5.001, …)	37	0.632	43	0.7806	32	0.5832
F45	10	(0,…,0)	545	1.3652	503	1.2277	514	1.2451
F45	50	(0,…,0)	791	3.3633	794	3.4645	793	3.372
F46	50	(2,…,2)	22	0.0546	22	0.0591	22	0.069
F46	100	(2,…,2)	-	-	24	0.0635	24	0.0641
F47	10	(1,…,1)	28	0.0689	28	0.0699	28	0.0806
F47	50	(1,…,1)	-	-	-	-	30	0.0851
F48	100	(0,…,0)	84	0.2112	84	0.2016	84	0.2023
F48	1,000	(0,…,0)	699	3.4645	699	3.5657	699	3.3704
F49	1,000	(2,…,2)	1	0.0613	1	0.0544	1	0.0543
F49	10,000	(2,…,2)	1	0.517	1	0.52	1	0.5126
F50	2	(-1, -1)	3	0.0176	2	0.0078	3	0.0112
F50	2	(4,4)	1	0.0063	1	0.0062	1	0.0055
F51	2	(-5,0)	4	0.0215	3	0.0148	5	0.0173
F51	2	(0, -5)	-	-	-	-	-	-
F52	2	(1,1)	-	-	27	0.0665	16	0.0505
F52	2	(-2, -2)	20	0.0569	20	0.0589	20	0.0562
F53	2	(1,1)	29	0.081	29	0.0795	29	0.0774
F53	2	(-2, -2)	12	0.0398	12	0.0394	12	0.0317
F54	2	(1,1)	1	0.0057	1	0.0062	1	0.0054
F54	2	(-2, -2)	1	0.0067	1	0.004	1	0.0058
F55	2	(1,1)	20	0.0544	17	0.0521	20	0.0563
F55	2	(-2, -2)	28	0.0774	22	0.0637	25	0.0627
F56	2	(1,1)	4	0.0147	4	0.0165	4	0.0152
F56	2	(-2, -2)	5	0.4364	5	0.0244	5	0.0202
F38	5,000	(-1,…,-1)	9	0.1155	9	0.1172	9	0.1168
F38	10,000	(-1,…,-1)	10	0.2377	10	0.2336	10	0.2316
F39	2	(1,1)	1	0.0055	1	0.0074	1	0.0061
F39	2	(-1, -1)	1	0.0097	1	0.0063	1	0.0061
F40	2	(-1, -1)	1	0.0054	1	0.0052	1	0.0053
F40	2	(7,7)	1	0.0116	1	0.0074	1	0.0062
F41	2	(2,2)	1	0.0063	1	0.0067	1	0.005
F41	2	(7,7)	1	0.0119	1	0.007	1	0.0065
F42	1,000	(1,…,1)	9	0.0789	12	0.0991	12	0.0877
F42	10,000	(1,…,1)	9	0.3475	13	0.4566	13	0.4565
F43	200	(0.8,…,0.8)	5	0.0561	5	0.0526	5	0.0489
F43	900	(0.8,…,0.8)	6	0.7623	6	0.6413	5	0.4221
F44	10	(5.001, …)	36	0.0946	35	0.0857	30	0.0757
F44	10,000	(5.001, …)	37	0.632	43	0.7806	32	0.5832
F45	10	(0,…,0)	545	1.3652	503	1.2277	514	1.2451
F45	50	(0,…,0)	791	3.3633	794	3.4645	793	3.372
F46	50	(2,…,2)	22	0.0546	22	0.0591	22	0.069
F46	100	(2,…,2)	-	-	24	0.0635	24	0.0641
F47	10	(1,…,1)	28	0.0689	28	0.0699	28	0.0806
F47	50	(1,…,1)	-	-	-	-	30	0.0851
F48	100	(0,…,0)	84	0.2112	84	0.2016	84	0.2023
F48	1,000	(0,…,0)	699	3.4645	699	3.5657	699	3.3704
F49	1,000	(2,…,2)	1	0.0613	1	0.0544	1	0.0543

No	Dimension	Initial Points	RMIL		RMIL+		hSNM	
			NOI	CPU	NOI	CPU	NOI	CPU
F49	10,000	(2,…,2)	1	0.517	1	0.52	1	0.5126
F50	2	(-1, -1)	3	0.0176	2	0.0078	3	0.0112
F50	2	(4,4)	1	0.0063	1	0.0062	1	0.0055
F51	2	(-5,0)	4	0.0215	3	0.0148	5	0.0173
F51	2	(0, -5)	-	-	-	-	-	-
F52	2	(1,1)	-	-	27	0.0665	16	0.0505
F52	2	(-2, -2)	20	0.0569	20	0.0589	20	0.0562
F53	2	(1,1)	29	0.081	29	0.0795	29	0.0774
F53	2	(-2, -2)	12	0.0398	12	0.0394	12	0.0317
F54	2	(1,1)	1	0.0057	1	0.0062	1	0.0054
F54	2	(-2, -2)	1	0.0067	1	0.004	1	0.0058
F55	2	(1,1)	20	0.0544	17	0.0521	20	0.0563
F55	2	(-2, -2)	28	0.0774	22	0.0637	25	0.0627
F56	2	(1, 1)	4	0.0147	4	0.0165	4	0.0152
F56	2	(-2, -2)	5	0.4364	5	0.0244	5	0.0202
F44	10,000	(5.001, …)	37	0.632	43	0.7806	32	0.5832
F45	10	(0,…,0)	545	1.3652	503	1.2277	514	1.2451
F45	50	(0,…,0)	791	3.3633	794	3.4645	793	3.372
F46	50	(2,…,2)	22	0.0546	22	0.0591	22	0.069
F46	100	(2,…,2)	-	-	24	0.0635	24	0.0641
F47	10	(1,…,1)	28	0.0689	28	0.0699	28	0.0806
F47	50	(1,…,1)	-	-	-	-	30	0.0851
F48	100	(0,…,0)	84	0.2112	84	0.2016	84	0.2023
F48	1,000	(0,…,0)	699	3.4645	699	3.5657	699	3.3704
F49	1,000	(2,…,2)	1	0.0613	1	0.0544	1	0.0543
F49	10,000	(2,…,2)	1	0.517	1	0.52	1	0.5126
F50	2	(-1, -1)	3	0.0176	2	0.0078	3	0.0112
F50	2	(4,4)	1	0.0063	1	0.0062	1	0.0055
F51	2	(-5,0)	4	0.0215	3	0.0148	5	0.0173
F51	2	(0, -5)	-	-	-	-	-	-
F52	2	(1,1)	-	-	27	0.0665	16	0.0505
F52	2	(-2, -2)	20	0.0569	20	0.0589	20	0.0562
F53	2	(1,1)	29	0.081	29	0.0795	29	0.0774
F53	2	(-2, -2)	12	0.0398	12	0.0394	12	0.0317
F54	2	(1,1)	1	0.0057	1	0.0062	1	0.0054
F54	2	(-2, -2)	1	0.0067	1	0.004	1	0.0058
F55	2	(1,1)	20	0.0544	17	0.0521	20	0.0563
F55	2	(-2, -2)	28	0.0774	22	0.0637	25	0.0627
F56	2	(1,1)	4	0.0147	4	0.0165	4	0.0152
F56	2	(-2, -2)	5	0.4364	5	0.0244	5	0.0202

To show the efficiency of all methods, the authors use the performance profile by Dolan and Mor'e [32], which can help to standardize the comparison of methods. Let us suppose the n_m method and the n_p problem and consider the performance measure, either the number of iterations or the CPU time. The authors let $b_{p,m}$ be the number of iterations or the CPU time needed to solve the problem by the method m. To compare the performance on problem p by a solver s with the best performance by other solvers in this problem, the authors use the performance ratio $r_{p,s}$ that is defined as follows:

$$r_{p,s} = \frac{b_{p,s}}{\min\{b_{p,s}: s \in S,\ p \in P\}},$$

where S is the solver set and P is the problem set. Next, let $\rho_s(\tau)$ is the probability for solver $s \in S$ that a performance ratio $r_{p,s}$ is within a factor $\tau \in \mathbb{R}^+$ of the best possible ratio, where $\rho_s(\tau)$ is formulated as

$$\rho_s(\tau) = \frac{1}{n_p} \text{size}\,\{p \in P: \log_2 r_{p,s} \leq \tau\}.$$

According to their rules, the solver with the large probability $\rho_s(\tau)$ is the best solver.

From the numerical results in Table 5, the authors can plot the performance profile based on several iterations in Figure 2 and CPU time in Figure 3. Both figures show that the value of $\rho_s(\tau)$ of the hSNM method is larger than other methods. This indicates that the proposed hSNM method is efficient than RMIL and RMIL+ methods.

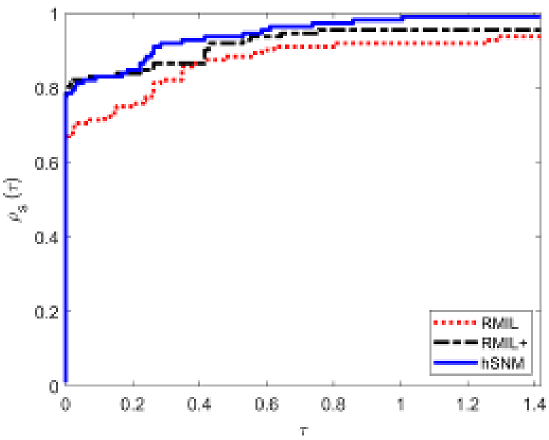

Fig. 2 Performance profile concerning several iterations

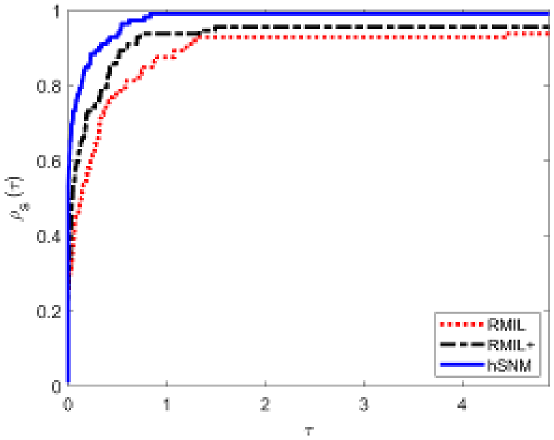

Fig. 3 Performance profile concerning CPU time

5. Application in Portfolio Selection

Many problems in financial economics involve large dimensions of nonlinear equations. Among the popular issues is to minimize risks in portfolio selections, where Harry Markowitz first introduced portfolio selection theory in his article [33]. A portfolio is a collection of financial investments such as bonds, stocks, cash, and deposits, including closed funds and funds traded on stock exchanges. People generally believe that stocks are the core of a portfolio. Therefore, this paper studies portfolio of stocks. Portfolio selection is useful as a guide for investors to consider investing. Of course, an investor wants maximum profits and small risks. Many theories have been developed to measure the risk of an investment. One way is to use variance. Therefore, the authors consider portfolio selection by minimizing variance.

The portfolio selection process involves two stages. In the first stage, several suitable stocks are selected, and then in the second stage, the percentage of the total investment for each stock is identified [34]. The stocks to be used are PT Bank Rakyat Indonesia (Persero) Tbk (BBRI), PT Unilever Indonesia Tbk (UNVR), PT Telekomunikasi Indonesia Tbk (TLKM), PT Indofood CBP Sukses Makmur Tbk (ICBP) and PT Bank Mandiri (Persero) Tbk (BMRI) and using the closing price of stock. All data is obtained from http://finance.yahoo.com/ over three years (Jan 1, 2018 - Dec 31, 2020).

As described in [35], return of stock s_i is defined as

$$T_i = \frac{C_t - C_{t-1}}{C_{t-1}},$$

where C_t is the closing price at time t and C_{t-1} is the closing price at time $t-1$. Meanwhile, the variance and expected return of the portfolio is formulated as follows respectively:

$$\sigma^2 = Var \sum_{i=1}^{n} w_i T_i = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov\left(T_i, T_j\right) \tag{25}$$

$$\mu = E\left(\sum_{i=1}^{n} w_i T_i\right) = \sum_{i=1}^{n} w_i \mu_i \tag{26}$$

where w_i is the proportion of invested stock i, μ_i is the mean of return of stock i, n is several stocks, and $Cov\left(T_i, T_j\right)$ is the covariance of return between stock i and j.

The main goal is to minimize risks, and the risk considered is the variance of the return portfolio. Furthermore, the problem can be formulated as follows:

$$\begin{cases} \min \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov\left(T_i, T_j\right) \\ \text{subject to} \displaystyle\sum_{i=1}^{n} w_i = 1 \end{cases}. \tag{27}$$

Based on the closing price data, the authors obtain the return of each closing stock price and have the mean, variance, and covariance of return in Table 6 and Table 7, respectively.

Table 6 Mean and variance of return stocks

Stock	Mean	Variance
UNVR	0.00311	0.00127
BBRI	0.00033	0.00273
TLKM	0.00247	0.00166
ICBP	0.00047	0.00142
BMRI	0.000277	0.00309

Table 7 Covariance of return stocks

Stock	UNVR	BBRI	TLKM	ICBP	BMRI
UNVR	0.00127	0.00058	0.00053	0.00062	0.000906
BBRI	0.00058	0.00273	0.00091	0.00059	0.00235
TLKM	0.00053	0.00091	0.00166	0.00048	0.001101
ICBP	0.00062	0.00059	0.00048	0.00142	0.000807
BMRI	0.00091	0.00235	0.00110	0.00081	0.00309

For $n = 5$, the authors can change problem (27) into an unconstrained optimization problem by setting $w_5 = 1 - w_1 - w_2 - w_3 - w_4$ and using the data in Tables 6 and 7. So that, the authors have an unconstrained minimization problem as follows:

$$\begin{aligned} \min_{(w_1,\dots,w_4)\in\mathbb{R}^4} (0.36e - 3w_1 - 0.33e - 3w_2 - 0.38e - 3w_3 - 0.29e \\ - 3w_4 + 0.91e - 3)w_1 \\ + (-0.177e - 2w_1 + 0.38e - 3w_2 - 0.144e \\ - 2w_3 - 0.176e - 2w_4 + 0.235e - 2)w_2 \\ + (-0.57e - 3w_1 - 0.19e - 3w_2 + 0.56e \\ - 3w_3 - 0.62e - 3w_4 + 0.110e - 2)w_3 \\ + (-0.19e - 3w_1 - 0.22e - 3w_2 - 0.33e \\ - 3w_3 + 0.61e - 3w_4 + 0.81e - 3)w_4 \\ + (-0.218e - 2w_1 - 0.74e - 3w_2 - 0.199e \\ - 2w_3 - 0.228e - 2w_4 + 0.309e - 2)(1 - w_1 \\ - w_2 - w_3 - w_4. \end{aligned}$$

The authors use the hSNM method with an initial point $(w_1, \dots, w_4) = (0.1, 0.2, 0.3, 0.4)$ to solve the abovementioned problem. The results obtained are

$w_1 = 0.4022, w_2 = 0.2343, \quad w_3 = 0.2716, w_4 = 0.3139$

And $w_5 = -0.222$. By substituting the value of $w_1, \dots, w_5$ to (25) and (26), then $\sigma^2 = 0.00078$ and $\mu = 0.00208$. In conclusion, in this case, an investor who wants to invest with minimal risks can choose a portfolio consisting of five stocks with each of the following proportions: UNVR 40.22%, BBRI 23.43%, TLKM 27.16%, ICBP 31.39%, and BMRI -22.2% where the value of risk is 0.00078 and the expected return is 0.00208. Note that a negative sign in proportion indicates that investors need to short selling.

As another consideration, the application of the conjugate gradient method in portfolio selection can be seen in [16, 36-38].

6. Conclusion

Modeling autism spectrum disorders using an optimization method is an area that needs to be explored by researchers in this area. This is because of the importance of this topic in the current research trend. Therefore, this study constructs an autistic optimization model using cases of autism spectrum disorder in Malaysia and proposes a Hybrid Conjugate Gradient (HCG) method to solve the model. The proposed method was further extended to solve unconstrained optimization problems and an application problem in portfolio selection. The authors showed that the proposed method satisfies the sufficient descent condition under exact line search and established the convergence prove under some suitable conditions. Preliminary numerical results show that the new hybrid method is efficient and promising and can find wider applications in other fields. This work is limited to solving optimization models and parameterized autistic models using exact line search. However, researchers in this area can try solving the proposed method using other line searches such as the strong-Wolfe and Standard-Wolfe line search techniques. In addition, future studies can combine the proposed method with a feed-forward neural network for improving the training process and produce efficient training multilayer algorithms. The time needed for training neural networks will be reduced whenever the training samples are massive.

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