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Optimal Design of a Revised Double Sampling $\bar{X}$ Chart Based on Median Run Length

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Abstract: In process control, it is very important to have a tool that is able to detect small shifts of a process mean. The revised double sampling $\bar{X}$ chart is more effective than a standard Shewhart chart in detecting small to moderate shifts of a process mean. To optimize this type of chart, the average run length is widely used because of its simplicity and consistency. Nevertheless, the skewness of the run length distribution changes along with the process mean shift. Furthermore, its average is confusing and not necessarily a good representation of control chart performance. Because this distribution is highly right-skewed, especially when the shift is small, it is argued that its median provides a more intuitive and fair representation of the central tendency of the distribution. Based on an in-control median run length and average sample size, this paper investigates the behavior of control limit parameters to obtain the optimal parameters with a minimized out-of-control median run length. The parameters obtained were used to construct an example of this chart that was illustrated with real data.

Keywords: median run length, process mean shift, revised double sampling $\bar{X}$ chart, run length distribution.

基于中值游程长度的修正双抽样 X¯ 图的优化设计

摘要：在过程控制中，拥有一种能够检测过程均值微小变化的工具非常重要。修订后的双抽样 X¯ 图在检测过程均值的小到中等变化方面比标准休哈特图更有效。为了优化这种类型的图表，平均游程长度因其简单性和一致性而被广泛使用。然而，行程长度分布的偏度随着过程均值偏移而变化。此外，它的平均值令人困惑，不一定能很好地代表控制图性能。因为这个分布是高度右偏的，特别是当偏移很小时，有人认为它的中位数提供了分布集中趋势的更直观和公平的表示。本文基于控制中位数游程长度和平均样本大小，研究控制限参数的行为，以获得具有最小失控中位数游程长度的最佳参数。获得的参数用于构建该图表的示例，该示例用真实数据进行说明。

关键词：中值运行长度、过程均值偏移、修正双抽样 X¯ 图、运行长度分布。

1. Introduction

Statistical process control (SPC) is a collection of powerful problem-solving tools to improve process capability and achieve process stability [1]. Here, a control chart is one of the most useful techniques to reduce variability in key parameters and produce conforming products. The Shewhart $\bar{X}$ chart is a popular chart that is extensively used to detect large process mean shifts in industrial applications. However, the weakness of this chart is that it is less sensitive in detecting small and moderate shifts of a process mean.

To overcome this problem, Daudin [2] proposed a double sampling (DS)-$\bar{X}$ chart, which is a modified Shewhart $\bar{X}$ chart that incorporates double-sampling plans. This chart combines the ideas of a variable

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sampling interval (VSI) and variable sample size (VSS). Unlike a VSI, in the DS procedure, two successive samples are taken without any intervening time. It means that the first and second samples of the DS chart are taken from the same population.

Daudin [2] proposed an optimization model to minimize the in-control average sample size (ASS_0), while Irianto and Shinozaki [3] constructed an optimization model to minimize the out-of-control average run length (ARL_1). Many researchers are rapidly developing DS methods because they outperform other Shewhart, exponentially weighted moving average, cumulative sum, VSI, and VSS charts [4], [5].

Furthermore, Irianto [6] proposed a revision of the DS-$\bar{\boldsymbol{X}}$ chart by eliminating the out-of-control limit for the first stage of sampling. As the result, the out-of-control process can only be determined after the second stage of sampling. This elimination was carried out because the optimization proposed by Daudin [2] produced an out-of-control limit after a first sample as high as 3.6 standard deviations. In many manufacturing companies, a high out-of-control limit is meaningless because the opportunities for an out-of-control condition are very small. Consequently, based on its power, the revised DS-$\bar{\boldsymbol{X}}$ chart turns out to be more efficient than the original one.

The ARL has been commonly used as a single measure of a chart's performance. However, sole dependence on the ARL is potentially confusing and has been criticized by some researchers [7], [8], [9], [10]. *Because the skewness of the run length (RL) distribution changes along with the process mean shift*, an interpretation based on ARL alone could be misleading in evaluating a control chart's performance. However, the median run length (MRL), as an alternative, is a more accurate measure of a chart's performance because it is less affected by the skewness of the RL distribution [11], [12], [13].

For example, if the revised DS-$\bar{\boldsymbol{X}}$ chart with the selected parameters has an in-control ARL (ARL_0) of 370 and the in-control MRL (MRL_0) is 256, this means that about 63% of all the RLs were less than 370, while about 50% of all the RLs were less than 256. In other words, a practitioner can claim that a false alarm will occur by the 256th sample in half of the time. Moreover, when the revised DS-$\bar{\boldsymbol{X}}$ chart with the parameters stated above has an out-of-control MRL (MRL_1) of 19 at shift $\delta = 0.4$, it means that for this particular shift, there is a 50% chance that an out-of-control signal will be produced no later than the 19th sample.

The MRL is considered an alternative performance criterion to design control charts, as mentioned in the recent literature. For example, Teoh et al. developed an optimal design of MRL-based DS-$\bar{X}$ control charts in the case of known and estimated parameters [14, 15] and investigated the optimal design of an MRL-based VSS-$\bar{X}$ chart [16]. Furthermore, You et al. [17] and Qiao et al. [18] also contributed work in this area.

As mentioned, Teoh et al. [15] developed the optimal design of an MRL-based *DS*-$\bar{X}$ chart by Daudin [2] (Fig. 1); however, this paper proposes an optimal design of an MRL-based revised DS-$\bar{X}$ chart by Irianto [6] (Fig. 2). As previously explained, based on Irianto [6], the revised *DS*-$\bar{X}$ chart uses two control limits by eliminating the out-of-control limits for the first stage of the DS-$\bar{X}$ chart. Therefore, this paper proposes a procedure to estimate the control limit parameters (i.e., L_1 and L_2) of the revised *DS*-$\bar{X}$ chart (Fig. 2) by minimizing the MRL. Furthermore, the values of the optimal parameters obtained are presented in tabular form, so that they can be used as a reference by practitioners. Additionally, an application of the revised *DS*-$\bar{X}$ chart is provided in a case study.

This paper is organized as follows. In Section 2, the revised DS-$\bar{X}$ chart's procedure and its RL properties are briefly introduced. In Section 3, the main contributions of this paper are presented, which are to propose an optimization design for the revised DS-$\bar{X}$ chart by minimizing the MRL and to provide an optimal combination for the specific in-control MRL (MRL_0) and ASS_0. The application of the optimal revised DS-$\bar{X}$ chart based on the MRL is illustrated by a case study in Section 4 through a resampling dataset from a heavy equipment company. Finally, conclusions are drawn in the last section.

2. The Run Length (RL) Properties of the Revised DS-$\bar{X}$ Chart

Assume that the quality characteristics X are independently and identically distributed by normal $N(\mu_0, {\sigma_0}^2)$ random variables, where μ_0 and ${\sigma_0}^2$ are the in-control process mean and variance, respectively. Let $I_1 = [-L_1, L_1]$, $I_2 = \mathbb{R} - I_1 = (-\infty, -L_1) \cup (L_1, +\infty)$, $I_3 = [-L_2, L_2]$, and $I_4 = (-\infty, -L_2) \cup (L_2, +\infty)$. Here, $\mathbb{R}$ is a set of real numbers, $L_1 > 0$ is the warning limit based on the first stage and $L_2 > 0$ is the control limit based on the second stage. Referring to Fig. 1, the procedure of the Revised DS-$\bar{X}$ chart is as follows [6]:

(1) Given n, n_1, and n_2, then determine L_1 and L_2.

(2) Take the first sample of size n_1, X_{1j}, for $j = 1,2,\dots,n_1$ and the second sample of size n_2, X_{2j}, for $j = 1,2,\dots,n_1$ from process with mean value μ_0 and standard deviation σ_0.

(3) Calculate the sample mean $\bar{X}_1 = \sum_{j=1}^{n_1} X_{1j}/n_1$. Then calculate $Z_1 = (\bar{X}_1 - \mu_0)\sqrt{n_1}/\sigma_0$.

(4) If $Z_1 \in I_1$, the process is considered to be in-control, but if $Z_1 \in I_2$, then observe the second sample.

(5) Calculate the second sample mean $\bar{X}_2 = \sum_{j=1}^{n_1} X_{2j}/n_2$ and the total (combined) sample mean, $\bar{X} = (n_1\bar{X}_1 + n_2\bar{X}_2)/(n_1 + n_2)$. Then calculate $Z = (\bar{X} - \mu_0)\sqrt{n_1 + n_2}/\sigma_0$.

(6) If $Z \in I_3$, the process is considered to be in-control. Otherwise the process is considered to be out-of-control.

Let RL be the run length of the Revised DS-$\bar{X}$ chart. Here, run length is the number of samples that must be observed until the first out-of-control signal occurs. Montgomery [1] stated that the RL distribution of a Shewhart chart is geometric when the plotted statistics are independently and identically distributed random variables and the control limits are known constants. Since the Revised DS-$\bar{X}$ chart could be viewed as a two-stage Shewhart-$\bar{X}$ chart, thus, all the RL properties of the Revised DS-$\bar{X}$ chart can be characterized by geometric distribution.

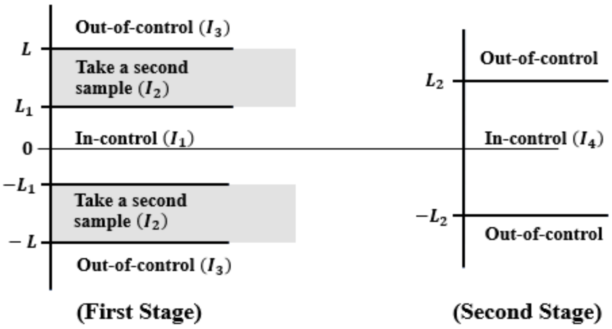

Fig. 1 Graphical procedure of the *DS*-$\bar{X}$ chart. A second sample will be observed when the statistics value plotted in interval I_2

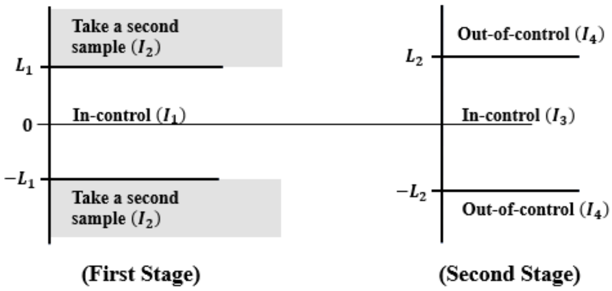

Fig. 2 Graphical procedure of the Revised DS-$\bar{X}$ chart. A second sample will be observed when the statistics value plotted in interval I_2

Hence, the cumulative distribution function (CDF) of RL, i.e., $F_{RL}(\ell)$, is equal to

$$F_{RL}(\ell) = P(RL \le \ell) = 1 - {p_a}^{\ell}, \quad (1)$$

where $\ell \in \{1,2,3\,...\}$ and $p_a = p_{a1} + p_{a2}$ is the probability in which the process is considered as in-control. Here, p_{ak} be the probability in which the process is considered as in-control at stage k with $k \in \{1,2\}$ where

$$p_{a1} = P(Z_1 \in I_1|\delta) = \Phi\left(L_1 + \delta\sqrt{n_1}\right) - \Phi\left(-L_1 + \delta\sqrt{n_1}\right) \quad (2)$$

and

$$p_{a2} = P(Z_1 \in I_2 \text{and } Z \in I_3|\delta) = (1 - p_{a1})\{\Phi(L_2 + \delta\sqrt{n_1 + n_2}) - \Phi(-L_2 + \delta\sqrt{n_1 + n_2})\} \quad (3)$$

where $\delta = |\mu_1 - \mu_0|/\sigma_0$ is the magnitude of the standardized mean shift with the out-of-control mean μ_1, and $\Phi(.)$ is the CDF of the standard normal random variable.

According to Equation (1), then the MRL of the Revised DS-$\bar{X}$ chart is equal to [19, 20].

$$P(RL \le MRL - 1) \le 0.5 \text{ and } P(RL \le MRL) > 0.5. \quad (4)$$

Based on the Revised DS-$\bar{X}$ chart procedures, Irianto [6] showed that the average sample size (ASS) at each sampling time is equal to

$$ASS = n_1 + n_2 p_2 \quad (5)$$

where

$$p_2 = P(Z_1 \in I_2|\delta) = 1 - p_{a1}. \quad (6)$$

3. Optimal Design of MRL-Based Revised *DS*-$\bar{X}$ Chart

In this section, an optimal design of the Revised *DS*-$\bar{X}$ chart is proposed. MRL value was used to evaluate this chart's performance. Suppose the out-of-control MRL is called MRL_1 and the out-of-control ASS is called ASS_1, the proposed optimization is to minimize $MRL_1(\delta_{\text{opt}})$. Here, δ_{opt} represents the size of a process mean shift, for which an immediate detection is required.

A control chart is considered to outperform its competitors if it has the smallest MRL_1 value, when MRL_0, ASS_0, and δ are fixed. Similarly, the Revised DS-$\bar{X}$ chart for optimally detecting the desired shift is obtained when the optimal parameters giving the lowest MRL_1 are identified from all the possible (n_1, n_2), and (L_1, L_2) combinations. The model of optimization is defined as follows:

$$\min_{n_1, n_2, L_1, L_2} MRL_1(\delta_{\text{opt}}) \quad (7)$$

Subject to

i) $MRL_0 = \tau$, where τ is the expected in-control MRL;

ii) $ASS_0 = n$, where n is the expected in-control ASS; and

iii) $1 \le n_1 < n < n_1 + n_2 \le n_{max}$, where n_{max} is the upper bound of $n_1 + n_2$.

$n_{max} = 15$ is set in this paper. The rationale behind this is that small and moderate sample sizes have been adopted in the industry.

The optimization of Equation (7) based on Constraints (i)–(iii) is carried out for finding the optimal combination of (n_1, n_2) and (L_1, L_2) using the following procedure:

(a) Set the desired τ, n, n_{max}, and δ_{opt} values.

(b) For each (n_1, n_2) pair selected based on Constraint (*iii*), L_1 can be obtained using Constraint (*ii*) and Equation (5), that is,

$$L_1 = \Phi^{-1}\left(\frac{n_1+2n_2-n}{2n_2}\right), \quad (8)$$

where $\Phi^{-1}(.)$ is the inverse CDF of the standard normal.

(c) For the fixed L_1, the value of L_2 is calculated using Constraint (i), Equation (2), and Equation (3), that is,

$$L_2 = \Phi^{-1}\left(1 - \frac{\alpha}{4\{1-\Phi(L_1)\}}\right) \quad (9)$$

Consequently, from step (b) and (c), a set of all possible combinations (n_1, n_2) and (L_1, L_2) that satisfy Constraints (i)–(iii) when $\delta_{opt} = 0$ are obtained,

(d) For any out-of-control condition ($\delta_{opt} \neq 0$), find the optimal combinations (n_1, n_2) and (L_1, L_2) that minimize $MRL_1(\delta_{\mathrm{opt}})$ from the set of parameter combinations found in step (b) and (c). Since MRL is an integer as the RL is a discrete random variable, there may exist several optimal combinations (n_1, n_2) and (L_1, L_2) for which the MRL_1 value is minimum at a specific $\delta_{opt} \neq 0$.

(e) In such a case, the optimal combinations of (n_1, n_2) and (L_1, L_2) having the smallest value of ASS_1 are preferred.

Note that, mathematically, the optimization procedure to estimate the control limit parameters of the Revised *DS*-$\bar{X}$ chart is said to be more simple than that of the *DS*-$\bar{X}$ chart because of MRL_0 and ASS_0 values being fixed, given the pair of sample sizes (n_1, n_2). The unique value of (L_1, L_2) will be obtained on the Revised *DS*-$\bar{X}$ chart. While, on the *DS*-$\bar{X}$ chart, an optimization procedure is needed to estimate the three parameters of the control limit, so there are several possible combinations for the values of (L_1, L, L_2). Consequently, in terms of calculation time to perform optimization results, the Revised *DS*-$\bar{X}$ chart is more efficient than the *DS*-$\bar{X}$chart. Nevertheless, the weakness of the optimal design of the Revised *DS*-$\bar{X}$ chart is that this chart could not detect the sample as an out-of-control signal at the first stage, especially if there are samples that are extremely not good in process control, because the out-of-control limits have been eliminated for the first stage.

The optimal combinations (n_1, n_2), (L_1, L_2) and their corresponding (MRL_1, ASS_1) values for different combinations of MRL_0, ASS_0, and δ_{opt} are shown in the first, second, and third rows of each cell in Table 1. These new optimal parameters facilitate the implementation of the Revised DS-$\bar{X}$ chart for practitioners, for example, considering a continuous manufacturing process in which an immediate detection is desired at a shift with $\delta_{opt} = 0.8$. If $MRL_0 = 250$ and $ASS_0 = 3$ are selected, Table 1 suggests using ($n_1 = 2, n_2 = 8$) and ($L_1 = 1.5341, L_2 = 2.2878$) as the optimal parameters to detect such a shift. Based on the suggested parameters, it means that 50% of the time, a shift with $\delta_{opt} = 0.8$ is detected no later than the third sample (i.e., $MRL_1 = 3$) and the chart requirements 4.7793 observations (i.e., $ASS_1 = 4.7793$) on the average to detect such a shift.

Additionally, as expected, Fig. 3 shows that the sensitivity of the Revised DS-$\bar{X}$ chart increases in detecting a certain shift as ASS_0 increases, especially for the small shift. However, there is a note that this chart has almost the same minimum MRL_1 value regardless of the ASS_0 used for moderate and large shifts ($\delta_{opt} \geq 1$). These are some properties of the Revised DS-$\bar{X}$ chart optimized based on MRL.

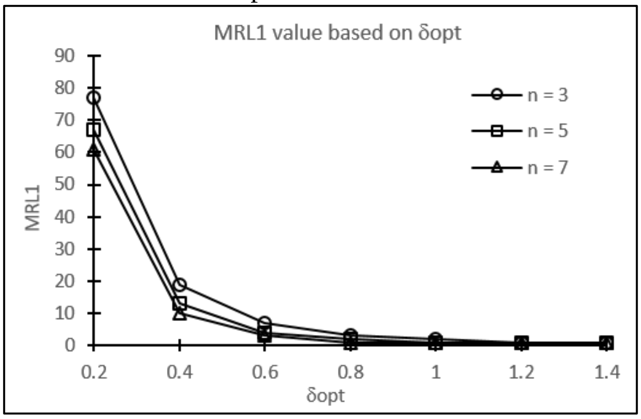

Fig. 3 MRL_1 values of the Revised DS-$\bar{X}$ chart based on shift size δ_{opt} for $MRL_0 = 250, ASS_0 \in \{3,5,7\}$. MRL_1 values of this chart are almost the same for moderate and large shifts ($\delta_{opt} \geq 1$)

4. Case Study

In this section, the optimal design of the Revised DS-$\bar{X}$ chart is illustrated by applying it to real industrial data associated with the sprocket packaging material provided by a heavy equipment company. Sprocket packaging is a process that uses bolts to place sprocket gear onto a sprocket hole.

Here, for the confidentiality of the company's data, the resampled dataset for the torque measurement (in kg/m) of bolts in sprocket packaging are considered with an in-control mean and standard deviation of the torque of $\mu_0 = 52.2$ kg/m and $\sigma_0 = 4.5$, respectively. The resampled dataset comprised 30 samples of size 9 ($n_1 = 3$ and $n_2 = 6$); measurements for the first 26 sampled times (t = 1 to 26) were generated based on an in-control condition, while measurements for the subsequent sampled times (t = 27 to 30) were generated with $\delta_{opt} = 1.0$. The dataset used in this example supports the findings that are available from the corresponding author upon request.

According to the dataset, process monitoring was conducted using the Shewhart-$\bar{X}$ chart with $n = 5$ (the first five of nine were selected for each sample). After that, it was performed using the Revised DS-$\bar{X}$ chart. From Table 1, for $n = 5$ and $\delta_{opt} = 1.0$, the optimal chart parameters for the Revised DS-$\bar{X}$ chart, which means $MRL_0 = 250$ and $ASS_0 = 5$, are $(n_1, n_2) = (3, 6)$ and $(L_1, L_2) = (0.9674, 2.6394)$. These optimal chart parameters were used for monitoring the process. The results of utilizing process monitoring, using the Shewhart-$\bar{X}$ chart and the Revised DS-$\bar{X}$ chart, are plotted in Fig. 4.

Fig. 4 (a) shows that the process is in control, though there was a mean shift of $\delta_{opt} = 1.0$ that started from the sampled time of t = 26. In this case, Shewhart-$\bar{X}$ chart is not sensitive in detecting the occurrence of the mean shift. Meanwhile, Fig. 4 (b) shows that the Revised DS-$\bar{X}$ chart detected the out-of-control signal at the sampled time of t = 30 as Z_{30} = 2.7542 > L_2 = 2.6394. Results describe that based on MRL, the Revised DS-$\bar{X}$ chart remains better than Shewhart-$\bar{X}$ in detecting small shift mean.

Table 1 Optimal parameter combinations (n_1, n_2), (L_1, L_2) and (MRL_1, ASS_1) values of the Revised DS-$\bar{X}$ chart when $MRL_0 = 250$, $ASS_0 \in \{3,5,7\}$ and $\delta_{opt} \in \{0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4\}$

δ_{opt}		ASS_0		
		3	**5**	**7**
0.2	(n_1, n_2)	(1, 14)	(1, 14)	(1, 14)
	(L_1, L_2)	(1.4652, 2.3381)	(1.0676, 2.5867)	(0.7916, 2.7234)
	(MRL_1, ASS_1)	(77, 3.1116)	(67, 5.1340)	(61, 7.1283)
0.4	(n_1, n_2)	(1, 13)	(4, 11)	(6, 9)
	(L_1, L_2)	(1.4261, 2.3657)	(1.6906, 2.1641)	(1.5932, 2.2427)
	(MRL_1, ASS_1)	(19, 3.4225)	(13, 6.1124)	(10, 8.4736)
0.6	(n_1, n_2)	(1, 12)	(2, 13)	(6, 7)
	(L_1, L_2)	(1.3830, 2.3952)	(1.1984, 2.5122)	(1.4652, 2.3381)
	(MRL_1, ASS_1)	(7, 4.1198)	(4, 6.9863)	(3, 9.5241)
0.8	(n_1, n_2)	(2, 8)	(1, 14)	(6, 8)
	(L_1, L_2)	(1.5341, 2.2878)	(1.0676, 2.5867)	(1.5341, 2.2878)
	(MRL_1, ASS_1)	(3, 4.7793)	(2, 6.9560)	(1, 11.3199)
1.0	(n_1, n_2)	(1, 7)	(3, 6)	(1, 10)
	(L_1, L_2)	(1.0676, 2.5867)	(0.9674, 2.6394)	(0.5244, 2.8328)
	(MRL_1, ASS_1)	(2, 4.4468)	(1, 7.6874)	(1, 8.4652)
1.2	(n_1, n_2)	(2, 5)	(1, 6)	(1, 7)
	(L_1, L_2)	(1.2816, 2.4613)	(0.4307, 2.8663)	(0.1800, 2.9449)
	(MRL_1, ASS_1)	(1, 5.3128)	(1, 5.9836)	(1, 7.5095)
1.4	(n_1, n_2)	(1, 4)	(1, 5)	(1, 7)
	(L_1, L_2)	(0.6745, 2.7740)	(0.2533, 2.9235)	(0.1800, 2.9449)
	(MRL_1, ASS_1)	(1, 4.1398)	(1, 5.6168)	(1, 7.6207)

Note: L_1 and L_2 are obtained using equations (8) and (9), respectively

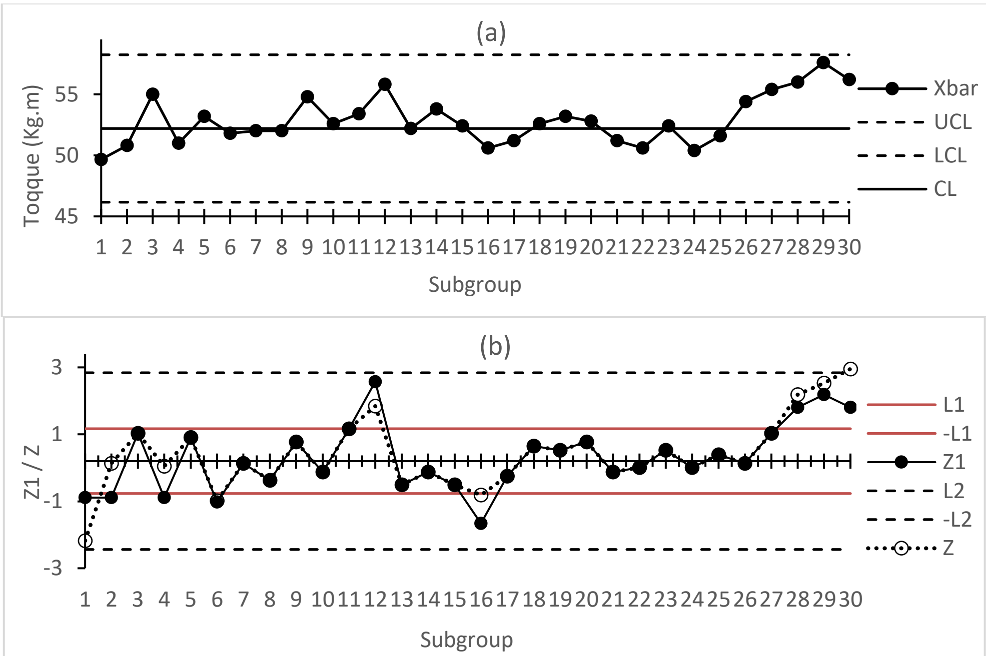

Fig. 4 Monitoring process for torque inspection: (a) Monitoring using Shewhart $\bar{X}$ chart, (b) Monitoring using Revised DS-$\bar{X}$ chart detecting the out-of-control signal at sampling time t = 30

5. Conclusion

In practice, understanding the control chart in use is very important for engineers or practitioners. MRL is easier to understand and interpret than ARL because of the distribution of run-length changes *along with the process mean shift***.** The performance of MRL and ARL are equal when the process has a large shift; the sample size will take a significant role when the shift becomes smaller. In addition, MRL provides practitioners with more credible information. For a process with a large shift ($\delta_{opt} \geq 1$), the obtained MRL values are equalized although the sample size is various. When the shift is moderate to small, the larger sample size will give the smaller MRL. Thus, the out-of-control signal can be detected earlier. Moreover, based on MRL, the specific optimal parameters of the Revised DS-$\bar{X}$ chart are provided to assist practitioners in a real application.

As a limitation of the study, this paper only discusses the optimal design for the known parameter case, meaning that it is assumed that the process mean is known. Further research can be developed regarding the optimal design for the estimated parameter case.

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References

[1] MONTGOMERY D. C. *Introduction to Statistical Quality Control*. 7th ed. John Wiley & Sons, New York, 2013.

[2] DAUDIN J. J. Double sampling $\bar{X}$ charts. *Journal of Quality Technology*, 1992, 24(2): 78–87. https://doi.org/10.1080/00224065.1992.12015231

[3] IRIANTO D., & SHINOZAKI N. An Optimal Double Sampling $\bar{X}$ Control Chart. *International Journal of Industrial Engineering: Theory, Applications, and Practice*, 1998, 5(3): 226-234.

[4] COSTA A. F. B. $\bar{X}$ charts with variable sample size. *Journal of Quality Technology*, 1994, 26(3): 155-163. https://doi.org/10.1080/00224065.1994.11979523

[5] HE D., & GRIGORYAN A. Construction of double sampling *s*-control charts for agile manufacturing. *Quality and Reliability Engineering International*, 2002, 18(4): 343-355. https://doi.org/10.1002/qre.466

[6] IRIANTO D. A Revised Double Sampling Control Chart. Proceedings of the 14th Asia Pacific Quality Organization International Conference, 2008.

[7] GOLOSNOY V., & SCHMID W. EWMA control charts for monitoring optimal portfolio weights. *Sequential Analysis*, 2007, 26(2): 195–224. https://doi.org/10.1080/07474940701247099

[8] DAS N. A comparison study of three non-parametric control charts to detect shift in location parameters. *International Journal of Advanced Manufacturing Technology*, 2009, 41: 799–807. https://doi.org/10.1007/s00170-008-1524-3

[9] LEE M. H., & KHOO M. B. C. Optimal designs of multivariate synthetic |S| control chart based on median run length. *Communications in Statistics - Theory and Methods*, 2017, 46(6): 3034–3053. https://doi.org/10.1080/03610926.2015.1048884

[10] TANG A., CASTAGLIOLA P., SUN J., and HU X. Optimal design of the adaptive EWMA chart for the mean based on median run length and expected median run length. *Quality Technology & Quantitative Management*, 2019, 16(4): 439–458. https://doi.org/10.1080/16843703.2018.1460908

[11] MARAVELAKIS P. E., PANARETOS J., and PSARAKIS S. An examination of the robustness to non-normality of the EWMA control charts for the dispersion. *Communications in Statistics – Simulation and Computation*, 2005, 34(4): 1069–1079. https://doi.org/10.1080/03610910500308719

[12] CHAKRABORTI S. Run length distribution and percentiles: The Shewhart X chart with unknown parameters. *Quality Engineering*, 2007, 19(2): 119-127. https://doi.org/10.1080/08982110701276653

[13] TEOH W. L., KHOO M. B. C., CASTAGLIOLA P., and LEE, M. H. The exact run length distributionand design of the Shewhart chart with estimated parameters based on median run length. *Communications in Statistics - Simulation and Computation*, 2016, 45(6): 2081-2103. https://doi.org/10.1080/03610918.2014.889158

[14] TEOH W. L., KHOO M. B. C., and TEH S. Y. Optimal Design of the Median Run Length Based Double Sampling $\bar{X}$ Chart for Minimizing the Average Sample Size. *PLoS ONE*, 2013, 8(7): e68580. https://doi.org/10.1371/journal.pone.0068580

[15] TEOH W. L., KHOO M. B. C., CASTAGLIOLA P., and CHAKRABORTI S. Optimal Design of the Double Sampling $\bar{X}$ Chart with estimated parameters based on median run length. *Computer & Industrial Engineering*, 2014, 67: 104-115. https://doi.org/10.1016/j.cie.2013.11.001

[16] TEOH W. L., CHONG J. K., KHOO M. B. C., and YEONG W.C. Optimal designs of the variable sample size $\bar{X}$ chart based on median run length and expected median run length. *Quality and Reliability Engineering International*, 2017, 33(1): 121–134. https://doi.org/10.1002/qre.1994

[17] YOU H. W., KHOO M. B. C., CASTAGLIOLA P., and QU L. Optimal exponentially weighted moving average $\bar{X}$ charts with estimated parameters based on median run length and expected median run length. *International Journal of Production Research*, 2016, 54(17): 5073-5094. https://doi.org/10.1080/00207543.2016.1145820

[18] QIAO Y. L., SUN J., CASTAGLIOLA P., and HU X. Optimal design of one-sided exponential EWMA charts based on median run length and expected median run length. *Communications in Statistics - Theory and Methods*, 2020. https://doi.org/10.1080/03610926.2020.1782937

[19] GAN F.F. An optimal design of EWMA control charts based on median run length. *Journal of Statistical Computation and Simulation*, 1993, 45(3-4): 169-184. https://doi.org/10.1080/00949659308811479

[20] GAN F. F. An optimal design of cumulative sum control chart based on median run length. *Communications in Statistics – Simulation and Computation*, 1994, 23(2): 485-503. https://doi.org/10.1080/03610919408813183

参考文:

[1] MONTGOMERY D. C. 统计质量控制导论。第 7 版。约翰威利父子公司，纽约，2013。

[2] DAUDIN J. J. 双采样 X¯图表。质量技术杂志，1992，24（2）：78-87。https://doi.org/10.1080/00224065.1992.12015231

[3] IRIANTO D., & SHINOZAKI N. 最佳双采样 X¯控制图。国际工业工程杂志：理论、应用和实践，1998，5(3)：226-234。

[4] COSTA A. F. B. (X)¯图表，样本大小可变。质量技术杂志，1994，26(3): 155-163。https://doi.org/10.1080/00224065.1994.11979523

[5] HE D., & GRIGORYAN A. 为敏捷制造构建双抽样秒控制图。国际质量与可靠性工程，2002，18(4)：343-355。https://doi.org/10.1002/qre.466

[6] IRIANTO D. 修订后的双抽样控制图。2008 年第 14 届亚太质量组织国际会议论文集。

[7] GOLOSNOY V., & SCHMID W. EWMA 控制图，用于监控最佳投资组合权重。顺序分析，2007，26（2）：195-224。https://doi.org/10.1080/07474940701247099

[8] DAS N. 三个非参数控制图的比较研究，以检测位置参数的变化。国际先进制造技术杂志，2009，41：799-807。https://doi.org/10.1007/s00170-008-1524-3

[9] LEE M. H., & KHOO M. B. C. 多元合成的最优设计 |S|基于中值游程长度的控制图。统计通讯 - 理论与方法，2017，46（6）：3034–3053。https://doi.org/10.1080/03610926.2015.1048884

[10] TANG A., CASTAGLIOLA P., SUN J., 和 HU X. 基于中位数游程长度和预期中位数游程长度的均值自适应 EWMA 图表的优化设计。质量技术与定量管理，2019，16(4)：439–458。https://doi.org/10.1080/16843703.2018.1460908

[11] MARAVELAKIS P. E.、PANARETOS J. 和 PSARAKIS S. 对 EWMA 控制图离散性非正态性稳健性的检验。统计通讯——模拟与计算，2005，34(4)：1069–1079。https://doi.org/10.1080/03610910500308719

[12] CHAKRABORTI S. 运行长度分布和百分位数：具有未知参数的休哈特 X 图。质量工程，2007，19(2): 119-127。https://doi.org/10.1080/08982110701276653

[13] TEOH W. L., KHOO M. B. C., CASTAGLIOLA P., 和 LEE, M. H. 休哈特图的精确游程分布和设计以及基于中位数游程长度的估计参数。统计通信 - 模拟与计算，2016, 45(6): 2081-2103。https://doi.org/10.1080/03610918.2014.889158

[14] TEOH W. L.、KHOO M. B. C. 和 TEH S. Y. 用于最小化平均样本量的基于中值游程长度的双采样 X¯图表的优化设计。公共图书馆一号，2013，8(7): e68580。https://doi.org/10.1371/journal.pone.0068580

[15] TEOH W. L.、KHOO M. B. C.、CASTAGLIOLA P. 和 CHAKRABORTI S. 双抽样 X¯图表的优化设计，其估计参数基于中位数游程长度。计算机与工业工程，2014，67：104-115。https://doi.org/10.1016/j.cie.2013.11.001

[16] TEOH W. L.、CHONG J. K.、KHOO M. B. C. 和 YEONG W.C. 基于中位数游程长度和预期中位数游程长度的可变样本量 X¯图表的优化设计。国际质量与可靠性工程，2017，33(1)：121–134。https://doi.org/10.1002/qre.1994

[17] YOU H. W., KHOO M. B. C., CASTAGLIOLA P., 和 QU L. 最优指数加权移动平均 X¯图表，估计参数基于中位数游程长度和预期中位数游程长度。国际生产研究杂志，2016, 54(17): 5073-5094。https://doi.org/10.1080/00207543.2016.1145820

[18] QIAO Y. L., SUN J., CASTAGLIOLA P., 和 HU X. 基于中位数游程长度和预期中位数游程长度的单边指数 EWMA 图的优化设计。统计通讯 - 理论与方法，2020。https://doi.org/10.1080/03610926.2020.1782937

[19] 甘法夫基于中值游程长度的 EWMA 控制图的优化设计。统计计算与模拟杂志，1993，45(3-4)：169-184。https://doi.org/10.1080/00949659308811479

[20] GAN F. F. 基于中值游程长度的累积总和控制图的优化设计。统计通讯——模拟与计算，1994，23(2)：485-503。https://doi.org/10.1080/03610919408813183