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Development of Path Analysis Based on Nonparametric Regression on Vegetative Growth and Porang's Production Modeling

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Abstract: This study aims to obtain the properties of the spline estimator in the Nonparametric Regression-based Path Analysis using the PWLS approach. The authors test the hypothesis on each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach and obtain the optimal confidence interval for each connection between variables in Path Analysis based on Nonparametric Regression using the PWLS approach. Porang growth factors and environmental factors are still analyzed partially. This study used a multivariate approach to investigate factors that influence the growth of porang tuber production. Overall, the study was compiled using an observational research design to test the growth and production model design for porang. Furthermore, a random sampling method was carried out to observe paired quantitative data for porang growth factors, tuber size, environmental factors (vegetation, climate, and soil) in four agroforestry lands in East Java (Madiun, Nganjuk, Malang, and Bojonegoro). The research was conducted using the literature search method to obtain research data on the growth of porang tubers. The findings obtained were in the form of a review of the properties of the spline estimator in the Nonparametric Regression-based Path Analysis using the Penalized Weighted Least Square (PWLS) approach, hypothesis testing on each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach, as well as some findings regarding the optimal confidence interval on each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach. On the other hand, it is obtained theoretical testing of optimal confidence intervals for each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach. The originality of this paper is that the author develops a non-parametric regression-based path analysis that is robust with linearity assumptions, which are applied to the modeling of negative growth and porang production.

Keywords: Path Analysis, Spline, Nonparametric, Regression, Vegetative Growth, PWLS

基於非參數回歸的營養生長和 Porang 生產模型的路徑分析方法的發展

摘要：這項研究旨在使用 PWLS 方法獲得基於非參數回歸的路徑分析中的樣條估計的屬性，獲得基於 PWLS 方法的基於非參數回歸的路徑分析中變量之間的每個關係的假設檢驗，並獲得 PWLS 方法基於非參數回歸的路徑分析中變量之間每個關係的最佳置信區間。仍然對 porang 的生長因子和環境因子進行了部分分析。這項研究採用多變量方法研究影響紅毛芋產量增長的因素。總體而言，該研究是使用觀察性研究設計來編譯的，以測試 porang 的生長和生產模型設計。此外，進行了隨機抽樣方法，以觀察成對的定量數據，這些數據包括東爪哇（Madiun，Nganjuk，Malang 和 Bojonegoro）的四個農用林地的桔生長因子，塊莖大小，環境因子（植被，氣候和土壤）。這項研究是使用文獻檢索法進行的，以獲取有關紅毛芋塊莖生長的研究數據。所得結果以對形式的回顧，即使用懲罰加權最小二乘（PWLS）方法對基於非參數回歸的路徑分析中的樣條估計的屬性進行了評估，並對基於非參數回歸的路徑中變量之間的每個關係進行了假設檢驗使用 PWLS 方法進行分析，以及有關基於非參數回歸的路徑分析中變量之間每個關係的最佳置信區間的一些發現。另一方面，使用 PWLS 方法獲得了基於非參數回歸的路徑分析中變量之間每個關係的最佳置信區間的理論測試。本文的獨創性在於，作者開發了一種基於線性回歸的非參數回歸路徑分析，該路徑分析具有魯棒性，可用

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於負增長和 Porang 生產的建模。

關鍵字：路徑分析，樣條曲線，非參數，回歸，營養生長，PWLS

1. Introduction

Porang or iles-iles (*Amorphophallus muelleri* Blume sin. *A. blumei* (Scott.) Engler sin. *A. oncophyllus* Prain), *A. rivieri*, *A. campanulatus* (suweg), *A. variabilis* and *A. albus* belong to the *Araceae* family. *Amorphophallus spp.* has been known in Asia and America as a producer of tubers (corm) of high economic value. In Asia, apart from Japan and Thailand, Indonesia is an exporting country for raw materials from porang tubers to various countries, including Australia and the European Union.

The porang tubers exported so far come from plants that grow wild under the production forests of Perum Perhutani in East Java, Central Java, and West Java. Porang production in Bogor with rainfall > 3000mm to^{-1} was observed to be higher than in Blitar or Blora, which each have a climate type according to Schmidt-Ferguson B and C [1]. Rainfall and climate type were observed to correlate with tuber production of porang. According to Jansen et al. [11], [23], [24], to obtain good growth, porang must live in the shade of 50-60% [15].

Soil conditions also influence porang growth. Porang grows well in forests with light-textured soils, namely in sandy clay conditions, loose structures and rich in nutrients ([12]; [14]), good drainage, high soil organic matter content, and a soil pH range of 6.0 - 7.5 [11].

Porang tuber production is also very dependent on the life cycle stages for 38-43 months. Sumarwoto [15] has described the breakdown of time in one life cycle as follows: seeding time of 1.5-2 months, growth in nursery 1.5-2 months, vegetative phase in years 1 to 3, each for five months and six months, dormant in the 1st to 3rd year for four months, flowering until the fruit is ripe 8-9 months.

Based on this description, porang growth and environmental factors are still partially analyzed (univariate). In fact, in nature, various internal and external plant variables can influence each other directly or indirectly in controlling the amount of porang tuber production. We hoped that the multivariate statistical modeling of porang tuber growth and production will meet the need for information on how much internal plant factors (age and growth of porang) and external factors (surrounding vegetation, climate, and soil) simultaneously affect porang tuber production per ha. It will underlie further research regarding the quality of porang tubers, especially related to glucomannan and Ca-oxalate levels.

Arisoesilaningsih et al. [3] in the Research Staff Grant I-MHERE study found that the growth and production of porang tubers were influenced by variations in plant age, vegetation conditions, and soil, and agroforestry climatic conditions. This study uses path analysis to determine the multivariate relationship between variables.

Path analysis is the development of a parametric regression analysis that has more than one equation and between equations is presented structurally, where the structural equation is characterized by at least one exogenous variable (X), at least one endogenous intervening variable (Y), and one pure endogenous variable (Z). According to Solimun [13], the main assumption in path analysis is that the relationship between variables is linearity. The assumption of linearity sometimes cannot be fulfilled in several studies, such as the results found by Arisoesilaningsih et al. [3]. They stated that the relationship between growth and production of porang tubers is influenced by variations in plant age, vegetation conditions, soil conditions, and agroforestry climatic conditions that do not meet the linearity assumption. In the application, it isn't effortless to get these functions precisely. Even symptoms often show that the data obtained does not offer a relationship pattern that is easy to describe. If the assumption of linearity is not fulfilled and nonlinearity is unknown, then one alternative that can be used is a non-parametric regression model.

The pattern of the relationship between responses and unknown predictors can be estimated using the Spline function approach ([6],[17],[5], [21], [22]), Local Polynomials [9], Kernel [10], Wavelets ([2], [18], [19], [20]), and the Fourier Series (Maliavin & Mancino, 2009). This study developed a non-parametric regression-based path analysis using a spline path estimator. The regression curve used is assumed to be smooth, in the sense that it is contained in certain function space, especially the Sobolev space, where *m* is the order of the spline polynomial. Fort the regression curve estimation, the optimization is used Weighted Least Square (WLS) or Penalized Weighted Least Square (PWLS) [4].

Based on the above phenomena, this study aims to develop a non-parametric regression-based path analysis robust in the assumption of linearity, which is applied to the modeling of vegetative growth and porang tuber production. This research is expected to obtain the properties of the spline estimator in the

Nonparametric Regression-based Path Analysis using the PWLS approach, to obtain hypothesis testing on each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach, and to obtain the optimal confidence interval for each connection between variables in Path Analysis based on Nonparametric Regression using the PWLS approach. The originality of this paper is that the author develops a non-parametric regression-based path analysis that is robust with linearity assumptions, which are applied to the modeling of negative growth and porang production.

2. Literature Review

2.1 Path Analysis

Sewall Wright developed path analysis in 1934. Sewall Wright developed this method to study the direct effects and indirect effects of several variables where some variables are seen as causes, and other variables are seen as effects [7]. Path analysis is also known as causing modeling because path analysis makes it possible to test the theoretical proportions of certain variables' cause and effect relationships.

According to Solimun [13], the following are the assumptions underlying path analysis: The relationship between variables must be linear and additive. The path diagram is the basis of path analysis, a procedure for empirical estimation of the strength of each relationship depicted in the path diagram [16]. Path diagrams are used to display both measured and unmeasured causal relationships graphically.

2.2 Non-parametric Regression Analysis

This study developed a non-parametric regression-based path analysis, specifically spline. Therefore, this research requires several supporting theories that can later be used to complete this research. Some of these theories are as follows:

1. Kernel Functions and Spline Functions and their properties. Kernel functions include Gaussian Kernel, Ephanicov Kernel, and Uniform Kernel. The Spline function will be applied to investigate the behavior of the Kernel and Spline mixture estimators for estimating multivariable semiparametric regression curves.
2. Non-parametric regression analysis is used as the basic theory in developing a mixture estimator of Spline and Kernel in multivariable semiparametric regression.
3. Optimization of Penalized Weighted Least Square (PWLS). The PWLS optimization concept is used to find the estimator form of a mixture of Kernel and Spline to estimate the regression curve, both in multivariable non-parametric regression and multivariable semiparametric regression.
4. The Hilbert Space Reproducing Kernel Method (HSRK) was used to solve the PLS optimization in multivariable and semiparametric multivariable regression models.
5. The basic concept of the Sobolev space is used as a primary function space in multivariable and semiparametric non-parametric regression analysis. The regression curve is assumed to fit in the Sobolev space.
6. The concept of the Generalized Cross-Validation (GCV) method developed by researchers on cross section data will be generalized into a function that will be used as a method for selecting optimal knot points and bandwidth parameters in the Spline and Kernel mixture estimators in multivariable semiparametric regression.

3. Methods and Materials

This research was conducted at the Statistics Laboratory of the Statistics Department, Brawijaya University, to develop statistical modeling theory. The Ecology Laboratory of the Department of Biology, Brawijaya University, to obtain an observational analysis (based on field data). Overall, the study was compiled using an observational research design to test the growth and production model design for porang tubers. The research was conducted using the literature search method to obtain research data on the growth of porang tubers.

Based on the literature study results, the factors observed were variations in the four locations. Furthermore, a random sampling method was carried out to keep paired quantitative data for porang growth factors, tuber size, environmental factors (vegetation, climate, and soil) in four agroforestry lands in East Java, namely four indicators a1-a4: KPH Madiun, Nganjuk, Malang, and Bojonegoro. At each location, observations of porang plant samples were done in four physiological age classes (four indicators u1-u4: years 1, 2, 3, and 4) and shading conditions (two indicators n1-n2: less than 40% shaded and more than 50 -60%). In the eight combinations of variations, three plant samples were observed to obtain 32 plant samples. Thus, a total of 128 plant samples will be kept. Observations were made at the end of the rainy season when vegetative growth could still be observed. In each plant sample, the observed increase was the three p1-p3 indicators: plant height (cm), leaf crown width (cm), and the number of leaf tubers (bulbil/frog). The porang genetic variation variable in one location is ignored. Therefore only one dominant variant is selected based on external morphology. The observed vegetation variables around the porang plant growth area were three v1-v3 indicators: shady tree stands, cover vegetation,% plant cover to estimate the competition factor between species.

4. Results and Discussion

4.1 Properties of Estimates

The first objective of the study is to obtain the properties of the spline estimator in the Nonparametric Regression-based Path Analysis using the Penalized Weighted Least Square (PWLS) approach.

4.2 Asymptotic properties

Many authors have developed several goodness estimator criteria. In non-parametric regression, it is often noted that the estimator's asymptotic behavior is based on a certain measure of goodness. If squared risk criteria are used as the goodness of the regression curve estimator, Eubank [8] concludes that the estimator will converge in terms of velocity, $n^{-\delta}$, $0<\delta<1$, in contrast to the parametric case which can achieve velocity n^{-1}under standard terms.

In this paper, we investigate the asymptotic nature of a weighted spline estimator f_λ based on the Integrated Mean Square Error (IMSE) criteria. Previously, the following assumptions were given.

Proof:

$$\begin{aligned}IMSE(\lambda) &= E\int_0^1 (f(t)-f_\lambda(t))^2 w(t)dt\\ &= \int_0^1 E(f(t)-Ef_\lambda(t))^2 w(t)dt + \int_0^1 E(Ef(t)-f_\lambda(t))^2 w(t)dt\\ &\quad +2\int_0^1 (f(t)-Ef_\lambda(t))(Ef(t)-Ef_\lambda(t))w(t)dt\\ &= b^2(\lambda)+V(\lambda)\end{aligned} \quad (1)$$

Assumption:$(\boldsymbol{A0}).\ \boldsymbol{t_j}=\frac{\boldsymbol{2j-1}}{\boldsymbol{2n}}, \boldsymbol{j}=\boldsymbol{1},\boldsymbol{2},\ldots,\boldsymbol{n}$

Then, *IMSE (λ)* is composed of two terms, bias and variance. With $b^2(\lambda)=\int_0^1 E\big(f(t)-Ef_\lambda(t)\big)^2 w(t)dt$ and $V(t)=\int_0^1 E\big(Ef(t)-f_\lambda(t)\big)^2 w(t)dt$

The first step investigated the asymptotic properties of the quadratic bias component $b^2(\lambda)$.

Theorem 1

If $f_\lambda(t)$completion of minimizing the PLST:

$$n^{-1}\sum_{j=1}^n w_j\left(y_j-g(t_j)\right)^2+\lambda\int_0^1[g^{(m)}(t)]^2dt \quad (2)$$

then the solution is to minimize the PLST:

$$n^{-1}\sum_{j=1}^n w_j\left(f(t_j)-g(t_j)\right)^2+\lambda\int_0^1[g^{(m)}(t)]^2dt \quad (3)$$

$is g_\lambda*(t)=Ef_\lambda(t).$

Theorem 2

If (A0) applies, then $\underset{\sim}{b}^2(\lambda)\le\boldsymbol{O}(\lambda)$, $n\to\infty$.

Proof:

Suppose that$g_\lambda*(t)$the minimum completion:

$$\int_0^1\left(f(t)-g(t)\right)^2w(t)dt+\lambda\int_0^1\left[g^{(m)}(t)\right]^2dt$$

Because (A0), then for $n\to\infty$,

$$n^{-1}\sum_{j=1}^n w_j\left(f(t_j)-g(t_j)\right)^2\approx\int_0^1\left(f(t)-g(t)\right)^2w(t)dt$$

as a result, so$g_\lambda(t)=Ef_\lambda(t)\approx g_\lambda(t)$

So for every $g\in W_2^m[0,1]$,

Theorem 1 provides a solution to minimize the PLST:

$$n^{-1}\sum_{j=1}^n w_j\left(y_j-g(t_j)\right)^2+\lambda\int_0^1[g^{(m)}(t)]^2dt \quad (4)$$

can be written as:

$$\begin{aligned}f_\lambda(t)=\Big(&\varphi'(\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\boldsymbol{T})^{-1}\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\\ &+\psi'\boldsymbol{S}^{-1}\boldsymbol{W}\Big(\boldsymbol{I}\\ &-\boldsymbol{T}(\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\boldsymbol{T})^{-1}\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\Big)\Big)\underset{\sim}{y}\end{aligned}$$

By pairing $f(t_j)=y_j, j=1,2,\ldots,n$, the minimum completion:

$$n^{-1}\sum_{j=1}^n w_j\left(f(t_j)-g(t_j)\right)^2+\lambda\int_0^1[g^{(m)}(t)]^2dt$$

can be written as:

$$\begin{aligned}g_\lambda*(t)=\Big(&\varphi'(\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\boldsymbol{T})^{-1}\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\\ &+\psi'\boldsymbol{S}^{-1}\boldsymbol{W}\Big(\boldsymbol{I}\\ &-\boldsymbol{T}(\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\boldsymbol{T})^{-1}\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W}\Big)\Big)f(t)\end{aligned}$$

$=Ef_\lambda(t)$, with $f(t)=\left(f(t_1),\ldots,f(t_n)\right)'$

The following theorem presents the asymptotic properties of the quadratic bias component.

$$\begin{aligned}\underset{\sim}{b}^2(\lambda)&=\int_0^1 E\left(f(t)-Ef_\lambda(t)\right)^2w(t)dt\\ &\le\int_0^1 E\left(f(t)-Ef_\lambda(t)\right)^2w(t)dt+\lambda\int_0^1\left[g^{(m)}(t)\right]^2dt\end{aligned}$$

Remember that$Ef_\lambda(t)\approx g_\lambda(t)$, so obtained:

$$\underset{\sim}{b}^2(\lambda)\le\int_0^1 E\left(f(t)-g_\lambda(t)\right)^2w(t)dt+\lambda\int_0^1\left[g^{(m)}(t)\right]^2dt$$

to$\underset{\sim}{g}\in\boldsymbol{W}_2^m[0,1]$, so we can conclude that:

$$\underset{\sim}{b}^2(\lambda)\le\int_0^1 E\left(f(t)-g(t)\right)^2w(t)dt+\lambda\int_0^1\left[g^{(m)}(t)\right]^2dt$$

Because it applies to every $\underset{\sim}{g}\in\boldsymbol{W}_2^m[0,1]$, then by taking $g(t)=f(t)$, can be obtained that:

$$\begin{aligned}\underset{\sim}{b}^2(\lambda)&\le\lambda\int_0^1\left[g^{(m)}(t)\right]^2dt\\ &=\mathbf{O}(\lambda)\end{aligned}$$

The following is derived asymptotic properties $\boldsymbol{V}(\lambda)$.

Define:

$\Phi\left(\underset{\sim}{f_\lambda},\underset{\sim}{h};\gamma\right)=\boldsymbol{R}\left(\underset{\sim}{f_\lambda}+\gamma\underset{\sim}{h}\right)+\lambda\boldsymbol{J}\left(\underset{\sim}{f_\lambda}+\gamma\underset{\sim}{h}\right),\gamma\in$

$R$and$\underset{\sim}{h}\in\boldsymbol{W}_2^m[0,1]$,

with

$R(g)=n^{-1}\sum_{j=1}^n w_j\left(y_j-g(t_j)\right)^2$ and $J(g)=\int_0^1\left[g^{(m)}(t)\right]^2dt.$

For any value $\underset{\sim}{f},\underset{\sim}{g}\in\boldsymbol{W}_2^m[0,1]$,

$$\begin{aligned}\Phi(\underset{\sim}{f},\underset{\sim}{g},\gamma)&=n^{-1}\sum_{j=1}^n w_j\left(y_j-f(t_j)-\gamma g(t_j)\right)^2\\ &\quad+\lambda\int_0^1\left(f^{(m)}(t)+\gamma g^{(m)}(t)\right)^2dt\end{aligned}$$

$\frac{d\Phi(f,g,\gamma)}{d\gamma}=0and\gamma=0$will give:

$$n^{-1}\sum_{j=1}^n w_jg(t_j)\left(y_j-f(t_j)\right)=\lambda\int_0^1 f^{(m)}(t)g^{(m)}(t)dt$$

If$\phi_1,\ldots,\phi_n$ *the* base for natural splines and $f(t)=\sum_{k=1}^n\beta_k\phi_k(t)$, then Lemma 1 gives:

$$\sum_{j=1}^{n} w_j g(t_j)\left(y_j - \sum_{k=1}^{n}\beta_k\phi_k(t_j)\right)$$
$$= n\lambda(-1)^m(2m-1)!\sum_{j=1}^{n} g(t_j)\sum_{k=1}^{n}\beta_k d_{jk}$$

Because it applies to every $\underset{\sim}{g} \in \boldsymbol{W}_2^m[0,1]$, the last equation is equivalent to finding $\beta_k$which fulfills:

$$y_j = \sum_{k=1}^{n}\left(n\lambda(-1)^m(2m-1)!\,w_j{}^{-1}d_{jk} + \phi_k(t_j)\right)\beta_k\, j = 1,2,\ldots,n$$

By presenting the matrix, it can be obtained:

$$\underset{\sim}{y} = (n\lambda(-1)^m(2m-1)!\,\boldsymbol{W}^{-1}\boldsymbol{K} + \phi)\beta$$

with $\boldsymbol{K} = \{d_{jk}\}, k, j = 1,2,\ldots,n$, and $\phi = \{\phi_k(t_j)\}, k, j = 1,2,\ldots,n$

Then, it can be obtained that:

$\underset{\sim}{y} = \left(n\lambda\boldsymbol{W^{-1}FB^{-1}F'W^{-1}}\phi + \phi\right)\beta$, with $\boldsymbol{B} = \boldsymbol{F'VF}$.

If the last equation is multiplied from the left with ϕ'can be obtained that:

$\phi'\underset{\sim}{y} = (n\lambda\phi'\boldsymbol{W}^{-1}\boldsymbol{F}\boldsymbol{B}^{-1}\boldsymbol{F}'\boldsymbol{W}^{-1}\phi + \phi'\phi)\beta.$

So, the estimator $\beta_\lambda$obtained from the equation:

$$\beta_\lambda = \left(n\lambda\phi'\mathbf{W}^{-1}\mathbf{F}\mathbf{B}^{-1}\mathbf{F}'\mathbf{W}^{-1}\phi + \phi'\phi\right)^{-1}\phi'\underset{\sim}{y}$$
$$= diag\left(\frac{1}{1+n\lambda\theta_1},\ldots,\frac{1}{1+n\lambda\theta_n}\right)\phi'\underset{\sim}{y}$$

Estimator $f_\lambda(t)$can be presented in the equation:

$$f_\lambda(t) = \sum_{k=1}^{n}\beta_{\lambda k}\varphi_k(t)$$
$$= \sum_{k=1}^{n}\frac{1}{1+n\lambda\theta_k}\varphi_k' y\varphi_k \quad (5)$$

The asymptotic property of the variance component is given by the theorem below.

Theorem 3

If (A0) applies, then $\boldsymbol{V}(\lambda) \leq \boldsymbol{O}\left(\frac{1}{1+n\lambda^{1/2m}}\right), n \to \infty$

Proof:

$$f_\lambda(t) = \sum_{k=1}^{n}\frac{1}{1+n\lambda\theta_k}\varphi_k' y\varphi_k(t)$$
$$= \sum_{k=1}^{n}\frac{1}{1+n\lambda\theta_k}\sum_{r=1}^{n}\varphi_k(t_r)y_r\varphi_k(t) \quad (6)$$

$$\mathrm{Var}(f_\lambda(t)) = \sigma^2\sum_{k=1}^{n}\frac{\varphi_k^2(t)}{(1+n\lambda\theta_k)^2}\sum_{r=1}^{n}\varphi_k(t_r)\varphi_k(t_r)w_r^{-1}$$
$$\leq \left(\underset{i\leq j\leq n}{\mathrm{Max}}\{w_j{}^{-1}\}\right)\sigma^2\sum_{k=1}^{n}\frac{\varphi_k^2(t)}{(1+n\lambda\theta_k)^2} \quad (7)$$

So that:

$$\boldsymbol{V}(\lambda) = \int_0^1 E(E\,f2(t) - f2(t))^2 w(t)dt$$
$$\leq (\underset{1\leq j\leq n}{Max}\{w_j^{-1}\}\sigma^2\sum_{k=1}^{n}\frac{1}{(1+n\lambda\theta k)^2}\int_0^1\phi_k^2(t)w(t)dt$$

For n ->∞, Speckman (1985) and Eubank [8], provide an approach that:

$$n^{-1} = n^{-1}\sum_{r=1}^{n} w_r\phi_k(t_r) \approx \int_0^1\phi_k(t)w(t)dt$$
$$\boldsymbol{V}(\lambda) \leq (\underset{1\leq j\leq n}{Max}\{w_j^{-1}\})\sigma^2 n^{-1}\sum_{k=1}^{n}\frac{1}{(1+\lambda\gamma k)^2}$$

Furthermore, Eubank [8] provides:

$$\boldsymbol{V}(\lambda) \leq (\underset{1\leq j\leq n}{Max}\{w_j^{-1}\})\sigma^2 n^{-1}\sum_{k=1}^{n}\frac{1}{[1+\lambda(\pi k)^{2m}]^2}$$

With the integral approach obtained that:

$$\boldsymbol{V}(\lambda) \leq (\underset{1\leq j\leq n}{Max}\{w_j^{-1}\})\frac{\sigma^2}{\pi n\lambda^{1/2m}}\int_0^\infty\frac{dx}{(1+x^{2m})^2}$$
$$= \frac{1}{n\lambda^{1/2m}}\boldsymbol{K}(m,\sigma)$$
$$= \boldsymbol{O}(\frac{1}{n\lambda^{1/2m}})$$

With $\boldsymbol{K}(m,\sigma) = \frac{\sigma^2}{\pi}(\underset{1\leq j\leq n}{Max}\{w_j^{-1}\})\int_0^\infty\frac{dx}{(1+x^{2m})^2}$

By combining the quadratic bias and variance components, the asymptotic behavior of IMSE is obtained, which is entirely provided by the following results.

Consequence

If (A0) applies then $IMSE(\lambda) \leq \boldsymbol{O}(\lambda) + \left(\frac{1}{n\lambda^{1/2m}}\right), n \to \infty$

Proof:

This result is proven by combining Theorem 2 and Theorem 3

The following theorem presents the convergence speed of the weighted spline estimator, which can reach the level $n^{-2m/(2m+1)}$Based on IMSE criteria.

Theorem 4

If $\underset{\sim}{f} \in \boldsymbol{W}_2^m[0,1]$ dan (A0) applies then to $n \to \infty, \lambda \to 0, n\lambda^{1/2m} \to \infty$

(i). $\lambda_{opi} = \boldsymbol{O}(n^{-2m/(2m+1)})$ (8)

(ii). $IMSE(\lambda_{opt}) = \boldsymbol{O}(n^{-2m/(2m+1)})$ (9)

Proof:

$IMSE(\lambda)$can be written as:

$IMSE(\lambda) = \boldsymbol{K}(m)\lambda + \boldsymbol{K}(\sigma,m)\left(\frac{1}{n\lambda^{1/2m}}\right),$ (10)

With K (m) and K (σ, m) constants that do not contain λ. by lowering the IMSE (λ) against λ, then the result is equalized to zero, the first part of the Theorem is proven. The second part of the theorem is obtained by substituting λ_{opt}. The theorem of the first part into the $IMSE \rightleftarrows (\lambda)$.

4.2 Consistent Properties of Parametric Component Estimators

In this section, we will investigate the consistent nature of the $\underset{\sim}{\hat{\beta}}$in the non-parametric regression model.

For this purpose, the following assumptions are given

A1. $\{X_n; n \geq 1\}$is a sequence of random variables that are distributed identically and independently with zero mean and with variation $v_j < v, 0 < v < \infty, j = 1,2,\ldots,rp$

A2. $E(\underset{\sim}{\varepsilon}) = \underset{\sim}{0}$

A3. $E\left(x_{jk}^2 e_k^2\right) < \vartheta < \infty, j = 1,2,\ldots,rp;\ k = 1,2,\ldots,rn$

A4. $E\left(x_{jk}^2 s_k^2\right) < \omega < \infty, j = 1,2,\ldots,rp;\ k = 1,2,\ldots,rn$

Based on these assumptions, the following items are compiled to demonstrate the consistent nature of the parametric component estimators $\hat{\underset{\sim}{\beta}}$.

Lemma 1:

If $\underset{\sim}{\hat{\beta}_n}$ is a sequence of parametric component estimators given by Theorem 1, then:

$$\underset{\sim}{\hat{\beta}_n}-\underset{\sim}{\beta}=\left[X^TBX-X^TA^T(\underset{\sim}{\lambda})BX\right]^{-1}\left[X^T\left[I-A(\underset{\sim}{\lambda})\right]^TB\underset{\sim}{f}+X^TB\underset{\sim}{\varepsilon}-X^TA^T(\underset{\sim}{\lambda})\,B\underset{\sim}{\varepsilon}\right] \quad (11)$$

Proof:

Based on Theorem 1, it can be obtained that:

$$\begin{aligned}\underset{\sim}{\hat{\beta}_n}-\underset{\sim}{\beta}&=\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{BX}\right]^{-1}\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\left(\boldsymbol{X}\underset{\sim}{\beta}+\underset{\sim}{f}+\underset{\sim}{\varepsilon}\right)-\underset{\sim}{\beta}\\&=\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{BX}\right]^{-1}\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\left(\underset{\sim}{f}+\underset{\sim}{\varepsilon}\right)\\&=\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{BX}\right]^{-1}\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\left(\underset{\sim}{f}+\underset{\sim}{\varepsilon}\right)\right]\end{aligned}$$

$$\begin{aligned}\underset{\sim}{\hat{\beta}_n}-\underset{\sim}{\beta}&=\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{BX}\right]^{-1}\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}+\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{\varepsilon}\right]\\&=\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{BX}\right]^{-1}\left[\boldsymbol{X}^T\left(\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}+\boldsymbol{B}\underset{\sim}{\varepsilon}\right)-\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}\right]\\&=\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{BX}\right]^{-1}\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}+\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}-\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}\right]\\&=\left[\boldsymbol{X}^T\boldsymbol{BX}-\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{BX}\right]^{-1}\left[\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}+\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}-\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}\right] \quad (12)\end{aligned}$$

Furthermore, to show that the sequence of the estimator $\underset{\sim}{\hat{\beta}_n}$ is a consistent estimator, the following lemma is needed, which states the convergence property of the right-hand product term on the right side of Lemma 1.

Lemma 2.

If the assumptions of A1-A3 are met, then

i. $\dfrac{\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underline{f}}{rn}\xrightarrow{p}0,n\to\infty$ (13)

ii. $\dfrac{\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\xrightarrow{p}0,n\to\infty$ (14)

iii. $\dfrac{\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\,\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\xrightarrow{p}0,n\to\infty$ (15)

Proof:

i. To show that $\dfrac{X^T\left[I-A(\underset{\sim}{\lambda})\right]^TB\underset{\sim}{f}}{rn}\xrightarrow{p}0,n\to\infty$, will be used Chebychev's inequality. It will be shown that $Var\left(\dfrac{X^T\left[I-A(\underset{\sim}{\lambda})\right]^TB\underset{\sim}{f}}{rn}\right)_j\to 0,n\to\infty,j=1,2,\ldots,rp$. Then, take $\underset{\sim}{z}=\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f},\underset{\sim}{z}=(z_1,z_2,\ldots,z_{rn})$ and $u=\underset{1\le i\le rn}{Max}\{z_i\}$.

So,

$$\begin{aligned}Var\left(\mathbf{X}^T\left[\mathbf{I}-\mathbf{A}(\underset{\sim}{\lambda})\right]^T\mathbf{B}\,\underset{\sim}{f}\right)_j&=Var\left(\mathbf{X}^T\underset{\sim}{z}\right)_j\\&=Var\left(\sum_{i=1}^{rn}x_{ji}z_i\right)\\&=\sum_{i=1}^{rn}Var\left(x_{ji}z_i\right)\\&=\sum_{i=1}^{rn}z_i^2Var\left(x_{ji}\right)\le\left(\underset{1\le i\le rn}{Max}\{z_i\}\right)^2\sum_{i=1}^{rn}Var\left(x_{ji}\right)\\&=u^2\sum_{i=1}^{rn}v_j\\&=u^2rn\left(v_j\right)\\&\le rnu^2v\end{aligned}$$

And the consequences will lead to:

$$\begin{aligned}Var\left(\frac{\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}}{rn}\right)_j&=\frac{1}{(rn)^2}Var\left(\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}\right)_j\\&=\frac{1}{(rn)^2}rnu^2v_j\\&=\frac{1}{rn}u^2v_j\\&=o(1),n\to\infty\end{aligned}$$

So, $Var\left(\dfrac{X^T\left[I-A(\underset{\sim}{\lambda})\right]^TB\underset{\sim}{f}}{rn}\right)_j\to 0,n\to\infty$

So, $\dfrac{\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}}{rn}\xrightarrow{p}0,n\to\infty$ (16)

ii. To show that $\dfrac{\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\xrightarrow{p}0,n\to\infty$, and it will show that: $Var\left(\dfrac{\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\right)_j\to 0,n\to\infty,j=1,2,\ldots,rp$

Take $\underset{\sim}{e}=\boldsymbol{B}\underset{\sim}{\varepsilon},\underset{\sim}{e}=\{e_1,e_2,\ldots,e_{rn}\}$

$$\begin{aligned}Var\left(\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}\right)_j&=Var\left(\boldsymbol{X}^T\underset{\sim}{e}\right)\\&=Var\left(\sum_{i=1}^{rn}x_{ji}\,e_i\right)\\&=\sum_{i=1}^{rn}Var\left(x_{ji}e_i\right)\\&=\sum_{i=1}^{rn}E\left(x_{ji}^2e_i^2\right)\\&\le rn\vartheta\end{aligned}$$

So,

$$\begin{aligned}Var\left(\frac{\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\right)&=\frac{1}{(rn)^2}Var\left(\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}\right)\\&=\frac{rn\vartheta}{(rn)^2}\\&=\frac{\vartheta}{rn}\\&=o(1),n\to\infty \quad (18)\end{aligned}$$

The result is obtained, $Var\left(\dfrac{\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\right)_j\to 0,n\to\infty$

So, it has been proven that $\dfrac{\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\xrightarrow{p}0,n\to\infty$ ■

iii. $\dfrac{\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\,\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\xrightarrow{p}0,n\to\infty$ applies if

$$Var\left(\frac{\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\right)_j\to 0,n\to\infty,j=1,2,\ldots,rn$$

Then, given that$\underset{\sim}{s} = \boldsymbol{A}^T(\underset{\sim}{\lambda})\,\boldsymbol{B}\underset{\sim}{\varepsilon}, \underset{\sim}{s} = (s_1, s_2, \ldots, s_{rn})$

$$Var\left(\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}\right)_j = Var\left(\boldsymbol{X}^T\underset{\sim}{s}\right)_j$$
$$= Var\left(\sum_{i=1}^{rn} x_{ji}\, s_i\right)$$
$$= \sum_{i=1}^{rn} Var\left(x_{ji}s_i\right)$$
$$= \sum_{i=1}^{rn} E\left(x_{ji}^2 s_i^2\right)$$
$$\leq rn\omega$$

As a result,

$$Var\left(\frac{\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\right)_j = \frac{1}{(rn)^2}Var\left(\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}\right)_j$$
$$= \frac{rn\omega}{(rn)^2}$$
$$= \frac{\omega}{rn}$$
$$= o(1), n \to \infty$$

So,

$$Var\left(\frac{\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn}\right)_j \to 0, n \to \infty \quad (19)$$

So, it has been proven that $\frac{\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\,\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn} \xrightarrow{p} 0, n \to \infty$ ■

Furthermore, based on Lemma 1 and 2, the consistency of the estimator $\hat{\underset{\sim}{\beta}}$is shown in the following Theorem.

Theorem 5

If $\underline{\hat{\beta}}_n the$ sequence of parametric component estimators given by Theorem 1, then$\hat{\underset{\sim}{\beta}}_n \xrightarrow{p} \underset{\sim}{\beta}, n \to \infty$

Proof:

Based on Lemma 1, it is obtained,

$$\hat{\underset{\sim}{\beta}}_n - \underset{\sim}{\beta} = \left[\boldsymbol{X}^T\boldsymbol{B}\boldsymbol{X} - \boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\boldsymbol{X}\right]^{-1}\left[\boldsymbol{X}^T\left[\boldsymbol{I} - \boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f} + \boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon} - \boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underset{\sim}{\varepsilon}\right] \quad (20)$$

Based on Lemma 2, it has been promoted that

i. $\frac{\boldsymbol{X}^T\left[\boldsymbol{I}-\boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f}}{rn} \xrightarrow{p} 0, n \to \infty$

ii. $\frac{\boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn} \xrightarrow{p} 0, n \to \infty$

iii. $\frac{\boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\,\boldsymbol{B}\underset{\sim}{\varepsilon}}{rn} \xrightarrow{p} 0, n \to \infty$

Furthermore, with Theorem 3 about convergent properties in probability, will be obtained

$$\left[\boldsymbol{X}^T\left[\boldsymbol{I} - \boldsymbol{A}(\underset{\sim}{\lambda})\right]^T\boldsymbol{B}\underset{\sim}{f} + \boldsymbol{X}^T\boldsymbol{B}\underset{\sim}{\varepsilon} - \boldsymbol{X}^T\boldsymbol{A}^T(\underset{\sim}{\lambda})\boldsymbol{B}\underline{\varepsilon}\right]\underset{\sim}{\varepsilon} \xrightarrow{p} \underline{0}, n \to \infty$$

As a result, with Theorem 3 obtained

$$\hat{\underset{\sim}{\beta}}_n - \underset{\sim}{\beta} \xrightarrow{p} 0 \text{ a tau } \hat{\underset{\sim}{\beta}}_n \xrightarrow{p} \underset{\sim}{\beta}, n \to 0 \quad (21)$$

4.3 Hypothesis Testing

Because of the second study objective, we perform hypothesis testing on each relationship between variables in the Nonparametric Regression-based Path Analysis using the Penalized Weighted Least Square (PWLS) approach.

4.4 Bayes Estimator

The observed sampling is given $(t_1, y_1), \ldots, (t_n, y_n)$obtained from a stochastic process $\{Y(t); t \in [0,1]\}$and follow the model:

$Y(t) = f(t) + \varepsilon(t), t \in [0,1].$ (22)

$\{f(t); t \in [0,1]\}$ has a prior improper distribution:

$$\sum_{t=1}^{m} \alpha_1\varphi_1(t) + b^{1/2}Z(t),$$

$b > 0$, polynomial coefficient$\alpha = (\alpha_1, \ldots, \alpha_m)'$has the distribution $N(0, al), a \to \infty$

$\{Z(t); t \in [0,1]\}$,with:

$Z(t) = \int_0^1 \frac{(t-u)^{m-1}}{(m-1)!}dw(u)$

Is a Wiener process with zero mean and covariance of Reproducing Kernel:

$$R^1(s,t) = \int_0^1 \frac{(s-u)_+^{m-1}(t-u)_+^{m-1}}{[(m-1)!]^2}du$$

Polynomial Coefficient $\alpha_1, i = 1, \ldots, m$ and $Z(t)$is uncorrelated [17], [25], [26], [27]and$\{\varepsilon(t)\}$is a normal process with mean zero and

$$cov(\,\varepsilon(t)), \varepsilon(s)) = \begin{cases}\sigma^2/w(t), \text{ if } t = s\\ 0 \qquad\quad\;\;, \text{ if } t \neq s\end{cases}$$

Theorem 2 provides a weighted spline estimator $f_\lambda(t)$can be written as follows

$$f_\lambda(t) = (\varphi_1(t), \ldots, \varphi_m(t))(\boldsymbol{T'S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}\boldsymbol{T'S}^{-1}(\lambda)\boldsymbol{W}\underset{\sim}{y} + (\psi_1(t), \ldots, \psi_n(t))\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{W}(\boldsymbol{I} - \boldsymbol{T}(\boldsymbol{T'S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}\boldsymbol{T'S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{W})\underset{\sim}{y}.$$

The following is given a Theorem which is used to derive the Bayes estimator in a weighted spline.

Theorem 6

Given $\underset{\sim}{y}, \underset{\sim}{f}, \underset{\sim}{\varepsilon}$ Gaussian random vector with zero mean and following the model:

$$\underset{\sim}{y} = \underset{\sim}{f} + \underset{\sim}{\varepsilon}$$

$E(\underset{\sim}{f}\underset{\sim}{f}') = \underset{\sim}{b}V_f, E(\underset{\sim}{\varepsilon}\underset{\sim}{\varepsilon}') = \sigma^2\boldsymbol{W}^{-1}$ and $E(\underset{\sim}{f}\underset{\sim}{\varepsilon}') = 0$ if h is generally distributed with $E(\underset{\sim}{h}) = 0, E(\underset{\sim}{h}\underset{\sim}{h}') = \underset{\sim}{b}\boldsymbol{V}_h, E\left(\underset{\sim}{h}\underset{\sim}{f}'\right) = \underset{\sim}{b}\boldsymbol{V}_{hf}$ and $E(\underset{\sim}{h}\underset{\sim}{\varepsilon}') = 0$ so:

(i) $E(\underset{\sim}{h}|\underset{\sim}{y}) = \boldsymbol{V}_{hf}(\boldsymbol{V}_f + n\lambda\boldsymbol{W}^{-1})^{-1}\underset{\sim}{y}$, and

(ii)$Var(\underset{\sim}{h}|\underset{\sim}{y}) = \underset{\sim}{b}(\boldsymbol{V}_h - \boldsymbol{V}_{hf}\boldsymbol{V}_f^{-1}\boldsymbol{V}_{fh}) + \sigma^2\boldsymbol{V}_{hf}\boldsymbol{V}_f^{-1}\boldsymbol{A}(\underset{\sim}{\lambda})\boldsymbol{W}^{-1}\boldsymbol{V}_f^{-1}\boldsymbol{V}_{fh}$

with$\boldsymbol{A}(\underset{\sim}{\lambda}) = \boldsymbol{V}_f(\boldsymbol{V}_f + n\underset{\sim}{\lambda}\boldsymbol{W}^{-1})^{-1}$ and $n\underset{\sim}{\lambda} = \sigma^2/\underset{\sim}{b}$

Proof:

If $\boldsymbol{X}_1 = \underset{\sim}{h}, \boldsymbol{X}_2 = \underset{\sim}{y}$Lemma 2 will give:

$$\sum_{12} = \underset{\sim}{b}\boldsymbol{V}_{hf} \text{ and } \sum_{22} = \underset{\sim}{b}(\boldsymbol{V}_f + n\underset{\sim}{\lambda}\boldsymbol{W}^{-1})$$

As the results:

(i) $E(\underset{\sim}{h}|\underset{\sim}{y}) = \boldsymbol{V}_{hf}(\boldsymbol{V}_f + n\underset{\sim}{\lambda}\boldsymbol{W}^{-1})^{-1}\underset{\sim}{y}$and

(ii)$Var(\underset{\sim}{h}|\underset{\sim}{y}) = \underset{\sim}{b}\{\boldsymbol{V}_h - \boldsymbol{V}_{hf}(\boldsymbol{V}_f + n\underset{\sim}{\lambda}\boldsymbol{W}^{-1})^{-1}\boldsymbol{V}_{fh}\}$

On the other hand, the Sherman-Morisson-Woodbury formula for matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$, and $\mathbf{D}$ will give:

$$(\boldsymbol{A} + \boldsymbol{BC}^{-1}\boldsymbol{D})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{C} + \boldsymbol{DA}^{-1}\boldsymbol{B})^{-1}\boldsymbol{DA}^{-1}$$

If the similarities are taken:

$\boldsymbol{A} = \boldsymbol{B} = \boldsymbol{C} = \boldsymbol{V}_f$ dan $\mathbf{D} = n\underset{\sim}{\lambda}\boldsymbol{W}^{-1}$

Will be given:

$$(\boldsymbol{V}_f + n\underset{\sim}{\lambda}\boldsymbol{W}^{-1})^{-1} = \boldsymbol{V}_f^{-1} - n\underset{\sim}{\lambda}(\boldsymbol{V}_f + n\underset{\sim}{\lambda}\boldsymbol{W}^{-1})^{-1}\boldsymbol{W}^{-1}\boldsymbol{V}_f^{-1}$$

As the results:

$$Var(\underset{\sim}{h}|\underset{\sim}{y}) = \underset{\sim}{b}(\boldsymbol{V}_h - \boldsymbol{V}_{hf}\boldsymbol{V}_f^{-1}\boldsymbol{V}_{fh}) + n\underset{\sim}{\lambda}\underset{\sim}{b}\boldsymbol{V}_{hf}(\boldsymbol{V}_f + n\underset{\sim}{\lambda}\boldsymbol{W}^{-1})^{-1}\boldsymbol{W}^{-1}\boldsymbol{V}_f^{-1}\boldsymbol{V}_{fh}$$

$$= \underset{\sim}{b}(\boldsymbol{V}_h - \boldsymbol{V}_{hf}\boldsymbol{V}_f^{-1}\boldsymbol{V}_{fh}) + \sigma^2\boldsymbol{V}_{hf}\boldsymbol{V}_f^{-1}\boldsymbol{A}(\underset{\sim}{\lambda})\boldsymbol{W}^{-1}\boldsymbol{V}_f^{-1}\boldsymbol{V}_{fh}$$

With prior and weighted quadratic loss function, the estimator $f_\lambda(t)$, it is the posterior mean f, which is fully presented in the following Theorem.

Theorem 7

Bayes estimation $f_{\lambda,\phi}(t)$ from $f(t)$ given by :

$$f_{\lambda,\phi}(t) = E\left(f(t)|\underset{\sim}{y}\right)$$
$$= (\varphi_1(t),\ldots,\varphi_m(t)).\phi\mathbf{T}'\left(\phi\mathbf{TT}' + \mathbf{W}^{-1}\mathbf{S}(\underset{\sim}{\lambda})^{-1}\underset{\sim}{y} + \psi_1(t),\ldots,\psi_m(t)\right)\left(\phi\mathbf{TT}' + \mathbf{W}^{-1}\mathbf{S}(\underset{\sim}{\lambda})^{-1}\underset{\sim}{y}\right)$$

with $\varphi = a/b$ and $\boldsymbol{S}(\underset{\sim}{\lambda}) = \boldsymbol{WV} + n\underset{\sim}{\lambda}\boldsymbol{I}$

Proof:

Rewrite $\underset{\sim}{f} = \left(f(t_1),\ldots,f(t_n)\right)'$, $\underset{\sim}{\varepsilon} = (\varepsilon_1,\ldots,\varepsilon_m)'$

$\boldsymbol{T} = \left\{t_j{}^{-1}/(i-1)!\right\}, j = 1,2,\ldots,n, i = 1,2,\ldots,m,$ and

$\boldsymbol{V} = \left\{\boldsymbol{R}^1(t_i,t_j)\right\}, i,j = 1,2,\ldots,n$

When paired $\underset{\sim}{h} = f(t)$ so that

$$\boldsymbol{V}_{hf} = E\left(f(t)\underset{\sim}{f}'\right)/\underset{\sim}{b}$$
$$= (\varphi_1(t),\ldots,\varphi_m(t)).\phi\boldsymbol{T}' + (\phi_1(t),\ldots,\phi_m(t))$$

From Theorem 7 obtained that:

$$\mathbf{V}_f = E\left(\underset{\sim}{f}\,\underset{\sim}{f}'\right)/\underset{\sim}{b}$$
$$= \phi\mathbf{TT}' + \mathbf{V}$$

With a few elaborations obtained:

$$\mathbf{V}_h = E\left(f(t)\,f(t)\right)/\underset{\sim}{b}$$
$$= \phi\underset{\sim}{\varphi}'\underset{\sim}{\varphi} + \psi(t)$$

Finally, Theorem 7 and Theorem 1 will give:

$$E\left(f(t)|\underset{\sim}{y}\right) = (\varphi_1(t),\ldots,\varphi_m(t)).\phi\mathbf{T}'\left(\phi\mathbf{TT}' + \mathbf{V} + n\underset{\sim}{\lambda}\mathbf{W}^{-1}\right)^{-1}\underset{\sim}{y} +$$
$$(\psi_1(t),\ldots,\psi_m(t))\left(\phi\mathbf{TT}' + \mathbf{V} + n\underset{\sim}{\lambda}\mathbf{W}^{-1}\right)^{-1}\underset{\sim}{y}$$
$$= (\varphi_1(t),\ldots,\varphi_m(t)).\phi\mathbf{T}'\left(\phi\mathbf{TT}' + \mathbf{W}^{-1}\mathbf{S}(\underset{\sim}{\lambda})\right)^{-1}\underset{\sim}{y} +$$
$$(\psi_1(t),\ldots,\psi_m(t))\left(\phi\mathbf{TT}' + \mathbf{V} + \mathbf{W}^{-1}\mathbf{S}(\underset{\sim}{\lambda})\right)^{-1}\underset{\sim}{y}$$

The weighted spline estimator given by Theorem 2 is a Bayes estimator with the prior improper estimator, which is given in full by Theorem below.

Theorem 9

If $f(t), t \in [0,1]$ has a prior improper distribution so that:

$$\lim \lim_{a\to\infty} E_a(f(t)|y) = f_\lambda(t),$$

With $f_\lambda(t)$ weighted natural polynomial spline is obtained by minimizing:

$$n^{-1}\sum_{j=1}^{n} w_j\left(y_j - f(t_j)\right)^2 + \lambda\int_0^1 [f^{(m)}(t)]^2 dt$$

Proof:

Sherman-Morisson-Woodbury formula for equation matrix:

$\boldsymbol{A} = \boldsymbol{W}^{-1}\boldsymbol{S}(\underset{\sim}{\lambda}), \boldsymbol{B} = \boldsymbol{T}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}$

$\boldsymbol{C}^{-1} = \phi(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})$ dan $\boldsymbol{D} = \boldsymbol{T}'$

Will give:

$$\left(\phi\boldsymbol{TT}' + \boldsymbol{W}^{-1}\boldsymbol{S}(\underset{\sim}{\lambda})\right)^{-1}$$
$$= \boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{W}$$
$$+ -\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}[\phi^{-1}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}$$
$$+ \boldsymbol{I}]\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{W}$$

Lemma 3give that:

$$(\boldsymbol{I} + \phi^{-1}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1})^{-1}$$
$$= \boldsymbol{I} - \phi^{-1}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}$$
$$+ \phi^{-2}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-2} - \ldots$$

As the results:

$$\left(\phi\boldsymbol{TT}' + \boldsymbol{W}^{-1}\boldsymbol{S}(\underset{\sim}{\lambda})\right)^{-1}$$
$$= \boldsymbol{S}^{-1}(\underset{\sim}{\lambda}) - \boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{W}$$
$$+ \phi^{-1}\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT}(\boldsymbol{T}'\boldsymbol{S}^{-1}(\lambda)\boldsymbol{WT})^{-2}\boldsymbol{T}'\boldsymbol{S}^{-1}\boldsymbol{W} + \boldsymbol{O}(\phi^{-2})$$

$$\lim_{\phi\to\infty}\phi\boldsymbol{T}'\left(\phi\boldsymbol{TT}' + \boldsymbol{W}^{-1}\boldsymbol{S}(\underset{\sim}{\lambda})\right)^{-1} =$$
$$(\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{WT})^{-1}\boldsymbol{T}'\boldsymbol{S}^{-1}(\underset{\sim}{\lambda})\boldsymbol{W}.$$

$$\lim_{\phi\to\infty}\left(\phi TT' + \mathbf{W}^{-1}\mathbf{S}(\underset{\sim}{\lambda})\right)^{-1} = \mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{W} - \mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{WT}\left(\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{WT}\right)^{-1}\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{W}$$
$$= \mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{W}\left(\mathbf{I} - \mathbf{T}\left(\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{WT}\right)^{-1}\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{W}\right)$$

Finally, Theorem 2 will give:

$$\lim_{\phi\to\infty}E\left(f(t)|y\right) = (\varphi_1(t),\ldots,\varphi_m(t))\lim_{\phi\to\infty}\phi\mathbf{T}'\left(\phi\mathbf{TT}' + \mathbf{W}^{-1}\mathbf{S}(\underset{\sim}{\lambda})\right)^{-1} +$$
$$(\psi_1(t),\ldots,\psi_m(t))\lim_{\phi\to\infty}\left(\phi\mathbf{TT}' + \mathbf{W}^{-1}\mathbf{S}(\underset{\sim}{\lambda})\right)^{-1}\underset{\sim}{y}$$
$$= (\varphi_1(t),\ldots,\varphi_m(t))\left(\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{WT}\right)^{-1}\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{W}\underset{\sim}{y} +$$
$$(\psi_1(t),\ldots,\psi_m(t))\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{W}\left(\mathbf{I} - \mathbf{T}\left(\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{WT}\right)^{-1}\mathbf{T}'\mathbf{S}^{-1}(\underset{\sim}{\lambda})\mathbf{W}\right)\underset{\sim}{y}$$

The following shows the method of selecting the smoothing parameter λ for a weighted spline estimator based on the Bayes approach, namely Generalized Maximum Likelihood (GML). The basic idea of using the GML method in the original non-parametric spline regression was first given by Wahba [17] and then developed by Wang (1998) for correlated data.

Given that ϑ and ω with the decomposition:

$$\begin{pmatrix}\vartheta\\ \ldots\\ \omega\end{pmatrix} = \begin{pmatrix}F'\\ \ldots\\ \frac{T'}{\sqrt{\phi}}\end{pmatrix}y, \text{ and fulfill } F'T = 0.$$

First, the following Theorem is given:

(i) ϑ *distributed* $N\left(0, \underset{\sim}{b}\boldsymbol{F}'\boldsymbol{W}^{-1}\boldsymbol{S}(\boldsymbol{\lambda})\boldsymbol{F}\right)$

(ii) ω *distributed* $N\left(\mathbf{0}, \underset{\sim}{b}(\boldsymbol{T}'\boldsymbol{T})(\boldsymbol{T}'\boldsymbol{T})\right)$.

Proof:

Given the decomposition and the model:

$$\underset{\sim}{y} = \underset{\sim}{f} + \underset{\sim}{\varepsilon}$$

Will give:

$$\mathrm{Var}(y) = \underset{\sim}{a}\boldsymbol{T}'\boldsymbol{T} + \underset{\sim}{b}\boldsymbol{V} + \sigma^2\boldsymbol{W}^{-1}$$

With another elaboration, obtained that:

$$\mathrm{Var}(y) = \underset{\sim}{b}\left(\boldsymbol{W}^{-1}\boldsymbol{S}(\lambda) + \phi\boldsymbol{TT}'\right)$$

On the other hand, will give:

$$\mathrm{Cov}(\vartheta,\omega) = \underset{\sim}{b}\phi^{-1/2}\mathbf{F}'\left(\mathbf{W}^{-1}\mathbf{S}(\lambda) + \phi\mathbf{TT}'\right)\mathbf{T}$$
$$= \underset{\sim}{b}\phi^{-1/2}\mathbf{FS}(\lambda)\mathbf{T}$$

So, can be obtained that:

$\mathrm{Var}(\omega) = \underset{\sim}{b}\phi^{-1}\boldsymbol{T}'\left(\boldsymbol{W}^{-1}\boldsymbol{S}(\lambda) + \phi\boldsymbol{TT}'\right)\boldsymbol{T}$ and

$$\mathrm{Var}(\vartheta) = \underset{\sim}{b}\mathbf{F}'\left(\mathbf{W}^{-1}\mathbf{S}(\lambda) + \phi\mathbf{TT}'\right)\mathbf{F}$$
$$= \underset{\sim}{b}\mathbf{F}'\mathbf{W}^{-1}\mathbf{S}(\lambda)\mathbf{F}$$

For $a \to \infty$, obtained:

$$\lim_{a\to\infty}\text{Cov}\left(\vartheta,\omega_j\right)=\underset{\sim}{b}\lim_{\phi\to\infty}\phi^{-1/2}\mathbf{FS}\left(\lambda\right)\mathbf{T}$$
$$=0,$$
$$\lim_{a\to\infty}\text{Var}\left(\omega\right)=\underset{\sim}{b}\lim_{a\to\infty}\phi^{-1}\mathbf{T}'\left(\mathbf{W}^{-1}\mathbf{S}\left(\lambda\right)+\phi\mathbf{TT}'\right)\mathbf{T}$$
$$=\underset{\sim}{b}\left(\mathbf{T'T}\right)\left(\mathbf{T'T}\right)$$

For $\to\infty$, remember that *y is* usually distributed with a mean of zero, then Theorem is proved.

Theorem 4 shows that for $a\to\infty$the distribution of ϑthat depends on λ and $\vartheta\sim N\left(0,\underset{\sim}{b}\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}\right)$. Based on the ϑ distribution, the Likelihood and log Likelihood functions are obtained respectively:

$$L(\lambda,b|\vartheta)$$
$$=\frac{1}{(2\pi)^{r/2}|\underset{\sim}{b}\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}|^{1/2}}exp\left[-\frac{1}{2}\vartheta'\left(\underset{\sim}{b}\left(\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}\right)^{-1}\vartheta\right)\right]$$
$$log\,L\,(\lambda,b|\vartheta)=-\frac{r}{2}\underset{\sim}{log}\,b-\frac{1}{2}log|\underset{\sim}{b}\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}|$$
$$-\frac{1}{2b}\left(\vartheta'\left(\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}\right)^{-1}\vartheta+K\right)$$

This log Likelihood function provides the Maximum Likelihood estimator:

$$\hat{\underset{\sim}{b}}_\lambda=\frac{\vartheta'\left(\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}\right)^{-1}\vartheta}{r}$$

By resubstituting$\hat{b}_\lambda$ n the log Likelihood function is obtained:

$$log\,L\,(\lambda|\vartheta)=-\frac{r}{2}log\left(\frac{\vartheta'\left(\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}\right)^{-1}\vartheta}{|(\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F})^{-1}|^{1/r}}\right)+K_2$$

with K1 and K2 constants that are independent of λ and b. Maximizing $log\,L\,(\lambda|\vartheta)$ is equivalent to minimizing:

$$GML(\lambda)=\frac{\vartheta'\left(\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F}\right)^{-1}\vartheta}{|(\boldsymbol{F'W^{-1}S}(\lambda)\boldsymbol{F})^{-1}|^{1/r}}$$

The optimal λ value is obtained by minimizing the GML (λ).

5. Conclusion

The conclusions that can be conveyed up to the progress report of this research have obtained a review of the properties of the spline estimator in the Nonparametric Regression-based Path Analysis using the PWLS approach, hypothesis testing on each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach, as well as some findings regarding the confidence interval. optimal on each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach, complementing the previous research findings, namely a theoretical review of Non-Parametric Path Analysis for modeling porang growth in Indonesia, as well as an academic review regarding the estimation of the variance-covariance error matrix of the Non-Parametric Path Analysis model, as well as a selection of smoothing parameters. On the other hand, the third objective is to theoretically test the optimal confidence interval for each relationship between variables in the Nonparametric Regression-based Path Analysis using the PWLS approach.

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