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Use of a MOOC and SCRATCH for Learning and Teaching Integer Addition

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Abstract: This article presents the results of research on mathematics teaching through the use of a MOOC (Massive Open Online Course), aimed at sixth-grade students from an urban educational institution to learn about integer addition, with computational support in SCRATCH. The main contribution of this study lies in the integration of a MOOC with SCRATCH programming to promote computational thinking and multi-representational understanding of integer addition. Unlike traditional ICT-based approaches, this model combines conceptual learning with algorithmic construction.The schooling process was carried out online and face-to-face. Programming was considered for the development of logical thinking. For the development of this work, a pedagogical design based on socioconstructivism and theories of autonomous, collaborative, and problem-based learning was proposed, in addition to using the Registers of Semiotic Representation. The work was carried out in three stages: the first was the design and creation of the MOOC; the second involved learning activities in which students programmed with SCRATCH; finally, students interacted with the programs they developed themselves to solve integer addition problems. This process revealed didactic contributions to the teacher as they transformed knowledge of integer addition into the SCRATCH programming scheme, thus offering another way of representing this knowledge, thereby increasing their specialized content knowledge. Regarding the students, they acquired other strategies to learn autonomously about integer addition when relating their mathematical knowledge with the computational knowledge they implemented in SCRATCH.

Keywords: MOOC; Integer Addition; Mathematics Education; Computational Thinking; Scratch Programming; ICT in Education; Semiotic Representation.

利用MOOC与Scratch进行整数加法的教与学

摘要：本文通过使用大规模开放在线课程（MOOC）并结合SCRATCH编程，呈现了关于数学教学的研究成果。研究对象为城市教育机构中的六年级学生，旨在支持其整数加法的学习。该研究的主要贡献在于将MOOC平台与可视化编程相结合，以促进计算思维的发展，并增强学生对整数加法的多表征理解。与传统基于信息与通信技术（ICT）的教学方法不同，该模型将概念性学习与算法构建相结合。

教学过程采用线上与线下相结合的方式进行，并通过编程活动促进学生逻辑思维的发展。本研究的教学设计基于社会建构主义理论，并融合自主学习、协作学习和基于问题学习的理论，同时引入符号表征系统理论（Registers of Semiotic Representation）。

研究分为三个阶段：第一阶段为MOOC的设计与开发；第二阶段为学生参与基于SCRATCH编程的学习活动；第三阶段为学生通过与自身开发的程序互动来解决整数加法问题。

研究结果表明，该教学过程对教师和学生均具有重要的教学意义。对于教师而言，该过程促进了将数学内容转化为计算表达形式，从而提升了其专业内容知识水平。对于学生而言，将数学知识与编程知识相结合，有助于发展其自主学习策略，并加深对整数加法的理解。

关键词：大规模开放在线课程（MOOC）；整数加法；数学教育；计算思维；Scratch编程；信息与通信技术；符号表征

1. Introduction

During learning and teaching processes, mathematical skills and competencies developed by students both inside and outside the classroom become evident. It happens that when they are acquired outside the classroom and initiated by prior concepts strongly attached to the development of mathematical thinking constructed autonomously in a real context, the traditional teaching object can be modified by a systematized pedagogical model for a specific educational community, considering that the characteristics and notions of students regarding what is known as mathematical activities can be diverse; in addition to the importance that the teacher should give to recognizing the concepts that students relate to daily life. In this way, the teacher will be a creator of meaningful learning spaces framed in the association of mathematical concepts similar to those learned in other contexts. Therefore, it is necessary to mention that school learning is an active process from the student's point of view, in which they construct, modify, enrich, and diversify their knowledge schemes with respect to different school contents based on the meaning and sense they can attribute to those contents and to the very act of learning them [1].

The teacher must transform discourse with innovative activities that impact the student in the teaching and learning process. For this reason, it is pertinent to include in the pedagogical design the use of ICT tools to develop, according to the Basic Learning Rights (DBA) Version 2, that the student must propose, compare, and use inductive procedures and algebraic language to formulate and test conjectures in various situations or contexts [2]. That is, the student represents numerical relationships through algebraic expressions. The teacher, for their part, implements the use of ICT supported by mathematics education theories through the development of didactic sequences on integer addition, with which it is intended to determine: What didactic contributions does the use of a MOOC and SCRATCH offer in teaching integer addition to sixth-grade students regarding the use and creation of educational material, adaptation of educational resources to the context, and communicative strategies in the classroom? For this purpose, a general objective and three specific objectives are proposed to guide the development of this research. This study contributes to mathematics education by proposing a structured pedagogical model that integrates MOOCs and block-based programming. The novelty of this approach lies in explicitly connecting conceptual understanding, computational thinking, and semiotic representations in the learning of integer addition.

2. State of the Art

The following research has been considered as a reference to consider insofar as it falls within the framework of mathematics education whose problem is in school contexts similar to those addressed in this work.

The article "Teaching and Learning in Problem Solving: Notable Products of Reflection" by Alfonso Jiménez, Laura Parra, and Hollman Camacho, derived from the partial results of the research "Mathematical Practices in Problem Solving on Factorization," aims to determine the particularities of teaching and learning practices in solving factorization and notable products problems; with the analysis of three school textbooks (Álgebra de Ortiz (2009), Álgebra de Bautista (2003), and Álgebra de Baldor (1986)) and a questionnaire with 11 closed and open questions with the intention of knowing the opinion of 191 eighth-grade students of the Institución Educativa Técnica Antonio Nariño de Moniquirá (Boyacá) regarding the algebra learning process with an onto-semiotic approach, a mixed methodology, and the content analysis method, showing a high degree of interest in the appropriation of knowledge and students' perception of the use of technology in algebra classes. In the results, it is notable that traditional models for teaching algebra, where master classes are prioritized, lack ICT implementation, individual activities without playfulness, and demotivation because students mention that they do not understand the proposed topics since they focus the class on a masterful explanation with some geometric representations and little implementation of support material that does not encourage student participation. In this sense, the pedagogical design of this work considers this article important because it evidences how insufficient intervention with only pencil and paper could be for the teaching and learning process of algebra, as it tends to disconnect factorization from other concepts and closes the possibility for the use of other representations that allow consolidating knowledge and rectifies that the teacher must incorporate the use of ICT.

The thesis for the Master's degree of Sandra Isabel Salazar Giraldo (2017) from UTP "Dynamic Hypermedial Aids (DHA) for Teaching Algebraic Expressions in Initiation to Algebra with Eighth-Grade Students of the Institución Educativa San Pablo de Pueblo Rico Risaralda" frames an innovative didactic proposal that is considered important for this work because the pedagogical design is mediated by ICT, understood as a didactic tool in the teaching and learning process; in addition to the implementation of activities with the creation of a didactic sequence that promotes collaborative learning from a socio-constructivist approach and the acquisition of knowledge of methodologies and didactic strategies for the teacher's teaching practice; they are based on autonomous learning and problem-based learning, where the pedagogical results of the DHA implementation are evidenced, such as strengthening teacher-student interaction in a participatory environment designed for learning algebraic expressions with representations, illustrations, examples, and demonstrations.

Regarding the use of MOOCs in mathematics education, several international studies have shown the effectiveness of these platforms for developing autonomous learning skills. Research by Siemens (2013) and Downes (2014) highlights that MOOCs facilitate access to quality educational content and promote self-regulated learning. In the Colombian context, studies by the Ministry of National Education have encouraged the incorporation of digital platforms to strengthen mathematical competencies in basic education [2].

Concerning SCRATCH as a didactic tool for mathematics learning, research by Resnick et al. (2009) at the MIT Media Lab demonstrates that programming in visual environments favors the development of logical thinking and problem-solving skills. In Latin America, studies by Calderón (2018) and Mendoza (2019) have shown that integrating SCRATCH into mathematics classrooms improves student motivation and understanding of abstract concepts such as integer operations.

Specifically on integer addition, research by Godino and Ruiz [8] indicates that understanding this operation requires overcoming several cognitive obstacles: the meaning of the negative sign, order on the number line, and sign rules. Traditional teaching often presents these concepts in a decontextualized manner, leading to rote learning without deep understanding. The combination of MOOCs for conceptual acquisition and SCRATCH for application and creation offers an innovative alternative to address these difficulties.

3. Theoretical framework

A. Socioconstructivism in Mathematics Teaching

This theoretical framework focuses on the constructs that directly support the research design, particularly semiotic representation (Duval), computational thinking, and technology integration models such as TPACK. General theoretical descriptions have been reduced to emphasize their role within the MOOC-SCRATCH integration.

Socioconstructivism provides the theoretical foundation for this study by emphasizing that mathematical knowledge is constructed through social interaction and mediation. In this research, learning is facilitated through the interaction between students, digital tools (MOOC and SCRATCH), and teacher guidance, allowing students to progressively internalize mathematical concepts such as integer addition through collaborative and guided activities.

B. MOOCs in Mathematics Education

A MOOC (Massive Open Online Course) is an online course aimed at unlimited participation and open access via the web. In addition to traditional course materials such as videos, readings, and problem sets, MOOCs provide interactive user forums that help build a community for students, professors, and teaching assistants [4]. For mathematics teaching, MOOCs offer the possibility of presenting content in multiple formats, allowing students to advance at their own pace and receive immediate feedback through automated evaluations.

According to Siemens [10], connectivism is the learning theory underlying MOOCs, where knowledge is distributed across a network of connections, and learning consists of the ability to construct and traverse those networks. This approach aligns with the autonomous learning that this research seeks to promote in sixth-grade students when studying integer addition.

C. SCRATCH as a Didactic Tool

SCRATCH is a visual programming environment developed by the MIT Media Lab that allows users to create interactive stories, games, and animations. It uses a block-based interface that simplifies programming, making it accessible to young people and beginners [5]. In mathematics education, SCRATCH has been used to develop logical thinking, problem-solving skills, and understanding of mathematical concepts through computational representation.

Research by Calderón [6] indicates that programming in SCRATCH allows students to "concretize" abstract mathematical concepts such as integers and their operations, as they can visualize the effects of their programs and receive immediate feedback on their reasoning.

D. Learning Theories

1) Autonomous Learning:

Autonomous learning is understood in this study as the student's ability to regulate their own learning process through the use of digital environments such as MOOCs. Students progress at their own pace, revisit content, and self-assess their understanding, which supports the development of responsibility and independence in learning integer addition.

2) Autonomous Learning in Mathematics Teaching

When establishing that the student's autonomous work is effective to the extent that there is harmonious development of other competencies, as well as habits of independent and self-directed work, institutions support the implementation of classroom work practices that seek to favor students' skills for searching for information, incorporating ICT, solving problems, analyzing, synthesizing, reasoning, arguing, among others [12]. In relation to the above, the teaching and learning process of mathematics must be coherent with the application of learning activities assigned to fulfill a learning objective.

3) Collaborative Learning:

Collaborative learning in this study is implemented through group work and interaction in both the MOOC environment and SCRATCH activities. Students share ideas, discuss problem-solving strategies, and co-construct knowledge, which enhances their understanding of mathematical concepts and promotes meaningful learning.

4) Collaborative Learning in Mathematics Teaching

For collaborative learning in mathematics teaching, the teacher must determine the order of activities, giving priority according to the context of the group members and collective objectives. Therefore, the teacher makes mathematical knowledge available to the teaching process to enhance new learning based on the student's own skills for group work [14]. Then, it is necessary for the teacher to communicate clearly the indications at the beginning, development, and end of learning activities to generate in students motivation to demonstrate in exchanges the ideas or interpretations they produce according to their prior knowledge.

5) Problem-Based Learning

Problem-based learning is applied through the use of contextualized mathematical tasks that require students to apply integer addition in meaningful situations. In this study, students solve problems by designing and programming solutions in SCRATCH, which fosters critical thinking and the application of mathematical concepts in real or simulated contexts.

6) Problem-Based Learning in Mathematics Teaching

For the application of a problem-based learning methodology, it is important to mention some aspects that should be considered in a method for the problem-solving work process, as expressed by Moust, Bouhuijs, and Schmidt [16]:

- Clarify concepts and terms: It aims to clarify possible terms in the problem text that are difficult (technical) or vague, so that the entire group shares their meaning.
- Define the problem: It is a first attempt to identify the problem that the text poses.
- Analyze the problem: In this phase, students contribute all the knowledge they possess about the problem as formulated, as well as possible connections that could be plausible.
- Make a systematic summary with several explanations for the analysis: Once the greatest number of ideas about the problem has been generated, the group tries to systematize and organize them.
- Formulate learning objectives: At this moment, students decide which aspects of the problem require further investigation and better understanding.

- Synthesis of the collected information and preparation of the report: The information provided by the different members of the group is discussed, contrasted, and finally, pertinent conclusions are drawn for the problem.

E. Registers of Semiotic Representation

Duval [17] defines a semiotic system as a register of representation when it enables three essential cognitive activities associated with semiosis: identification, treatment, and conversion. In teaching integer addition, different registers are used: the numerical register (numbers and signs), the graphical register (representation on the number line), the natural language register (verbal problems), and the computational register (programming in SCRATCH). The coordination between these registers allows students to construct a robust meaning of integer addition.

F. TPACK Model

The model called Technological Pedagogical Content Knowledge (TPACK) aims to group the types of knowledge that teachers need to master to adequately integrate ICT in the classroom [18]. These basic knowledge areas are: technological, pedagogical, and disciplinary, which allow the development of good pedagogical practices in teaching and learning processes.

The implementation of the TPACK model in educational processes is of great importance, integrating student and context knowledge; with the teacher being flexible for the design of pedagogical strategies so that the student acquires a sense of what they are learning. The model proposed by Punya Mishra and Matt Koehler is made up of three base knowledge domains: Content (CK), Pedagogy (PK), and Technology (TK), and they are closely related to the basic knowledge domains mentioned above: Pedagogical Content Knowledge (PCK), Technological Content Knowledge (TCK), and Technological Pedagogical Knowledge (TPK).

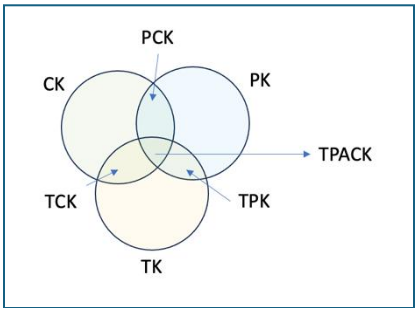

Figure 1. TPACK Model (Own creation)

G. Mathematical Knowledge for Teaching (MKT)

The model called MKT (Mathematical Knowledge for Teaching), developed by the research group coordinated by Deborah Ball at the University of Michigan, is defined as "the mathematical knowledge that the teacher uses in the classroom to produce instruction and growth in the student" [19]. That is, according to this model, the teacher must possess content knowledge to exercise mathematics teaching and is classified as follows:

- Common Content Knowledge
- Specialized Content Knowledge
- Knowledge of Content and Students
- Knowledge of Content and Teaching

H. Integer Addition: Conceptual Aspects

Integer addition is a fundamental operation in secondary school mathematics. According to Godino and Ruiz [8], understanding integer addition requires overcoming several cognitive obstacles: the meaning of the negative sign, the order on the number line, and the rules of signs. Traditional teaching often presents these concepts in a decontextualized manner, leading to rote learning without deep understanding.

Different models have been developed for teaching integer addition: the number line model, the counters model (positive and negative chips), and the contextualized problems model (debts and assets, temperatures, altitudes). This research incorporates these models in the design of the MOOC and in the programming activities with SCRATCH, allowing students to visualize and manipulate integer addition in different semiotic registers.

4. Methodology

A. Research Design

The methodology of this research is qualitative, of an analytical-interpretive type, to analyze the objectives and results related to the problem statement: What didactic contributions does the use of a MOOC and SCRATCH offer in teaching integer addition to sixth-grade students regarding the use and creation of educational material, adaptation of educational resources to the context, and communicative strategies in the classroom? In this way, the teacher's mathematics teaching knowledge in the application of a didactic sequence will be studied in practice, along with the significant contributions of using a MOOC and SCRATCH in the teaching and learning process.

B. Research Context

The research was conducted with sixth-grade students of an urban educational institution in Pereira, Risaralda. The sample consisted of 28 students between 10 and 12 years old, with diverse socioeconomic backgrounds and varying levels of prior knowledge in mathematics and technology use.

C. Research Techniques and Instruments

- Activity Log
- Video Recordings
- Participant Observation
- Semi-structured Interviews
- Analysis of SCRATCH projects developed by students
- MOOC platform interaction records

Data analysis was conducted using thematic coding. Observational data, interview responses, and SCRATCH projects were analyzed to identify patterns in student reasoning, common errors, and levels of conceptual understanding. SCRATCH projects were evaluated based on correctness, logical structure, and the use of representations.

Triangulation was applied across multiple data sources (observations, interviews, and digital artifacts) to ensure the validity of findings. Learning outcomes were inferred from improvements in performance, quality of student projects, and their ability to explain mathematical concepts.

D. Research Phases

1) Phase 1: Design and Creation of the MOOC

A MOOC was designed and created on a freely accessible platform, containing modules on integer addition: concept of integers, representation on the number line, addition of integers with equal signs, addition of integers with different signs, and contextualized problem-solving. The MOOC included explanatory videos, interactive activities, automated evaluations, and discussion forums.

2) Phase 2: Learning Activities with SCRATCH

Learning activities were designed in which students, after reviewing the concepts in the MOOC, programmed in SCRATCH to represent and solve integer addition problems. Activities included: creating a number line simulator, designing an interactive game for adding integers, and programming solutions to contextualized problems.

3) Phase 3: Interaction and Problem Solving

Students interacted with the programs they developed themselves to solve integer addition problems. This phase included socialization spaces where students explained their programs, the mathematical reasoning behind them, and the difficulties encountered during programming.

E. Pedagogical Design

The pedagogical design is based on the socio-constructivist approach and learning theories (autonomous, collaborative, and problem-based learning), with the incorporation of the Registers of Semiotic Representation [17]. The design establishes relationships between:

Autonomous Learning: Students advance through the MOOC modules at their own pace, review content as many times as necessary, and complete automated evaluations that provide immediate feedback.

Collaborative Learning: Students participate in discussion forums within the MOOC and work in pairs or small groups to develop SCRATCH projects, sharing ideas and solutions.

Problem-Based Learning: Students face challenging problems that require the application of integer addition, first in the MOOC and then in the creation of their own programs.

The pedagogical design contemplates the following elements:

Scaffolding: The teacher provides graduated support, from direct instruction in initial videos to guided exploration in SCRATCH activities.

Zone of Proximal Development: Collaborative activities and teacher mediation allow students to solve problems that they could not solve individually.

Adjusted Help: The teacher intervenes at key moments to clarify doubts, suggest strategies, and guide the programming process.

Co-construction of Shared Meanings: In socialization spaces, students share their creations and explain their reasoning, enriching collective understanding.

5. Design and implementation of the MOOC and SCRATCH activities

A. MOOC Structure

The MOOC was organized into five modules, each with the following structure:

- Introductory video: Presentation of concepts with concrete examples.
- Reading material: Expanded explanation with multiple representations (symbolic, graphical, verbal).
- Interactive activity: Exercises with immediate feedback.
- Discussion forum: Space to share doubts and solutions.
- Module evaluation: Automated quiz to verify learning.

B. SCRATCH Activities Design

Three progressive SCRATCH activities were designed:

- Activity 1: Number Line Simulator
  Students program a simulator that represents any integer on the number line and shows its absolute value. This activity reinforces the concept of integer and its graphical representation.

- Activity 2: Integer Addition Game
  Students design an interactive game that presents random integer addition problems and rewards correct answers. The game must show the operation symbolically and its representation on the number line.
- Activity 3: Contextualized Problem Solver
  Students program a solution to a contextualized problem (temperature changes, altitude variations, financial transactions) involving integer addition. The program must show the step-by-step reasoning.

C. Implementation Process

The implementation was carried out over eight weeks, with two weekly sessions of 90 minutes each, in the school's computer lab. The process was as follows:

- Week 1: Introduction to the MOOC platform, creation of user accounts, exploration of Module 1.
- Week 2: Development of Module 2, introduction to SCRATCH, first contact with the programming environment.
- Week 3: Development of Module 3, implementation of Activity 1 (Number Line Simulator).
- Week 4: Development of Module 4, implementation of Activity 2 (Integer Addition Game).
- Week 5: Development of Module 5, design of solutions to contextualized problems.
- Week 6: Implementation of Activity 3 (Contextualized Problem Solver).
- Week 7: Final project: students choose a problem and develop a complete solution in SCRATCH.
- Week 8: Project socialization, collective feedback, and final evaluation.

6. Results

The results are presented in two categories: teacher development and student learning outcomes, providing both qualitative and quantitative evidence.

A. Teacher's Didactic Contributions

a. Transformation of Content Knowledge: The teacher significantly transformed their knowledge of integer addition by adapting it to the MOOC format and the SCRATCH programming scheme. This process required decomposing the content into progressive modules, selecting appropriate semiotic representations, and designing activities that would allow students to construct meaning autonomously.

b. Development of Technological Pedagogical Content Knowledge (TPACK): The teacher acquired skills in:

Technological Knowledge (TK): Mastery of the MOOC platform and SCRATCH environment, ability to solve technical problems, creation of multimedia resources.

Technological Content Knowledge (TCK): Understanding of how integer addition can be represented and manipulated in a computational environment.

Technological Pedagogical Knowledge (TPK): Design of learning activities that effectively integrate technology, considering students' learning rhythms and styles.

TPACK: Integration of all knowledge to create meaningful learning experiences where technology, pedagogy, and content work synergistically.

c. Creation of Educational Material: The teacher developed original educational materials: explanatory videos, interactive guides, SCRATCH project templates, and automated evaluations, which can be reused in future teaching cycles.

d. Adaptation to Context and Contingencies: The teacher demonstrated flexibility in adjusting activities to the school context, available resources, and students' prior knowledge. During implementation, they faced and resolved unforeseen situations such as connectivity problems, differences in students' technological skills, and the need to adjust activity complexity.

e. Strengthening of Communicative Strategies: The teacher developed new strategies to communicate mathematical ideas in virtual and face-to-face environments, using multiple semiotic registers and promoting student participation in forums and socialization spaces.

B. Students' Learning Outcomes

a. Autonomous Learning: Students showed significant progress in their ability to self-regulate their learning. MOOC platform records indicated that 82% of students completed all modules, with an average access time of 4.5 hours per week. Students reviewed videos and activities as many times as necessary, evidencing autonomous learning strategies.

b. Understanding of Integer Addition: The combination of MOOC (conceptual) and SCRATCH (procedural/application) allowed students to construct a deeper understanding of integer addition. In the final evaluation, 75% of students correctly solved contextualized problems, compared to 42% in the initial diagnosis. Analysis of SCRATCH projects showed that students were able to model integer addition using variables and conditional structures. For example, several students created programs that represented movements on the number line, demonstrating a conceptual understanding beyond procedural calculation.

c. Development of Logical Thinking: Programming in SCRATCH contributed to the development of logical thinking skills. Students learned to sequence instructions, use conditional structures, handle variables, and debug errors—skills transferable to mathematical problem-solving.

d. Motivation and Engagement: Students showed high levels of motivation and engagement throughout the process. The gamified activities in SCRATCH and the autonomy offered by the MOOC generated sustained interest. In surveys, 89% of students stated that they preferred this learning modality over traditional classes.

e. Transfer of Knowledge: Students demonstrated the ability to transfer integer addition knowledge to new contexts. In the final projects, they created programs to solve diverse problems: temperature changes in different cities, altitude variations in the Colombian geography, control of points in games, among others.

These results indicate that the integration of computational environments contributes to deeper conceptual understanding and increased student engagement.

7. Discussion

The results obtained confirm the effectiveness of integrating MOOCs and SCRATCH in teaching integer addition, aligning with findings from previous research [3], [5], [6]. The pedagogical design based on socioconstructivism and learning theories proved to be appropriate for the educational context and the characteristics of sixth-grade students.

A. Regarding the Teacher's Didactic Contributions

The teacher's experience in this research coincides with what is proposed by the TPACK model [18] and MKT [19]. The need to transform integer addition content to be taught through a MOOC and SCRATCH forced the teacher to deepen their specialized content knowledge, recognizing aspects of the concept that are often overlooked in traditional teaching. Likewise, the integration of technology required developing new pedagogical strategies, evidencing growth in technological pedagogical knowledge.

B. Regarding Students' Learning

The significant improvement in student performance (from 42% to 75% correct answers) supports the thesis that multiple semiotic representations [17] favor the construction of robust mathematical meanings. Students not only learned the rules for integer addition but also understood their meaning and application in everyday contexts.

Programming in SCRATCH proved to be an effective strategy for "concretizing" abstract concepts, as suggested by Calderón [6]. By building programs that represent integer addition, students externalized their mental models, allowing them to identify and correct errors in their reasoning.

The high levels of motivation and engagement coincide with findings from studies on gamification and technology use in education [20]. The autonomy offered by the MOOC and the creativity stimulated by SCRATCH generated a positive attitude toward mathematics learning.

C. Regarding the Integration of MOOC and SCRATCH

The combination of MOOC (for conceptual acquisition) and SCRATCH (for application and creation) proved to be complementary. The MOOC provided a solid conceptual base that students could access at their own pace, while SCRATCH allowed them to apply and deepen that knowledge in creative and meaningful ways.

The discussion forums in the MOOC and the socialization spaces for SCRATCH projects favored collaborative learning and the co-construction of meanings, fundamental elements of the socio-constructivist approach.

From a pedagogical perspective, this study provides a replicable model for integrating programming into mathematics teaching. The approach is scalable to other mathematical topics and educational contexts, provided that teachers receive adequate training and technological resources are available.

8. Conclusions

The main contribution of this study is the design and validation of a pedagogical model that integrates MOOCs and SCRATCH to enhance the learning of integer addition through computational thinking and multiple representations.

The general objective of strengthening the teaching of integer addition through a MOOC and SCRATCH was achieved. The specific phases of design, implementation, and evaluation provided a clear structure for addressing the problem.

A. Main Conclusions

- Regarding the Teacher: The pedagogical design for teaching integer addition with a MOOC and SCRATCH allowed the teacher to adjust their knowledge about socioconstructivism and learning theories, addressing the concept of integer addition from multiple semiotic representations. This pedagogical process allowed the teacher greater dynamism between the teaching object, their content knowledge, and the students [19], along with technological and pedagogical content knowledge [18], in a learning environment defined by such design.
- Regarding the Didactic Sequence: The MOOC and SCRATCH activities allowed the teacher to plan the teaching of integer addition and select didactic strategies according to the theoretical approach of the methodological design. The teacher acquired skills in the use of ICT to innovate and guide the educational process, promoting spaces for

collective knowledge construction and recognizing students' prior knowledge.

- Regarding Feedback and Interaction: The application of the MOOC and SCRATCH for teaching integer addition allowed the teacher to promote feedback spaces for the discussion of mathematical arguments and showed their ability to guide and intervene in student errors using content knowledge and pedagogical technological knowledge.
- Regarding Teacher Knowledge Development: With the implementation of the pedagogical design with learning theories and the socio-constructivist approach, the teacher managed to approach the teaching of integer addition in a blended environment, developing skills in the transformation dimension and the contingency dimension. The creation of didactic strategies and multimedia resources allowed enhancing technological knowledge, pedagogical knowledge [21], and specialized content knowledge [19].
- Regarding Students: Students acquired autonomous learning strategies by relating their mathematical knowledge with the computational knowledge implemented in SCRATCH. They demonstrated significant improvement in understanding integer addition and its application in contextualized problems, as well as the development of logical thinking skills through programming.

B. Limitations and Recommendations

Limitations:

Connectivity problems in some students' homes affected continuous access to the MOOC.

Differences in prior technological skills required additional time for leveling.

The sample size (28 students) limits the generalization of results.

Recommendations:

Implement longer intervention periods to consolidate learning.

Include a control group in future studies for comparative analysis.

Develop teacher training programs in the use of MOOCs and SCRATCH for mathematics teaching.

Adapt the pedagogical design to other mathematical contents and educational levels.

Future research should include larger samples and comparative designs to further validate the effectiveness of this approach.

Declarations

Author Contributions

Conceptualization, D.G.B. and J.R.G.G.; Methodology, D.G.B.; Validation, D.G.B. and J.R.G.G.; Formal Analysis, D.G.B.; Investigation, D.G.B.; Resources, J.R.G.G. and J.F.A.M.; Data Curation, D.G.B.; Writing – Original Draft Preparation, D.G.B.; Writing – Review & Editing, J.R.G.G. and J.F.A.M.; Supervision, J.R.G.G. and J.F.A.M.; Project Administration, D.G.B. All authors have read and agreed to the published version of the manuscript.

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Ethical Approval

Ethical review and approval were waived for this study, as it involved standard educational practices and anonymized data analysis in accordance with institutional policies.

Informed Consent Statement

Informed consent was obtained from all subjects involved in the study.

Data Availability Statement

The data presented in this study are available on request from the corresponding author.

Conflicts of Interest

The authors declare no conflict of interest.

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