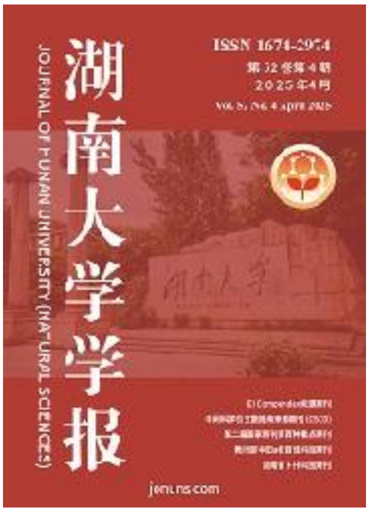


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# Efficiency of Colombian Educational Institutions: A StoNED and Order-m Approach

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**Abstract:** In Colombia, in accordance with the Ministry of National Education (MEN) and the guidelines of the National Development Plan, the importance of evaluating academic performance in upper secondary education institutions is widely recognized. This evaluation aimed to analyze the relationship between academic performance and educational policies implemented by local government authorities, particularly from an efficiency perspective. The objective of this study is to measure the academic performance of secondary education institutions from 2016 to 2019, analyze the variation in efficiency during this period, and determine whether institutional development contributes to improving the efficiency of the educational system. Specifically, two environmental variables were considered: one representing students' socioeconomic, academic, and cultural characteristics, and another reflecting the institutional context. For this analysis, a methodological approach known as Stochastic Non-parametric Envelopment of Data with Z-variables (StoNEZD) was implemented, complemented by an estimation based on the order-m model as a robustness check of the initial model. The results indicated that the efficiency levels of educational institutions fluctuated throughout the study period. Although a downward trend was observed between 2017 and 2018, the efficiency indicators gradually increased over the entire period. These findings suggest a positive trajectory in institutional management, highlighting improvements that could contribute to better educational system performance in the future.

**Keywords:** efficiency; StoNED; panel data; environmental variables; economic models.

# 哥倫比亞教育機構的效率：固定和有序方法

**摘要：**在哥倫比亞，根據國家教育部（MEN）和國家發展計劃的指導方針，評估高中教育機構學業成績的重要性已得到廣泛認可。本次評估旨在分析學業成績與地方政府當局實施的教育政策之間的關係，特别是從效率的角度。本研究的目的是逐年衡量2016年至2019年中等教育機構的學業表現，分析這段時期的效率變化，並確定機構發展是否有助於提高教育系統的效率。具體來說，考慮兩個環境變數：一個代表學生的社會經濟、學術和文化特徵，另一個反映制度背景。為了進行此分析，實施了一種稱為帶有 Z 變數的隨機非參數資料包絡 (StoNEZD) 的方法，並輔以基於 m 階模型的估計值作為初始模型的穩健性檢定。結果表明，在整個分析期內，教育機構的效率水準都在波動。儘管2017年至2018年期間呈現下降趨勢，但整個期間效率指標呈現逐步上升的趨勢。這些發現顯示機構管理呈現出正面的發展軌跡，凸顯了未來可能有助於提升教育系統績效的改善

**关键词**：效率; 停止; 面板數據; 環境變數; 經濟模型.

## 1. Introduction

The quality, effectiveness, and efficiency of education are globally significant because of their implications for a country's economic and social development, particularly in today's rapidly evolving world. In Colombia, following the student protests of 2011, two impactful proposals were presented to the Ministry of National Education (MEN) in August 2014. The first, introduced by the National Council of Higher Education (CESU), was titled the "Agreement for Higher Education 2034: Public Policy Proposal for Excellence in Higher Education in Colombia." The second, proposed by the All for Education movement, was called the "Grand National Agreement for Education." Both aim to reshape the future of education in the country [3].

Over time and in response to pedagogical, logistical, structural, political, and technological developments, successive national and departmental governments have proposed various strategies to advance education. However, government policies have not played an optimal role in shaping teaching-learning processes and the development of educational institutions in Latin America [4]. This is primarily due to issues such as a lack of clear direction in educational policy proposals; inadequate monitoring of education in terms of effectiveness and efficiency; and limited investment in infrastructure, technology, and staff training.

In Colombia, according to MEN regulations and the guidelines of the National Development Plan (PND), the evaluation of the educational system (ICFES) emphasizes measuring effectiveness rather than efficiency. Consequently, the importance of assessing academic performance in secondary education institutions has been recognized, particularly in relation to the efficiency of education policies implemented by local governments [5]. This study measures the efficiency of educational institutions in Colombia in 2016 as a baseline, coinciding with the start of the local government administration period. This naturally raises the following question: How did the efficiency of these institutions evolve over the years during this period? Answering this question requires considering external conditions, as various external factors can influence institutional performance [1, 6].

This study focuses on analyzing academic performance in secondary education institutions between 2016 and 2019, corresponding to the local government administration period. It incorporates relevant contextual factors [9, 17], such as the socioeconomic conditions of students and educational institutions, analyzed separately. This approach provides a differentiated perspective on the current situation by assessing institutional performance on a yearly basis. The study considers two levels of aggregation: individual educational institutions and departmental levels.

To conduct this analysis, we utilized the results of the SABER 11 tests from 2016 to 2019, corresponding to the tenure of local governments. The decision-making units (DMUs) are educational institutions that are evaluated based on students' test scores. The SABER 11 test has been widely employed in academic research [34], including in studies on efficiency [35].

The efficiency analysis proposed in this study follows an output-oriented approach by comparing observed and optimal values (inputs and outputs). Specifically, it assessed the maximum achievable output relative to the actual output for a given input level. The selected output was the overall student score [17], while the inputs included the number of teachers, enrolled students, and electronic equipment [27]. Additionally, two environmental variables account for the socioeconomic, academic, and cultural characteristics of students and institutions [2, 12, 14, 16].

The most commonly used methodologies in the literature to analyze contexts include non-parametric frontier models based on Data Envelopment Analysis (DEA) [26] and parametric frontier models based on Stochastic Frontier Analysis (SFA) [17]. However, this study employs an innovative methodology adopted in Finland since 2012, known as the Stochastic Nonparametric Envelopment of Data (StoNED). Introduced by Kuosmanen and Kortelainen [24], this semi-parametric frontier model combines the non-parametric DEA-type frontier, satisfying monotonicity and concavity, with the homoscedastic stochastic error term from the SFA. It follows a two-stage process: first, the non-parametric convex least squares (CNLS) estimates the frontier shape without functional form assumptions; second, the inefficiency term is estimated using CNLS residuals through the method of moments or quasi-likelihood techniques [24].

Given the context of the study, it is necessary to incorporate environmental variables reflecting students' conditions across different regions and the inherent variability in the year-to-year analysis. To achieve this, we employed the Stochastic Nonparametric Envelopment of Data with Z variables (StoNEZD) method [22]. In addition, a resampling process was implemented to generate random samples for a more computationally stable approximation. This approach, known as the order m method, involves drawing m observations randomly from the total sample, repeating the process B times, averaging the results, and enhancing the model robustness [11, 13]. The StoNEZD methodology offers two key advantages: (i) StoNED's suitability for "noisy" environments [25, 31], and (ii) its statistical consistency as a conditional non-parametric method with environmental variables, operating under less restrictive assumptions than conventional estimators [11, 22].

This research contributes significantly to the field, both methodologically and in its applications. By employing novel approaches, such as StoNEZD and the order m method, to evaluate the efficiency of educational institutions with environmental variables, this study advances the existing literature. Although these methods have been applied in other research areas, this study represents the first empirical application to assess the performance of educational institutions. Additionally, it seeks to establish a more robust framework for evaluating productive efficiency in Colombian educational institutions between 2016 and 2019 by analyzing year-by-year variations. By incorporating environmental variables into efficiency measurements, this study provides a valuable tool for assessing and improving educational institutions.

The remainder of the paper is structured as follows. Section 2 reviews the relevant literature supporting this study. Section 3 details the methodology, including data sources and variables. Section 4 presents the main results followed by a discussion. Finally, Section 5 concludes the study.

## 2. Literature review

This literature review focuses on three main aspects. First, the synthesis of the re-search line is related to efficiency in education. Second, we analyzed the use of environmental variables in the field of education. Finally, we present studies related to the StoNEZD method [22].

### 2.1. Educational Efficiency

Regarding production theory, Hanushek and Luque [20] extensively outline the relationship between this theory and educational performance, emphasizing both individual student achievement and institutional performance [20]. These studies analyzed educational production functions and examined the relationship between various inputs in the educational process and the outcomes obtained, whether at the individual or institutional level.

Since the Coleman Report [10], the performance of private and public schools has been a significant research topic in various educational contexts. The prevailing assumption is that private schools outperform public schools because of market competition, which drives them to use resources more efficiently. However, empirical studies comparing public and private schools have highlighted the importance of considering the socioeconomic conditions of students and institutions. Since then, educational research has been divided into two perspectives. The first focuses on the direct and indirect implications of academia in education and psychology, while the second emphasizes public economics, particularly technical efficiency and the evaluation of decision-making units such as educational institutions, in relation to different factors and resources (inputs) and their outcomes (outputs), often utilizing frontier models.

To address these issues, various studies have assessed the performance of Higher Education Institutions in Colombia by estimating their production frontier. This approach allows the evaluation of relative efficiency and its determinants, including external factors influencing the quality of education in Colombia [2, 3]. In recent years, efficiency analysis has increasingly incorporated environmental or contextual variables to account for discrepancies between measurement units (García et al., 2020). This explains why similar measurement units in the education sector do not necessarily yield comparable results. Conditions arising from adverse learning environments significantly affect the efficiency measurements. Some of these factors include differences in resource allocation within educational systems and the diverse financial contexts in which institutions operate [12]. Another crucial factor is the variation in students' socioeconomic levels, both individually and at the institutional level, which fluctuates annually [14].

Finally, an often-overlooked aspect is the academic and cultural background inherited by students from their parents or teachers, suggesting that the home environment, institutional settings, and regional contexts play a critical role in students' educational performance and, by extension, the efficiency of educational institutions.

### 2.2. Environmental Variables in Education

As mentioned previously, there is a need to use environmental variables because of the situation established in this study. In addition, the importance of measuring efficiency in educational settings cannot be overlooked. Failure to account for the environment in which educational institutions operate can render efficiency measurements meaningless [26]. For example, when attempting to evaluate the efficiency of an educational system, it is not fair or valid to compare institutions located in areas with significant socioeconomic differences. If the observations being evaluated are influenced by exogenous factors, the performance analysis must account for this diversity.

Thus, management strategies that enhance technical efficiency are implemented to improve the measurement of institutional performance across different contexts. A key factor is understanding the differences in the efficiency levels achieved by decision-making units, which are affected by various external and environmental conditions [6]. Analyses such as these have led to non-parametric conditional frontier models using exogenous or environmental variables that may influence the production process, impacting both input and output variables, as well as the measurement of efficiency [7].

Daraio et al. [13] developed processes based on efficiency measurements, with a focus on heterogeneous conditions. This approach has gained popularity owing to its direct and natural way of establishing conditional efficiency measures with the aim of addressing heterogeneity in performance evaluations [6]. Processes that involve environmental variables often present the advantage of not suffering from parametric specification assumptions and the limitations of non-parametric estimators, as is the case with other estimators, such as DEA and SFA. Furthermore, when conditional models are robust and involve resampling methods, the estimators are not affected by atypical sensitivity or the curse of dimensionality [18]. In addition, research has been conducted on the consistency and asymptotic properties of various conditional efficiency estimators. Owing to its many merits, the conditional efficiency approach is increasingly used to address research questions [26].

Studies have been conducted in the education sector using environmental variables, often through techniques associated with two-stage methodologies, such as DEA, SFA, and FDH. For example, Haelermans and Ruggiero [19] examined the impact of innovations in educational institutions on academic performance. This study used environmental variables related to students' academic achievement, instrumental implementation, and budget constraints.

Daraio et al. [13] addressed the paradox of university rankings, showing that while experts and social scientists criticize them for methodological issues, they are increasingly gaining attention in the media and influencing political decision-making. Therefore, they proposed creating a new generation of rankings that include more varied contextual information and use innovative classification techniques. They established a probabilistic development by considering four main approaches: one-dimensionality, statistical robustness, dependence on university size, and the combination of subjects.

Using contextual approaches to evaluate the educational effectiveness of Spanish students who participated in PISA 2009, Cordero et al. [12] managed to break down their overall inefficiency into different components, focusing particularly on the environmental differences between public and state-subsidized private schools. Similarly, to compare and break down the technical efficiency of primary schools in different educational contexts, Cordero et al. [12] adapted the metafrontier framework using an extension of the non-parametric DEA approach with conditional variables to assess the influence of environmental school variables, which represent the cultural values of each country. On the other hand, López-Torres and Prior [26] provide evidence of how strategic interaction between public schools affects the demand for school places. Through environmental variables and using a two-stage estimation method followed by a spatial econometric framework, they identified the factors that influence school demand when parents consider competition between schools.

Recently, Agasisti et al. analyzed how the characteristics of a territory and surrounding institutions can influence the efficiency of universities. By considering contextual factors such as the socioeconomic and cultural characteristics of the region and neighboring educational institutions, they gained a more comprehensive understanding of the determinants of university efficiency. Thus, by addressing the specific conditions of each university and its context, more effective policies tailored to local needs can be implemented, which in turn can lead to higher quality and efficiency in the higher education system [1, 4].

The studies mentioned above applied multiple techniques using parametric and non-parametric frontier estimators related to handling environmental variables. However, in the education sector, no study has been developed that involves a semi-parametric estimator [22] using the non-parametric conditional efficiency method [13]. This approach presents an unexplored scenario regarding efficiency in education, particularly concerning the performance measurement of educational institutions in Colombia.

### 2.3. StoNEZD Semiparametric Estimator

Johnson and Kuosmanen [22] argued that the use of the StoNED method to construct an estimator that combines the non-parametric axiomatic DEA-style frontier with the probabilistic SFA-style treatment of inefficiency and noise offers several advantages in estimating the effects of environmental variables. This estimator is statistically consistent under more general assumptions than those required by the other methods. It remains consistent even when the noise term is unbounded and the environmental variables are correlated with the inputs. Furthermore, it is asymptotically efficient for environmental variables, is normally distributed, and converges at a standard parametric rate.

Since its formalization [24], especially in recent years, this method has been implemented in various studies focused on efficiency analysis across different sectors. Table 1 summarizes the studies identified in the literature that utilize the StoNED method in various sectors.

**Table 1: Efficiency studies using the StoNED model**

| Sector | Authors | Year |
|---|---|---|
| Academia | Louca and Demosthenous | 2015 |
| | Arévalo, Giménez and Prior | 2022 |
| Agricultural | Shen and Lin | 2017 |
| | Chaovanapoonphol and Somyana | 2018 |
| Company | Liu and Kweh | 2022 |
| | Luo, Liu, Chen and Zeng | 2022 |
| | Liu, Li, Ye and Qin | 2023 |
| Finance | Eskelinen and Kuosmanen | 2013 |
| | Nguyen, Prior and Van Hemmen | 2020 |
| | Kweh, Ting, Lu and Le | 2022 |
| Health | Lee and Wang | 2019 |
| | Cordero, García-García, Lau-Cortés and Polo | 2020 |
| Utilities | Kuosmanen | 2012 |
| | Kuosmanen, Saastamoinen and | 2013 |
| | Sipiläinen | 2014 |
| | Cheng, Bjørndal and Bjørnda | 2014 |
| | Li, Kopsakangas-Savolainen, | 2016 |
| | Xiao, Tian, Yang and Wang | 2021 |
| | Molinos-Senante and Maziotis | 2022 |

*Source: Compiled by the authors*

By incorporating environmental variables to measure efficiency, the StoNED estimator avoids the main problem of the two-stage estimator approach, which ignores the correlations between inputs and environmental variables [22]. This gives rise to a one-stage semiparametric estimator called StoNEZD. This method combines the non-parametric DEA-style frontier with a regression model for environmental variables [22]. StoNEZD is statistically consistent in terms of the environmental variables under less restrictive assumptions than the two-stage DEA estimator. Furthermore, this estimator is unbiased, asymptotically efficient, normally distributed, and converges at a standard parametric rate [25].

The use of this method in empirical work began with Johnson and Kuosmanen [22]. The results show that the StoNEZD estimator produced significantly more accurate estimates than the two-stage DEA in most scenarios, regardless of the presence of noise. Moreover, Monte Carlo simulations demonstrated that the one-stage method is effective in jointly estimating frontier and environmental variables, paving the way for its practical use. Subsequently, Nieswand and Seifert [32] used Monte Carlo simulations to compare the performance of three estimators: non-parametric conditional DEA, latent parametric SFA class, and semiparametric StoNEZD. Their study focused on the ability to account for environmental factors when estimating production functions and efficiency. The simulation results provide insights into the properties of the three estimators in finite samples, and highlight the specific strengths and weaknesses of each estimator. StoNEZD was shown to be a highly competitive estimator in various scenarios, performing well with environmental variables in contextual situations and in the presence of noise.

Finally, in 2020, two studies using this estimator were published, confirming the stability of the results. One focused on the efficiency of technical assistance, specifically on the relationship between costs and their application in the tax administration of OECD countries. The StoNEZD estimator and conditional order are used to evaluate the performance of tax agencies across all countries [31]. Another study sought to assess the effectiveness of the most recently developed estimators: the non-parametric conditional DEA and StoNEZD approaches. To do so, they conducted a Monte Carlo experiment using three different data-generation processes to examine the performance of each model under various circumstances. The results indicated that the StoNEZD approach outperformed conditional DEA in all evaluated scenarios. The observed discrepancies in the correlation coefficients obtained can be explained by the definition of the production function in the presence of both noise and inefficiency. In such cases, the StoNEZD approach has an advantage over others because it incorporates a stochastic component in its mathematical formulation [11].

Finally, although the StoNEZD model offers good interpretability and stable results, efficient computational algorithms are needed to solve the CNLS formulation required for the model to be applicable to large-scale practical problems [11]. This presents a considerable disadvantage when attempting to conduct studies involving large datasets such as those that may arise in the efficiency analysis of educational contexts. For this reason, this paper proposes the use of the StoNED model, while first applying an order m treatment to simplify the computational difficulty generated by the number of calculations involved. This process makes the model more robust, especially if environmental variables are incorporated into the model, allowing the process to be concluded on the order of m. It should be noted that, to the best of our knowledge, there is no treatment similar to what is intended in this study, particularly with respect to educational institutions.

# 3. Methodology

This section comprises three parts. The first two sections present the methodology developed in this study. It begins by introducing the development of a model known as Non-Parametric Stochastic Data Envelopment Analysis with Z-variables (StoNEZD), introduced by Johnson and Kuosmanen [22], followed by the implementation of the order-m model [13] as a robustness approach to the initial model. In the third part, the data and sources used in the model are described, and the respective variables are employed, such as outputs, inputs, and environmental variables.

### 3.1. StoNED Model

Building on the StoNED estimator (semi-parametric stochastic data envelopment analysis) developed by [24] and Johnson and Kuosmanen [22] implemented a model called StoNEZD (semi-parametric stochastic data envelopment analysis with Z-variables). This new semiparametric estimator combines the axiomatic DEA-style production frontier with the parametric regression of environmental variables. It is based on the non-parametric least squares interpretation of DEA developed by Johnson and Kuosmanen [22] and the StoNED method of Kuosmanen and Kortelainen [24], which combines the axiomatic, non-parametric DEA-style frontier with the probabilistic treatment of inefficiency and noise in the SFA style.

Thus, this estimator relaxes some restrictive assumptions of the model. Additionally, it has been shown that this estimator exhibits consistency in its development, even when the noise term is unbounded and there is a correlation between inputs and environmental variables. Moreover, it is statistically consistent, asymptotically efficient, normally distributed, and converges at a standard parametric rate [22].

Based on, a multiple-input production model $\left(f(x_i)\right)$ with a single output $(y)$ is considered, relating random noise $(v_i)$, efficiency $(u_i > 0)$, and the effect of the exogenous factors $(z_i)$.

This model can be formally defined as follows:

$$y_i = f(x_i)\, e^{\varepsilon_i}, \quad \text{where} \quad \varepsilon_i = \delta z_i + v_i - u_i,$$
$$\text{for } i = 1,2,\dots,n.$$

In this way, it can be understood that $\delta z_i - u_i$ refers to the overall efficiency of a unit, where $\delta z_i$ represents the technical efficiency that can be explained by environmental variables common to all units, while $u_i$ measures management efficiency. It is assumed that environmental variables influence only the distribution of efficiency scores and not the location of the frontier. The StoNEZD method calculates the efficiency in two stages. In the first stage, the non-parametric convex least-squares method was used to obtain the shape of the frontier. This method does not assume a specific functional form, but relies on constraints such as monotonicity and convexity [11].

The estimator can be solved as the optimal solution to the following quadratic programming problem.

$$\operatorname*{Min}_{\alpha,\beta,\delta,\varepsilon} \sum_{i=1}^{n} \left(\varepsilon_i^{CNLS}\right)^2$$

subject to:

$$y_i = \alpha_i + \beta_i' x_i + \delta' z_i + \varepsilon_i^{CNLS}, \quad i = 1,2,\dots,n.$$
$$\alpha_i + \beta_i' x_i \le \alpha_h + \beta_h' x_i, \quad h,i = 1,2,\dots,n.$$
$$\beta_i \ge 0, \quad i = 1,2,\dots,n.$$

where $\varepsilon_i^{CNLS}$ refers to the residual errors of the regression and the estimated coefficients $\alpha_i$ and $\beta_i$ describe the hyperplanes tangent to the production technology $f(x_i)$ at point $x_i$. To ensure convexity, inequality restrictions are applied using Afriat's inequalities (Afriat, 1972) and the non-negativity constraint.

As pointed out by Johnson and Kuosmanen [22], in this method, the exogenous variables are included in the composite error term, suggesting that it affects technical efficiency but not the frontier. After obtaining $\varepsilon_i^{CNLS}$, the next stage is to separate inefficiency from noise to maintain the assumption that inefficiency and noise are normally distributed. To achieve this, the Method of Moments is used, where the second and third moments can be estimated as a function of the distribution of the residuals as follows:

$$\widehat{\sigma_u} = \sqrt[3]{\frac{\widehat{M_3}}{\sqrt{\frac{2}{\pi}}\,\left(1 - \frac{4}{\pi}\right)}}, \qquad \widehat{\sigma_v} = \sqrt{\widehat{M_2} - \left(\frac{\pi - 2}{\pi}\right)\sigma_u^2}.$$

The StoNEZD estimator has several attractive properties in this type of situation, as it has been shown to be unbiased, asymptotically efficient, normally distributed, and converges to the standard parametric rate √n, as detailed in the studies by Kuosmanen and others [25] and Agasisti and others [1]. Additionally, it is consistent under a few assumptions, allowing standard regression analysis techniques such as t-tests, $R^2$ or confidence intervals to be applied for inference [28].

### 3.2. Order-m Model

As previously identified, although the StoNEZD model exhibits satisfactory and stable behavior, it is evident that dense computational calculations are required to solve the necessary formulation for applying the model to problems containing a significantly large dataset [11]. This limitation presents a disadvantage in research related to the analysis of efficiency in educational contexts due to the large amount of data involved in the measurement units.

To address this challenge, the idea proposed by Simar and Wilson [33] studied the robustness of non-parametric methods, followed by Daraio et al. [13], who introduced a robust estimation of order m. This order-m approach seeks to construct a partial frontier that envelopes m observations $(m > 1)$ and randomly extracts $(\theta_{m_i})$ with replacement from the original database. This procedure is repeated B times, resulting in a series of measures $(\theta_{m_i 1}, \theta_{m_i 2}, \theta_{m_i 3}, \dots, \theta_{m_i B})$, from which the final order m efficiency measure is calculated as the simple average of these results $(\hat{\theta}_{m_i})$. Specifically, the order m efficiency measurement is derived from the following relationship:

$$\theta_m = E\left[\underset{i=1,\dots,m}{\text{Min}}\left\{\underset{j=1,\dots,k}{\text{Max}}\ \frac{x_{ij}}{x_j}\right\} \mid y_i \geq y\right],$$

where $(k)$ is the number of m-dimensional random variables $(x_1, x_2, \dots, x_m)$ that are repeatedly extracted at random from the conditional distribution of $X$, given that $(y_i \geq y)$. This estimator allows for the comparison of the efficiency of an observation with that of $m$ potential units that have production equal to or greater than $y$. As it does not include all observations when executing the model, it allows computational calculations to be performed without any issues. Additionally, it is less sensitive to outliers, extreme values, or noise in the data [11].

### 3.3. Data and Variables

In Colombia, before completing high school (secondary education), students must take the Saber 11 exam, a mandatory requirement for admission to undergraduate programs, as established by Law 30 of 1992 and Law 1324 of 2009. However, under the current regulations, an exam is not required to obtain a high school diploma. Nevertheless, Law 1324 of 2009 mandates that educational institutions present all their students for exams, except in exceptional circumstances.

This study analyzed the results of the Saber 11 exams from 2016 to 2019. The dataset comprised information from educational institutions for which complete data on all variables included in the model were available during this period. Consequently, an average of 498.000 students per year were analyzed, representing 8.562 educational institutions (5.945 public and 2.617 private) distributed across 1.092 municipalities in 33 departments of Colombia. Of these institutions, 5.832 were in urban areas and 2.730 were in rural regions.

The primary data source was the Colombian Institute for Educational Evaluation (ICFES) database, which provides open-access data for research and dissemination purposes. This repository contains extensive information about students, their families, and educational institutions. The second data source was the National Administrative Department of Statistics (DANE), specifically the National Data Archive (ANDA) catalog related to the formal education survey. This database includes multiple modules with detailed information on the students, teachers, administrative staff, and institutional characteristics.

In this study, educational institutions were considered the Decision-Making Units (DMUs). The data were aggregated at two levels: educational institutions and departments. Efficiency is assessed by comparing the observed and optimal values using an output-oriented approach, where the maximum achievable output is compared to the actual output for a given set of inputs. Additionally, two contextual variables were incorporated to account for environmental factors: one representing the institutional context and the other reflecting students' socioeconomic environment.

Following the literature and collected data, the model includes six variables: one output $(y)$, three inputs $(x_1, x_2, x_3)$, and two contextual variables $(z_1, z_2)$. In educational efficiency research, test scores are commonly used as the output variable [15, 17, 26, 34, 35]. Accordingly, this study employed the aggregated scores of all students who took the Saber 11 exam at each institution as the output variable. To maintain consistency, only first-time test-takers were included, ensuring that the results reflect the performance of students upon completing their secondary education at the respective institution.

The selected inputs align with standard practices in educational efficiency studies, and are available in the dataset. These inputs included the number of enrolled students, teachers, and electronic equipment. The number of students per institution is widely used as an input in efficiency analyses [2] given its relevance in shaping institutional characteristics and educational outcomes. Enrollment figures influence factors such as teacher availability, infrastructure, and space allocation, justifying their inclusion as an input.

Regarding human resources, literature commonly considers the proportion of tenured teachers as a key efficiency determinant [2, 26]. Some studies also include all institutional staff [11], whereas others classify personnel by role [19]. This study focuses on secondary education teachers as primary human capital input. Physical resources, particularly electronic devices, are also recognized as crucial inputs in efficiency studies [8, 28]. Therefore, this study considered the total number of functional computers, laptops, tablets, and other electronic devices in use at each institution.

Environmental factors, which are extensively discussed in the literature [17], play a critical role in educational efficiency assessments. Studies often incorporate socioeconomic and cultural indices as contextual variables [12, 14, 16]. Accordingly, this study includes two environmental indices: one capturing the socioeconomic and cultural conditions of students, and another representing the overall institutional environment. Both variables were treated as continuous.

These indices incorporate diverse factors, including parental education level, employment status, reading habits, access to communication devices, and educational tools at home for students. For institutions, variables such as location, security indices, and access to transportation and commerce were considered. This distinction allows for nuanced analysis of how individual and institutional environments shape educational outcomes [27]. Separating these effects facilitates the identification of their distinct impacts, as the family environment often influences critical thinking development, while peer interactions within the institution shape behavioral traits [30].

To construct socioeconomic and cultural indices, categorical and quantitative variables were processed through principal component analysis (PCA). This method enables the transformation of qualitative data into continuous indices by leveraging eigenvalues from the analysis. PCA has been widely applied in studies that require the integration of nonquantitative variables or datasets with categorical and discrete numerical variables [23].

It is worth noting that the statistics show non-ideal behavior in the educational process. While the average number of enrolled students decreased by 1,17%, the average overall score dropped by 5,04%. On the positive side, although the average number of teachers increased by 9,13% and the average number of electronic devices rose by 11,76%, these figures did not reach significant amounts in relation to the number of students being served.

Additionally, although some data remained at zero throughout the period and others experienced significant growth, for some variables, the variability of the presented data generated confidence as they had a standard deviation that remained stable or decreased during this period, except for one variable. Particularly, the variable in which the standard deviation increases during this period is the one corresponding to electronic devices, which is the same one that shows high variability in the data from year to year. A possible explanation for this situation is that, on one hand, various educational policy developments were implemented to provide technology to educational institutions, and on the other hand, the information could vary due to the arrival or absence of these devices at the institution, as well as whether they were reported or not due to some of them being damaged.

Finally, according to the methodological process mentioned above, and based on the data taken for the execution of this study, it is necessary to specify that the parameters taken for the realization of the order m are given by m=200 and B=1,000. This means that the StoNEZD method for each unit of measurement was processed in 1,000 different opportunities from 200 randomly chosen data in each opportunity.

## 4. Results and discussion

This section presents the results obtained from estimations corresponding to the stipulated methodological process. Based on these results, a discussion follows to establish the evolution of the efficiency of educational institutions in Colombia's departments between 2016 and 2019.

### 4.1. Results

To analyze the efficiency estimation, Table 2 presents the estimated variance parameters for the inefficiency and noise terms, along with the expected inefficiency values and corresponding determination coefficients for each year of the analyzed period. It is important to note that because of the construction of the StoNEZD method, use of moments, and its statistical properties, the determination coefficient is applicable for assessing the fit of the model [25].

**Table 2. Parameter estimation by year (Own elaboration)**

| Parameter | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|
| $\mu$ (expected value of the inefficiency term) | 0,2824 | 0,2412 | 0,2622 | 0,2375 |
| $\sigma$ (variance of the composite error term) | 0,1597 | 0,2642 | 0,1865 | 0,1532 |
| $\sigma_u$ (variance of the inefficiency term) | 0,0016 | 0,0009 | 0,0012 | 0,0008 |
| $\sigma_v$ (variance of the noise term) | 0,1581 | 0,2633 | 0,1853 | 0,1524 |
| Expected value of production efficiency | 71,76% | 75,88% | 73,78% | 76,25% |
| $R^2$ (coefficient of determination) | 0,8972 | 0,8951 | 0,8916 | 0,8968 |

It is important to emphasize that the efficiency measurement in this study was based on the students sent by each institution to take the test [5, 21]. This approach prevents penalizing institutions that do not present all eligible students for the exam for various reasons. Several key aspects were identified based on the information presented in Table 2. First, on average, the lowest expected value of the inefficiency term corresponds to the final year of the analyzed period (0,2375), suggesting that the expected efficiency reached its highest level at the end of the period. This trend may serve as a positive indicator of educational institutions' overall efficiency throughout Colombia's local administrative cycle. However, it is worth noting that efficiency did not improve consistently across the entire period, while it declined between the second and third years and subsequently increased in the fourth year. This fluctuation warrants further discussion as it reflects an initial setback followed by a corrective response that ultimately leads to improved performance.

Regarding the variance of the composite error term, it is important to note that this value results from the sum of the inefficiency and noise term variances, which makes it sensitive to individual fluctuations. Consequently, the variance does not follow a monotonic trend but instead exhibits irregular year-to-year changes. Ideally, a government policy framework would produce a steady decline in variance, reflecting progressive improvements over time. However, in this case, the variance increased by 65,44% from the first to the second year, before declining by 29,41% and 21,74% in the third and fourth years, respectively. Notably, the final year exhibited the lowest variance, reinforcing the positive trend observed for efficiency. Regarding the inefficiency and noise terms, the noise component remained dominant across all years, accounting for 98,99% and 99,65% of the total variance. This reinforces the reliability of the variance indicator by demonstrating lower volatility of inefficiency throughout the period.

The expected value of production efficiency, on average, presents an encouraging outlook, suggesting that efficiency performance during the analyzed governmental period was generally favorable across Colombia. Although there was room for further improvement, the efficiency ratio increased by 4.49 percentage points between the first and last years. The trend in this metric showed a strong initial improvement, followed by a temporary decline in the third year, and concluded with a rebound in the final year, surpassing the prior results. This pattern highlights both the anomaly observed in the second year and subsequent recovery at the end of the period. It is essential to acknowledge that these results provide a general perspective at the national level, indicating an overall trend toward greater efficiency. However, a more detailed analysis requires disaggregating the data by department, as this is the chosen level of aggregation.

Finally, the StoNEZD method, based on least-squares regression, allows the coefficient of determination ($R^2$) to serve as a valid measure of the model's empirical fit. The proportion of variance explained by the model for each year ranges from 0,8916 to 0,8972, confirming the statistical significance and strong explanatory power of the models. This finding further reinforces observations regarding efficiency trends and variance behavior over the analyzed period.

**Table 3. Percentage of institutions by department (Own elaboration)**

| Department | % Country | % Efficient Institutions | | | |
|---|---|---|---|---|---|
| | | 2016 | 2017 | 2018 | 2019 |
| Amazonas | 0,08% | 0,01% | 0,01% | 0,01% | 0,01% |
| Antioquia | 11,43% | 10,75% | 10,84% | 10,69% | 10,98% |
| Arauca | 0,60% | 0,06% | 0,06% | 0,05% | 0,07% |
| Atlántico | 5,13% | 3,59% | 3,63% | 3,56% | 3,69% |
| Bogotá | 11,75% | 11,28% | 11,37% | 11,22% | 11,51% |
| Bolívar | 4,50% | 2,79% | 2,82% | 2,77% | 2,88% |
| Boyacá | 3,96% | 2,61% | 2,64% | 2,59% | 2,69% |
| Caldas | 2,39% | 1,92% | 1,93% | 1,90% | 1,96% |
| Caquetá | 0,93% | 0,22% | 0,23% | 0,21% | 0,23% |
| Casanare | 1,09% | 0,20% | 0,20% | 0,19% | 0,21% |
| Cauca | 4,47% | 2,86% | 2,90% | 2,84% | 2,95% |
| Cesar | 2,21% | 0,69% | 0,71% | 0,68% | 0,73% |
| Chocó | 1,14% | 0,15% | 0,15% | 0,14% | 0,16% |
| Córdoba | 4,16% | 1,64% | 1,67% | 1,62% | 1,70% |
| Cundinamar | 7,35% | 5,00% | 5,05% | 4,96% | 5,14% |
| Guainía | 0,01% | 0,00% | 0,00% | 0,00% | 0,00% |
| Guaviare | 0,16% | 0,01% | 0,02% | 0,01% | 0,01% |
| Huila | 2,66% | 0,98% | 1,00% | 0,96% | 1,02% |
| La Guajira | 1,30% | 0,27% | 0,28% | 0,26% | 0,28% |
| Magdalena | 2,65% | 1,59% | 1,61% | 1,58% | 1,64% |
| Meta | 1,95% | 0,66% | 0,68% | 0,65% | 0,69% |
| Nariño | 3,87% | 1,93% | 1,97% | 1,92% | 1,99% |
| Norte de S | 2,66% | 1,19% | 1,21% | 1,18% | 1,23% |
| Putumayo | 0,91% | 0,14% | 0,15% | 0,14% | 0,15% |
| Quindío | 1,16% | 0,30% | 0,31% | 0,30% | 0,31% |
| Risaralda | 1,83% | 0,77% | 0,79% | 0,76% | 0,80% |
| San Andrés | 0,14% | 0,01% | 0,02% | 0,01% | 0,01% |
| Santander | 5,06% | 3,74% | 3,78% | 3,72% | 3,84% |
| Sucre | 2,21% | 0,63% | 0,65% | 0,62% | 0,66% |
| Tolima | 3,68% | 1,74% | 1,77% | 1,72% | 1,80% |
| Valle del C | 8,26% | 6,44% | 6,51% | 6,40% | 6,61% |
| Vaupés | 0,13% | 0,01% | 0,01% | 0,01% | 0,01% |
| Vichada | 0,18% | 0,01% | 0,01% | 0,01% | 0,01% |

Additionally, Table 3 provides a departmental breakdown, presenting the number of institutions analyzed and their respective representation at the national level. It also includes the percentage of institutions classified as efficient in each department.

### 4.2. Discussion

Considering the obtained results, we can identify various national circumstances that may have influenced the efficiency measurements of the evaluated educational institutions. To understand this, we revisit 2015, when certain events likely impacted the 2016 results. For instance, in 2015, the Single-Day program was launched to ensure access to comprehensive education, improve educational quality, reduce inequities in the Colombian education system, and address poor performance in international assessments. The progress in these areas has been gradual.

This initiative was established under Law 1753 of 2015, which introduced educational reforms that focused on enhancing quality, strengthening teacher training, promoting equity, and ensuring universal access to education. These reforms have led to technological advancements in education, increased time allocated to educational processes, the introduction of school meal programs, and expanded coverage. Such developments likely provided a solid foundation for the administrative period under evaluation, setting a baseline that could be improved, as evidenced by the results.

In 2017, the Todos a Aprender programme, initiated in 2012, was further strengthened. This program aimed to advance teacher education and training through pedagogical updates, new teaching methodologies, digital competency training, and other initiatives. Additionally, efforts have been made to improve access to education across all regions by constructing new schools and implementing scholarship programs to expand preschool, elementary, and middle school coverage. Technology in the Classroom program also contributed to technological learning advancements among the students. Furthermore, the historic peace agreement signed in late 2016 between the Revolutionary Armed Forces of Colombia (FARC) and the National Government had a significant impact. While its effects may not have been fully realized in 2017 or 2018, the influx of children into schools, particularly in rural areas, strained the educational infrastructure, affecting its operational efficiency.

This strain was evident in the efficiency measurements, with 2018 recording the lowest results of the period. However, the outcome was not catastrophic thanks to countermeasures such as peace education programs, expanded rural educational coverage, and intensified inclusion initiatives. These strategies helped maintain baseline performance, setting the stage for 2019, which saw the highest efficiency measurement of the period. Under the new National Government (2018–2022), outgoing departmental governors continued existing programs, strengthened institutions, and achieved the best results in their terms. Initiatives such as public-private partnerships for education, expanded rural coverage, teacher training, and technological improvements in marginalized areas contributed to these gains. By addressing key weaknesses in rural institutions, access, retention, and support for vulnerable students are ensured, significantly improving educational efficiency in these regions and, consequently, nationwide.

While national trends provide a general overview of departmental disparities, specific departmental characteristics reveal distinct patterns. On average, only 11 departments demonstrated technical efficiency coefficients higher than the national average, meaning just 33,33% of Colombia's departments managed their educational resources optimally. Factors such as expanded coverage, teacher training, increased technological resources, and strategic rural alliances have driven these improvements. However, only 55,71% of institutions in these areas showed significant efficiency gains, as just 17,76% of rural institutions (31,89% of the total) were in top-performing departments. Similarly, 41.74% of public educational institutions (69,43% of the total) were concentrated in these 11 departments, meaning 60,12% of public institutions improved their efficiency coefficients. This highlights that inefficiencies persist primarily in rural public institutions, suggesting a misalignment between these institutions and the guidelines set by departmental and national authorities.

The top-performing departments (Antioquia, Atlántico, Bogotá, Bolívar, Boyacá, Caldas, Cauca, Cundinamarca, Magdalena, Santander, and Valle del Cauca) included eight, with the highest number of institutions. This indicates that managing fewer institutions does not necessarily result in greater efficiency. Among them, six departments (Bogotá, Antioquia, Caldas, Valle del Cauca, Santander, and Atlántico) stood out, with Bogotá and Antioquia leading significantly. Conversely, Amazonas, Arauca, Guainía, Guaviare, San Andrés, Vaupés, and Vichada had the lowest efficiency levels, with Guainía, Vichada, and Vaupés at the bottom. In terms of overall variation during the evaluation period, the top six departments not only achieved the highest efficiency, but also showed the most improvement, underscoring their superior resource management. By contrast, the bottom three departments exhibited minimal variation, indicating little to no progress in managing educational resources.

Finally, the results suggest a possible centralization effect on Colombia's development process. The top performing departments are among the country's most prominent, with Bogotá and Antioquia leading by a significant margin. This may reflect better organization and structure in these regions, influenced by the effectiveness of their departmental authorities. While departments such as Guainía, Vichada, and Vaupés may have structured plans in place, their execution and implementation appear less effective, as reflected in their efficiency.

## 5. Conclusion

This study continues the application of the StoNED method, introduced by Kuosmanen and Kortelainen [24], to measure technical efficiency, particularly in the field of education. Additionally, the StoNEZD method, which incorporates environmental variables, was employed to enhance the analysis by accounting for specific contextual factors. Furthermore, the m-order technique was utilized to generate multiple random samples, ensuring a more stable, robust, and computationally efficient approximation [33]. Methodologically, this approach combines the strengths of StoNEZD and m-order, such as statistical consistency, homogeneity, and convergence, under less restrictive assumptions than other estimators, even in the presence of outliers and noise. Thus, the combination of these models means that the analysis performed in this study is much more robust than that of the other models used in similar studies. Similarly, it also means that the validation of the study's results is reliable, in accordance with the tests established for this type of model and in relation to the environmental developments analyzed.

This study identifies year-by-year trends in the efficiency of educational institutions from 2016 to 2019, corresponding to the administrative period of local governments in Colombia. On average, the results showed year-to-year variability, with a notable decline from 2017 to 2018. However, over the entire period, there is measured growth in efficiency indicators, suggesting the positive management of educational institutions. These findings align with various national developments during the evaluation period and demonstrate consistency with the evolution of administrative practices.

The departments of Bogotá and Antioquia emerge as the most efficient, followed - with a significant gap - by Caldas, Valle del Cauca, Santander, and Atlántico. In contrast, Guainía, Vichada, and Vaupés recorded the lowest efficiency coefficients, closely trailed by Guaviare, San Andrés, Amazonas, and Arauca, respectively. Contrary to the assumption that a higher number of institutions complicates administration, some departments with the most institutions achieved higher efficiency coefficients, whereas others with fewer institutions under their responsibility performed worse.

This discrepancy may be linked to the fact that departments with the largest number of institutions are among Colombia's most prominent ones. However, this does not mean that smaller departmentsin terms of territory, infrastructure, or populationnecessarily have lower efficiency coefficients. Some departments with fewer institutions have achieved higher efficiency than those with more institutions. Nonetheless, the results suggest that managing a larger number of institutions may foster better organizational and administrative structures. This could stem from accumulated experience, enhanced management capabilities over time, or a competitive drive to outperform that of other departments.

Finally, despite significant progress in managing educational institutions, particularly in rural areas, through expanded coverage, teacher training, technological resources, and strategic alliances, efficiency remains low in these regions. While 55,71% of the rural institutions showed substantial improvements in efficiency, public educational institutions fared slightly better, with 60,12% achieving higher efficiency coefficients. Overall, the national efficiency trends during the evaluation period indicate a gradual improvement, suggesting that management practices in most departments are advancing and are increasingly focused on achieving better results with available resources.

Despite the use of official sources (ICFES and DANE), this study has limitations associated with the quality and completeness of the databases. Potential errors in the recording and digitization of information were identified, as well as difficulties in accessing data from institutions in rural or vulnerable contexts. Additionally, the exclusion of institutions without complete information for the entire period between 2016 and 2019 and the lack of data on those that ceased operations affected the representativeness and continuity of the analysis.

Furthermore, future research should geared on a longitudinal analysis that allows for an examination of the evolution of institutional efficiency between 2016 and 2019, complementing the current perspective. Furthermore, it is proposed that the study be extended to subsequent periods, considering the impact of virtual education implemented during the pandemic. Additionally, comparisons were proposed between educational contexts differentiated by geographic location (urban vs. rural) and institutional nature (official vs. informal) to identify structural variations in the efficiency of the educational system.

## Declarations

*Author Contributions*

Conceptualization: A.A., V. G., and D.P.; methodology: A.A. and V.G.; software: A.A. and D.P.; validation: V.G. and D.P.; formal analysis: A.A.; investigation: A.A.; resources: A.A.; data curation: D.P.; writing—original draft preparation: A.A.;

writing—review and editing: V.G. and D.P.; visualization: A.A.; supervision: V.G. and D.P.; project administration: A.A.; funding acquisition: A.A., V.G., and D.P. All authors have read and agreed to the published version of the manuscript.

*Data Availability Statement*

Data available in a publicly accessible repository that does not issue DOIs: Publicly available datasets were analyzed in this study. These data can be found in the ICFES:
https://www2.icfes.gov.co/investigadores-y-estudiantes-posgrado/acceso-a-bases-de-datos
DANE:
http://microdatos.dane.gov.co/index.php/catalog/MICRODATOS/about_collection/25/2.

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*Conflicts of Interest*

The authors declare no conflicts of interest regarding the publication of this manuscript. In addition, ethical issues include plagiarism, informed consent, misconduct, data fabrication or falsification, and double publication or submission.

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