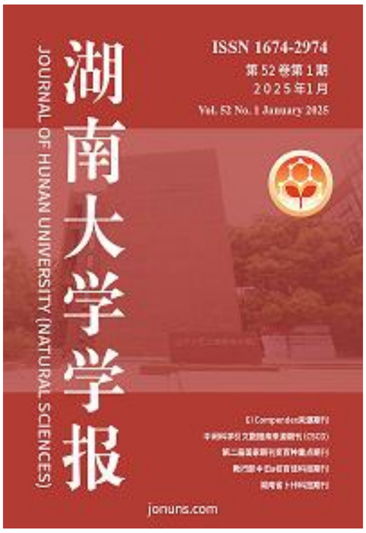

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Ritualized Non-Mathematics in Drawing Curves

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Abstract: This study aimed to investigate students' mathematical discourse related to the concavity, turning points, and maximum and minimum used in describing graphs on derivative use materials. More specifically, the characteristics of the mathematical discourse based on the commognition perspective is the focus of this study. Data were collected from 70 students who were given assignments on describing function graphs. The results of the students' assignment work are sorted. Every same answer is in one group. Likewise for different answers. Subjects were selected based on the two groups. Subjects who answered correctly and completely did not become research subjects. Furthermore, interviews were conducted with 11 selected students who were used as research subjects. A qualitative approach was used in this study. The results obtained from the 11 subjects interviewed were different discourses. The research findings revealed the existence of cognitive conflicts. The subject did not build a well-supported narrative. The researcher continued to explore the subject's ideas and knowledge about the task of describing curves on the derivative application material. There was a misalignment in their task completion and interview results. So that communitive conflicts, especially in ritualized non-mathematics. This is based on the incorrect procedure/statement given by the subject. When completing a task, keywords must first be understood and used properly. Furthermore, using definitions of keywords that have been agreed upon by experts in discourse. In addition, aspects of visual mediators and endorsed narrative are also needed. So that communitive conflict can be overcome.

Keywords: commognitive; ritualized non-mathematical; discourse; curve

绘制曲线时仪式化的非数学

摘要：本研究旨在调查学生在描述导数应用材料上的图形时，与凹度、转折点、最大值和最小值有关的数学话语。更具体地说，基于共同点视角的数学话语特征是本研究的重点。数据来自70名被分配描述函数图作业的学生。对学生作业的结果进行排序。每个相同的答案归为一组。不同的答案也是如此。根据两组选择受试者。回答正确和完整的受试者没有成为研究对象。此外，还对11名被选为研究对象的学生进行了访谈。本研究采用了定性方法。从11名被采访对象那里得到的结果是不同的话语。研究结果揭示了认知冲突的存在。受试者没有建立一个得到良好支持的叙述。研究人员继续探索受试者关于在导数应用材料上描述曲线的任务的想法和知识。他们的任务完成情况和访谈结果不一致。这样才能克服社区冲突，尤其是在仪式化的非数学领域。这是基于主体给出的不正确的程序/陈述。在完成任务时，首先必须理解并正确使用关键词。此外，在讨论中使用专家一致认可的关键词定义。此外，还需要视觉媒介和认可的叙述方面。这样才能克服社区冲突。

关键词：交流；仪式化非数学；话语；曲线

1. Introduction

Language has become the focus of inquiry in educational research, especially in mathematical education research. It has become the focus since language is considered as a window to indirectly see what is happening in the minds of students [1]. When there is a conceptualization of students that is considered incorrect, this is where language is often considered as a barrier to effective communication. Furthermore, in the “language mathematics education” research, several Psychology of Mathematics Education (PME) studies related to language have been discussed. Some examples are the focus on language and conceptualization, language in collective participation and embodiment, representation and use of symbols, and Vygotskian Semiotics. Furthermore, when there is silence, language also exists, which makes it technically everywhere [2]. Furthermore, according to [2], in mathematics education, attention to language is new, since researchers in this field used to investigate mathematical activities without thinking much about the language used in these activities.

This study uses the Sfard Approach, with a focus on the idea of participation in mathematical discourse. This forms the basis of individual mathematical commognition. As a result, learning becomes conceptualized as a change in discourse. Mathematical discourse is about mathematical objects and the relationships between them. Moreover, pedagogical discourse differs from mathematics in many ways. This paper focused more on the mathematical discourse. In connection with these discourse differences, ontic and deontic discourses are distinguished [3]. Ontic discourse talks about objects and the relationships between them in the physical world. The questions discussed in the discourse are about what is and how it works. Deontic discourse contains narratives about social context, which is about interactions between people. The narratives that make up this discourse answer the question of what is right to do and how to judge deontic discourse as it relates to the “good” or “right” way of treating learners [4].

Discourse-forming elements are a set of *routines* with repeated forms of communicative action. A repetitive pattern of communication is called a routine. Related to this, keywords and visual mediators that are used regularly in narratives are also called routine. Mathematical discourse includes routine, e.g., computation, problem-solving, and verification. In addition, it also includes new narratives that are proven [5] Activities involving mathematical objects and mathematical narratives are routine discourses [5]. Furthermore, mathematical discourse is divided into four aspects: narrative aspects, routines, keywords, and visual mediators. According to [6], two types of discursive routines are distinguished: exploration and ritual. Furthermore, three types of routines were identified in [5]. When working on mathematics, aspect routines are divided into *explorations, rituals, and* deeds [5]. As stated in [6], routine is the most basic thing where all creativity is rooted, or the media where students find their expression. Furthermore, mathematics learning distinguishes between *practical routines* and *discursive* routines (rituals and exploration). Routines can also be developed by individualizing processes [5-8]. Further developments demonstrated that aspects of routines can be classified into ritualized non-mathematics (incorrect procedure/statement), ritualized mathematics (correct procedure no justification) and exploratory (verification of narratives; working with unfamiliar tasks) [9]. This routine aspect focuses more on the student’s work process in completing the tasks given in this study.

Cognitively demanding learning, rich discourse, and student-centered learning have been the focus of various studies [10]. Broadly speaking, discourse change in mathematics learning can be divided into two types: Object-level learning and Meta-level learning. Explaining mathematical objects occurs in the object-level discourse. Here are some examples of object-level, namely, “2 + 3 = 5”, “the expression $2(x + y)$ can also be written as $2x + 2y$”, “$(a + b)^2 = a^2 + 2ab + b^2$” and “the sum of the angles in a triangle is 180°” which are mathematical objects. Another example in the field of geometry, with the narrative “The sum of the angles in a polygon with n sides is equal to $(n - 2)x180^{\circ}$.” At the meta-level discourse is discourse about discourse. At this level, there must be an explanation of why what is done is right. Furthermore, see the results, and even explain the obstacles obtained. The narrative at the meta-level is an implicit narrative in the learning process. Recent research has shown that problems with meta-rules may often lie in the different interpretations that students form of a particular task situation [11]. At this stage, meta-level learning includes learning to identify the given example as a particular mathematical object by checking whether it meets the mathematical definition. Students are central participants in certain discourses according to the communitive view. Furthermore, according to this view, a “misalignment” can be categorized into communitive conflicts. This happens if there are several students in different classes whose discourse cannot be compared. Cognitive conflict is a situation in which participants use the same word in different ways. Thus far, the conflict of cognition has been discussed more in the context of mathematical discourse and not in the context of pedagogical discourse.

Studies that emphasize the relevance of the context in which learning occurs based on Sfard’s cognitive approach have been numerous [12-16]. Cognitive theory assumes that learning is not a process in which a person changes certain cognitive structures in his mind, but rather a process of change in the participation routine in a particular community [17]. As a result, discourse analysis in the context of classroom learning includes clues about how learning occurs. As found in [4], cognitive conflicts in mathematics education research focused on students’ mathematical discourse. Therefore, thinking, according to the participatory framework, is conceptualized as communicating with oneself that is diachronic or synchronous. Communicate with others or with oneself, mostly verbally or with the help of other symbolic systems [5]. According to the psychology of communication about human thought, language is no longer a window for thinking, but is the determining element. Therefore, as long as thoughts are in language, both thoughts and speech certainly cannot be separated. This study focuses more on students’ mathematical routines and the use of mathematical language when giving explanations during interviews. This is also based on the tasks that have been done. Here the researcher uses communitive theory so that students’ mathematical communication can be clearly seen. can be done orally or in writing. Therefore, when students use keywords, visual mediators of narratives and routines can be used to measure their mathematical maturity and literacy [18].

2. Methods and Materials

2.1. Type of Study

The type of research conducted was exploratory research with a qualitative approach. Qualitative research has four characteristics: (1) focusing on the process, understanding and meaning; (2) researchers are the main instruments in data collection and analysis; (3) the research process takes place inductively; and (4) the presentation of data in descriptive form, namely in the form of words and images [19]. A total of 70 students were given assignments, as shown in Figure 1. From that number, only 50 students completed the task. The results of student work were all scanned to be checked one by one. Furthermore, eight students were selected to be the subjects of the study and interviews were conducted related to what they had done. Furthermore, to explore the subject commognition, the researcher added 3 more subjects. This was conducted by considering that the given answers were different. The total number of subjects interviewed was 11 students. In the data collection process, the supporting instruments used to obtain the research data were curve drawing tasks, interviews, and documentation.

> Discuss curves on $y = x^4 - 4x^3$ with respect to basins, turning points, and local maximums and minimums. Use this information to sketch curves (Stewart, 2010).

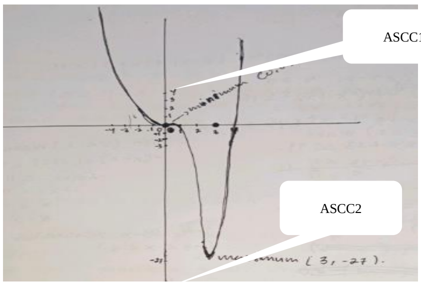
Figure 1. Task to describe the curve (developed by the authors based on [20])

The data analyzed in this study were interview transcript data, document data, and field notes. Data analysis used the interactive model analysis technique proposed in [21]. The quality of qualitative research was achieved by prolonged engagement with the subjects.

3. Results and Discussion

The assignment shown in Figure 1 was distributed to 70 second- and third-year college students. They were asked to understand the task. Each student was asked to mention an idea while working on a task describing the curve. Based on the findings of the subjects’ answers and interviews, there was a *cognitive conflict*. The subjects interviewed were given the initials AS, CHE, CC, EKA, FPT, FIT, LUH, NUR RA, RS. The following describes the findings of different subject discourses.

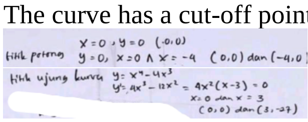
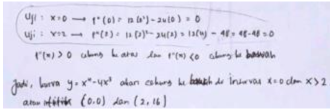
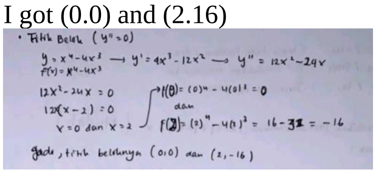
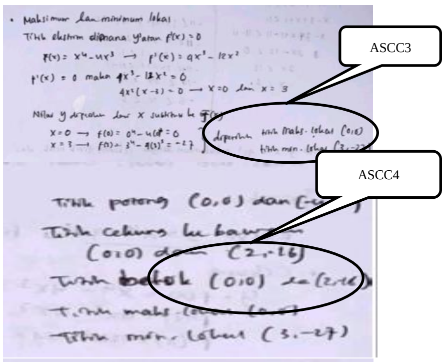
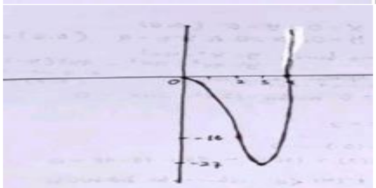
Table 1. Discussion transcript of the AS respondent (compiled by the authors)

Interview	Discussion
AS	Determine the turning point, $y'' = 0$ so that it is obtained $x = 0$ and $x = 2$
AS	To determine the turning point, we use the second derivative test
Researcher	How do I test it?
AS	Maximum and minimum local, we first complete $y' = 0$ I earn $x = 0$ and $x = 3$ Test the first derivative to determine whether these points are local maximums and minimums.  The minimum is at point (0.0), and the local maximum is at (3, -27).
Researcher	Analysis: does not have a local maximum or minimum on this question.

As seen from Table 1, the subject did not mention the word “concavity” which is clearly contained in the question. When re-confirmed, he said part of the turning point. To determine the turning point, we use the second derivative test. This means that the narratives mentioned by AS were not yet of true value. It should determine the inflection point using a second derivative equal to zero or usually written with $f''(x) = 0$. The next narrative, which was “test the first derivative to determine whether telepoints are local maximums and minimums” was true. This is a definition. Based on the researchers’ experience in the classroom, students often misunderstand the terms “first derivative test” and “second derivative test”. This results in students often using this incorrectly. Table 1 also shows that the subject identified the minimum at point (0.0), and the maximum local at (3, -27). When the picture in the table was seen, one function had no local minimum or local maximum in $fx = 0$.

The CHE respondent stated to determine the negligence with the second derivative test. Next, CHE said $f''(x) > 0$ concave up and $f''(x) < 0$ concave down. However, the subject was only limited in knowing the definition. He substituted the results of the $f''(x) = 0$ determination into the second derivative and produced a zero value.

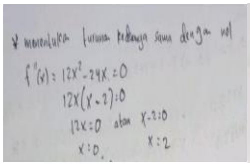
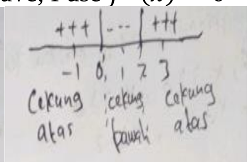
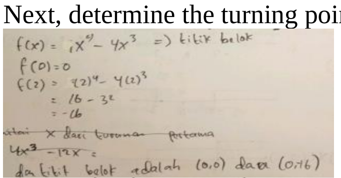
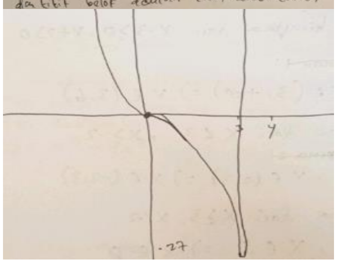
Table 2. Discussion with the CHE respondent (compiled by the authors)

Interview	Discussion
CHE	The curve has a cut-off point 
Researcher	Which end point of the curve?
CHE	Points (0.0) and (3, -27)
Researcher	Do you remember the endpoint definition?
	Silence
	Second derivative test’s concavity 
Researcher	At what interval is the concave?
CHE	$f''(x) > 0$Concave up and $f''(x) < 0$ concave down It is the concave downwards at intervals of x = 0 and $x > 2$ or points (0.0) and (2.16)
Researcher	How did it go? I did a trial-and-error, madam. The turning points, I define by $y'' = 0$ I got (0.0) and (2.16)  Next, we determine the local maximum and local minimum  
Researcher	Are the concave points and turning points the same?
CHE	I still doubt it, madam. I cannot describe it fully yet

The process that CHE did was deemed incorrect. CHE did not distinguish the definition of concave and inflection point, so in Table 2, he said the concave and inflection point were the same.

In Table 3, the CC subject determined the concavity using the formula. However, $f''(x) = 0$ was an incorrect narrative. After the interview, the subject meant that the points x = 0 and x = 2 were only used as a help point to determine the concavity. The respondent stated that the graph only had a local minimum at point (3, -27). The subject did not distinguish between determining the turning point and the concave.

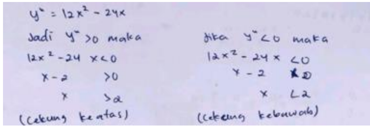
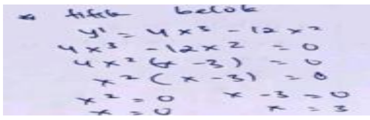
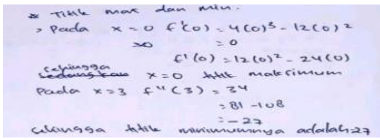
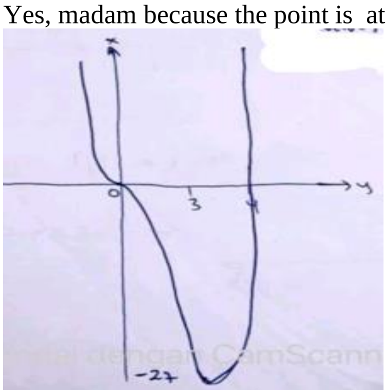
Table 3. The CC respondent's interview (compiled by the authors)

Interview	**Discussion**
CC	To determine the concave, I use $f''(x) = 0$ 
Researcher	What is the point $x = 0$ and $x = 2$ what do you mean by concave?
CC	No, madam. I use this point to help determine the concavity. Meanwhile, points -1, 1, and 3 are the test points. The upward concave I was marked with (+ + +) and the downward concave was marked with (--)
Researcher	Ok. Next, determine the turning point 
Researcher	Is point (0, -16) an inflection point?
CC	(check again). … I mean (2, -16) madam Next, we draw the chart. The minimum value of this chart at the moment $x = 3$ is point (3, -27)

Next, the EKA respondent described the function of approaching correctly. When examined one by one through interviews, there were many discrepancies between keywords and visual mediators. The interview of the EKA subject is shown in Table 4.

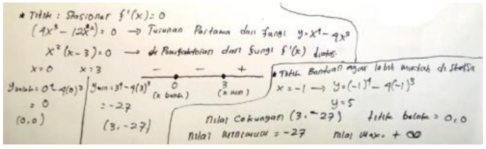
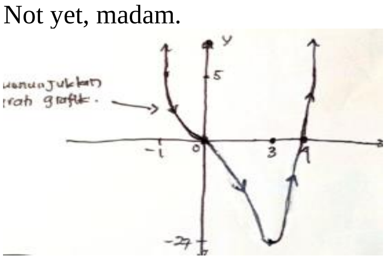
When determining the inflection point and the critical point, the EKA used $f'(x) = 0$. These are erroneous narratives. To determine the maximum and minimum values, he did not pay attention to traits or definitions. The maximum point placement was based only on point placement.

Table 4. The EKA respondent's interview (compiled by the authors)

Interview	Discussion
EKA	I began to determine the concurrency using the second derivative of the function. 
Researcher	Try to check again, why did you get $x - 2 >$ 0 and $x - 2 < 0$?
EKA	(silence and rewrite) Look, madam $12x(x - 2) < 0$, I $12x$ don't want to write it down because the value is zero right away. This will affect determining the curvature of the function
EKA	$x > 2$concave up and $x < 2$ concave down
EKA	The turning point is determined by the first derivative of the function 
Researcher	How to determine the critical point?
EKA	Same madam 
EKA	Maximum local at point (0.0) and minimum at (3, -27)
Researcher	What is the local maximum point (0.0)? Yes, madam because the point is at the top.  At point (3, -27) the local minimum.

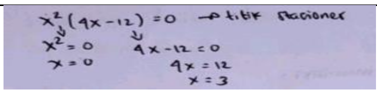
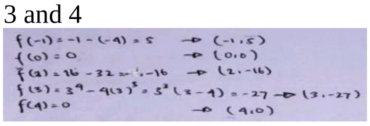
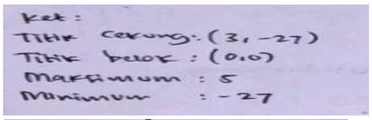
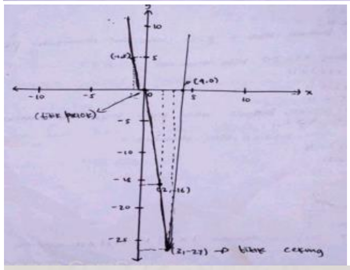
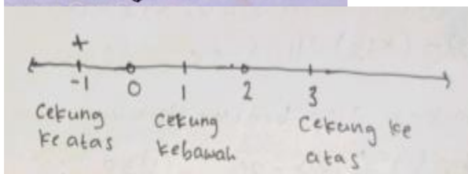
The FPT subject drew a curve that was almost correct, but it was not based on the definition or theorem on the application of derivatives. The subject used a number line as an aid to determine monotonicity. However, monotonicity was only determined by the arrow on the function. Point (3, -17) they say is the point of concavity and the minimum value. When asked, the FPT subject could not explain. The work process of this subject is categorized as ritualized non-mathematical because it gets an incorrect procedure/statement [9].

Table 5. The FPT respondent's interview (compiled by the authors)

Interview	Discussion
FPT	
Researcher	Why did you specify a stationary point?
FPT	The points x = 0 and x = 3 obtained can be used to facilitate drawing sketches and determine the maximum and minimum values.
	The number line you created, is it required?
FPT	I use it to help determine the monotony of the function.
	The (-) sign means monotonous down and (+) monotonous up
Researcher	Okay
Researcher	Point (3, -27) whether the concavity value or the minimum value
FPT	Same, madam.
Researcher	Why do you say the same?
Researcher	Could you please specify the definition of concavity?
	Not yet, madam.
	
Researcher	What do the arrows on the image mean?
	Marking the chart down and up madam.
	Hmm… monotony.

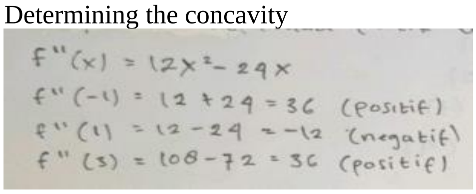
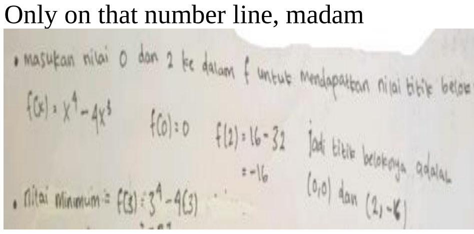
As shown in Table 6, the FIT respondent stated the concave points he obtained from taking points helped him draw the graph. The same is true for the maximum, minimum, and inflection point values. The subject could not base the argument on the nature, definitions, and theories contained in this material. This could be clearly observed in the subject's approach of trial-and-error.

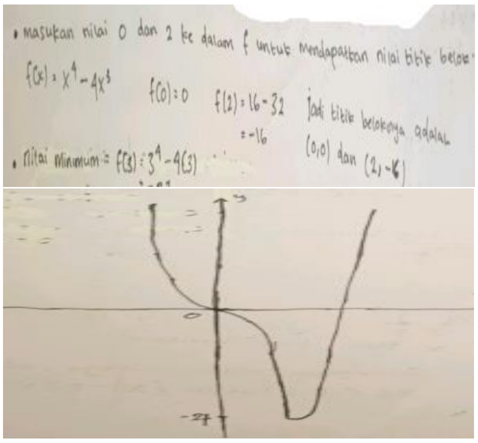
Table 6. The FIT respondent's interview transcript (compiled by the authors)

Interview	Discussion
FIT	
	First, I determined the stationary point, obtained at x = 0 and x = 3
Researcher	In the image caption, you got a concave point (3, -27). What is the process?
	I took other points, for example x = -1, 0.2, 3 and 4
FIT	
	Based on this, I put it on the coordinate axis.
FIT	  
Researcher	How do you determine the concave point, turning point, maximum, and minimum?
	I connected the points that I have placed on the coordinate axis.
Researcher	You don't use definitions or anything?
FIT	No, madam.

The LUH respondent only wrote a line to determine the concavity, but did not explain in which interval the concavity occurred. The graphs pictured appear to be true, but when asked, many are inappropriate.

Table 7 The LUH respondent's interview transcript (compiled by the authors)

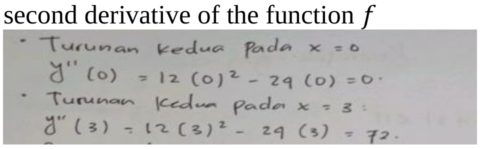
Interview	Discussion
LUH	First, I determined the critical point in the way that the first derivative f equals zero.
	I obtained points x = 0 and x = 2.
Researcher	Ok.
LUH	Determining the concavity
	
Researcher	Which are the concave intervals?
LUH	Only on that number line, madam
	
	The turning points were obtained by substituting points x = 0 and x = 2 into the function f



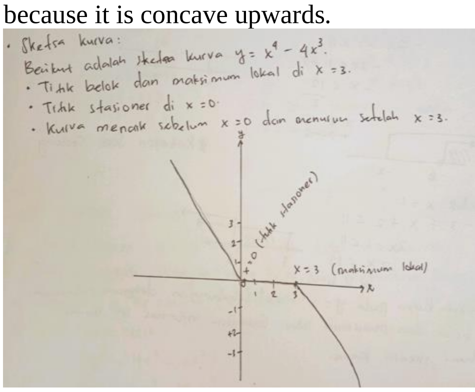
ILUH	Minimum value at f(3) = -27
Researcher	Where is the maximum value?
ILUH	The probability is zero if you look at the chart.

Table 8. The NUR respondent's interview transcripts with (compiled by the authors)

Interview	Discussion
NUR	Determining the stationary point, namely$y' = 0$, obtained x = 0 and x = 3. Subsequently, this point is subsumed into the second derivative of the function f



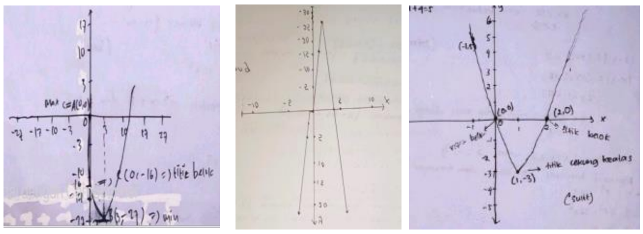
Researcher	What is it used for?
NUR	To determine the concavity.
	At the time, there is no information because the derivative of both is zero.$x = 0$
	At the moment $x = 3$, the concave is facing upward because $y''(3) = 72$.
	Turning points also occur here.
Researcher	Why do turning points also occur in $x = 3$?
NUR	Because the second derivative changes the sign from negative to positive.
	Which one changed the sign?
NUR	Definition of the turning point, madam
	Try defining it again?
NUR	The local maximum also occurs in $di\ x = 3$, because it is concave upwards.



	You also write the inflection point and local maximum point in $x = 3$
Researcher	Are you sure x = 3 is also the local maximum?
	Yes because the curve rises before x = 0 and decreases after x = 3
Researcher	Can you complete the caption on the image?
	No, madam.

As shown in Table 8, the NUR respondent argued that with $y''(0) = 0$ the value obtained, meaning that there was no concurrence at that point. There was an inflection point at x = 3 because the second derivative changed the sign of negative and positive. This means that the subject's narrative was not correct. However, the subject could not explain. At point x = 3, the subject said there was also an inflection point and a local maximum. Lack of understanding of a concept in mathematics can result in errors in solving mathematical problems [22].

After obtaining findings from eight subjects with the initials AS, CHE, CC, EKA, FPT, FIT, LUH, and NUR, the researcher added three more subjects to dig up information about describing the curve. The subjects that the researcher added were given the initials RA, RS and RU. Thus, the total sampling in this study included 11 respondents. The researcher found another form of the image that was made almost similar to the NUR respondent, as shown in Figure 2.



a) b) c)

Figure 2. (the authors elaboration)

a) Subject RA; b) Subject RS; c) Subject RU

As shown in Figure 2a, the RA respondent said that point (3, -37) was the minimum value. In Figure 2b, according to subject RS, there was no minimum value in the questions being worked on. Furthermore, the RU respondent in Figure 2c said that in the image made, point (3, -37) was a concave point upwards. These three subjects also differed in understanding point (3, -37) that they got. This happened to the previous eight subjects. The aspect of the respondent's ending narratives was not good because they did not try to build it. This can be seen from them only answering assignments according to what they understand. The researcher tried to continue to dig, but they did not know the answers. This agrees with the results of the study that students must try to build endorsed narratives [23]. Routines are pairs consisting of tasks and procedures [8]. If subjects perform routines according to their capacity without paying attention to whether the results are useful or not, this is called ritualistic or ritual [7]. In ritualistic participation, a student does not create new work. They imitate more procedures given by the teacher or based on textbooks. Sometimes they imitate via YouTube. This is because students' understanding of certain materials is still lacking. When to understand something means we are trying to build a convincing story about this thing. A meaningful story must be consistent, comprehensive, and cohesive [3].

Based on 11 subjects who had done interviews, they found differences in their discourse, which led to communitive conflicts. The existence of "misalignment" can be said to be a communitive conflict. This is in accordance with the keywords in the question, namely concave, turning point, and local maximum and local minimum. At this point, there were several different discourses. The subject found it difficult to distinguish between determining the concavity and determining the inflection point. Both of them they consider the same. It is also worth noting how the tangent turns when it moves from left to right along the chart. If the tangent turns steadily in the counterclockwise direction, the chart is *concave upwards*. If the tangent turns clockwise, the graph is said to be *concave downwards* [20, 24]. In the respondents' responses about the concave, some said the concave occurred at point (3, -37). Another subject stated that to determine the inflection point, the second derivative test should be used. It should determine the inflection point using the second derivative equal to zero or usually written with $f''(x) = 0$ or based on the change in the concavity. The turning point occurred at point (0.0) and point (2, -16) because there was a change in the concavity. A P-point on a curve $y = f(x)$ is called an inflection point if it is f continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward in P [20]. Mathematics is a unique language consisting of words, tables, and illustrations such as graphs and symbols [25].

At the time, the subjects drew the curve almost right, but based on interviews, they were not based on definitions or theorems. The narratives given by the subjects were not correct when determining the maximum and minimum local values. They tend to say that the maximum value was based on the high-low point on the curve. When used, the first derivative test had f no local maximum or local minimum at $x = 0$ When used, the second derivative test also found no information for the point$x = 0$. At the critical point $x = 3$, $f''(x) > 0$, $f(3) = -27$ which expresses the local minimum value. Under the first derivative test theorem, suppose c is the critical point of a continuous function$f$; (i) If f' changes from positive to negative in c, then it has a local maximum in c. (ii) If f' changes from negative to positive in c, then it has a local minimum in c. (iii) If f' does not change sign (for example, if it is positive on both sides or negative on both sides), then it has no local maximum or minimum in c [20]. In the second derivative test; suppose f'' is continuous near c. (i) If $f'(c) = 0$ and$f''(c) > 0$, then it has a local minimum at c. (ii) If $f'(c) = 0$ and $f''(c) < 0$and, then it has a local maximum at c. Based on the procedure carried out by the subject, it is included in the type of Ritualized non-mathematical, which means there are procedures and incorrect statements [9]. Research on the application of derivatives in mathematics education has shown promising results. However, misconceptions about derivatives still exist among students with various abilities, which often stem from weak prior knowledge and inadequate practice [26]. Furthermore, in line with this opinion, analyzing the visual mediators, routines, and meta-rules used in classroom discourse, but more importantly, how and when they are used, explains the mediation modes used in elementary calculus teaching [27].

4. Conclusion

Conflicts may occur within students because two overlapping discourses are already embedded in the students' heads. The existence of conflict in students' thinking will cause them to be doubtful and confused. If not resolved immediately, it will be embedded in their heads. Differences in mathematical discourse among students in studying a topic cause commoditize conflict. This occurs because of differences in the use of keywords, visual mediators, and routines of students' mathematical communication. Therefore, to overcome this problem, researchers suggest that in the classroom, teachers should not only pay attention to students' cognitive abilities but also their mathematical communication. Because the two cannot be separated. Misconceptions that often occur in the use of derivatives, such as in concavity analysis, inflection points, and extreme values, can be overcome with a deeper understanding and proper mathematical analysis. Expert opinion and related studies underline the importance of understanding the context of each concept and using appropriate derivative tests to ensure accurate results. Students often assume that a graph that goes up always shows a concave function upwards. Meanwhile, to determine concavity is the change in the rate of the slope of the graph, not the rise or fall off the graph. Likewise, for decreasing graphs, students often show concave functions downward. Furthermore, the inflection point always occurs at the point where the first derivative $f'(x)$ is equal to zero. In fact, the inflection point can occur even though the first derivative is not zero; what is important is the change in sign in the second derivative. Communitive conflict can be resolved by paying attention to how keywords are used. In this case (1) all participants must agree with the differences in keywords to be used, all can be used in general (2) Directly express the use of keywords. (3)

Agree with one use used by experts in a discourse. Commognition is an alternative to explain how humans develop to build their knowledge. This discourse cannot be separated from a person's thinking activity. Therefore, to understand how someone thinks mathematically can be done by understanding the discourse. Therefore, there is a term Incommensurable discourses in communitive conflict. This term is used when there are inconsistent keywords, as well as in visual mediators or routines when we communicate.

Declarations

Author Contributions

Conceptualization, R.L., and S.R.; methodology, R.L., S.R., and B.M.; software, M., and A.; validation, R.I., S.R., and BM.; formal analysis, R.L.; investigation, R.L., S.R., and B.M.; resources, R.L.; data curation, R.L.; writing—original draft preparation, R.L.; writing—review and editing, R.L., S.R., BM., and M.; visualization, M., and A.; supervision, R.L.; project administration, R.L.; funding acquisition, S.R. All authors have read and agreed to the published version of the manuscript.

Data Availability

The data presented in this study are available on request from the corresponding author.

Institutional Review Board Statement

Rigorous ethical guidelines were adhered to throughout the study to ensure participant privacy and data confidentiality, in compliance with institutional and national research standards.

Informed Consent Statement

Participation in the study was voluntary and informed consent was obtained from all participants prior to their involvement.

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

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