Journal of Hunan University

(Natural Sciences)

Vol. 51 No. 10

October 2024

Available online at

http://jonuns.com/index.php/journal/index

Open Access Article

https://doi.org/10.55463/issn.1674-2974.51.10.14

Development of Digital Tools to Improve Mathematical Argumentation Skills

Lia Budi Tristanti[1]*, Toto Nusantara[2], Syarifatul Maf'ulah[1]

([1] Mathematics Education Department, University of PGRI Jombang, East Java, Indonesia

[2] Mathematics Education Department, University Negeri Malang, East Java, Indonesia)

* Corresponding author: btlia@rocketmail.com

Article History:

Received: August 31, 2024
Revised: September 20, 2024
Accepted: September 29, 2024
Published: October 31, 2024

Abstract: This study aims to develop a digital tool to enhance mathematical argumentation skills. This study employs a Research and Development (R&D) approach. The development of this digital tool follows the ADDIE model, which consists of stages of analysis, design, development, testing, and evaluation. The subjects of the study were 120 second-semester students from the Mathematics Education Study Program at a university in Indonesia. The object of this study is a digital tool for argumentation. The research instruments included validation sheets, questionnaires for feedback from lecturers and students and tests. The research findings indicated that the digital tool for mathematical argumentation is valid, practical, and effective in improving students' mathematical argumentation skills. The creation of this digital tool is based on the integration of the Toulmin schema components and indicators of mathematical argumentation skills. This digital tool can be a good alternative to effectively address students' limitations in constructing, understanding, and presenting mathematical arguments. The novelty of this study is the development of a valid, practical, and effective digital device for mathematical argumentation to enhance students' mathematical argumentation skills.

Keywords: digital tools, mathematical argumentation, proof

开发数字工具以提高数学论证技能

摘要：本研究旨在开发一种数字工具来提高数学论证技能。本研究采用研发(R&D)方法。该数字工具的开发遵循ADDIE模型，该模型包括分析、设计、开发、测试和评估阶段。研究对象是印度尼西亚一所大学数学教育研究项目的120名第二学期学生。本研究的对象是一种数字论证工具。研究工具包括验证表、讲师和学生反馈问卷以及测试。研究结果表明，数学论证数字工具在提高学生的数学论证技能方面是有效、实用和有效的。该数字工具的创建基于Toulmin模式组件和数学论证技能指标的整合。该数字工具可以成为有效解决学生在构建、理解和提出数学论证方面的局限性的良好替代方案。本研究的新颖之处在于开发了一种有效、实用且有效的数字数学论证设备，以提高学生的数学论证技能。

关键词：数字工具、数学论证、证明

1. Introduction

The era of globalization, characterized by the outsized role of information and communications technology (ICT), refers to a period in which global interconnection, information exchange, and economic integration are increasing. In this era, information and communication technology (ICT) plays a very important role in almost every aspect of human life [1, 2]. Technology can be used for mathematics learning. A learning environment uses computer technology to support collaboration between individuals in understanding and working with dynamic mathematical objects created by users [3].

Along with advances in information technology, the use of multimedia has become a new way to teach mathematics [4]. However, mathematics learning methods in Indonesia are still based on traditional approaches, and the use of technology is still limited in daily mathematics learning activities [5]. As suggested in [6], integrating technology in mathematics learning, especially at the secondary school and college levels, seems very relevant and should be studied further in the context of education in Indonesia. Therefore, a digital mathematical argument tool was developed.

Digital mathematical argument tools are learning media that utilize electronic technology to demonstrate or prove the truth of a mathematical statement. These tools can take many forms, including interactive software, web applications, or online learning platforms specifically designed to help students understand mathematical concepts through logical steps. The digital tool for mathematical argumentation was developed as one of the suggestions by [7] and [8] that teachers are expected to not only guide students in understanding concepts and skills but also facilitate students’ ability to conduct investigations, build models, argue, and achieve certainty. in science and mathematics. Learning strategies combine collaborative elements and software technology to improve students’ argumentation abilities [3].

The findings of [9], which focus on the thinking process in constructing mathematical arguments, are closely related to the use of digital tools to enhance mathematical argumentation skills. Digital tools can help accelerate and strengthen the thinking process by providing visualization tools, real-time feedback, and collaborative platforms that enable the development of stronger and more structured arguments. Thus, technology plays an important role in supporting more interactive and effective mathematical learning, aligned with the thinking processes highlighted in [9].

Students’ argumentation skills must be improved because some students do not use deductive arguments [9-12]. In developing argumentation skills, students solving evidentiary problems must change non-deductive arguments into deductive arguments [13]. Therefore, the learning objective must gradually improve students’ argumentation skills to produce formal evidence [14].

Research related to the use of software to develop mathematical argumentation skills includes [3], the developing of student argumentation through small-group collaboration using the dynamic geometry software (DGS) environment. A web-based multimedia whiteboard system was developed in [15] to assist students in learning mathematical problem solving, where students are asked to criticize solutions and reply to other people’s arguments. Therefore, this study was conducted with the aim of developing digital tools to improve mathematical argumentation skills.

2. Literature Review

2.1. Digital Devices

Technology can provide equal access to mathematics for all students [16]. Technological tools, such as theorem proving, can play an important role in the development of mental and social flexibility skills, creativity, collaboration, etc. [17]. Educators must maximize the benefits of technology in mathematics education in the form of digital devices.

2.2 Mathematical Argument Ability

Building an argument is not limited to simply connecting premises and conclusions. To achieve a more in-depth argument analysis, the use of Toulmin's scheme is important [18]. The Toulmin scheme comprises data, claims, warrants, backing, disclaimers, and qualifiers. Data are information or facts used to support claims. A claim is a statement that will be proven to be true. Warrants are statements that link data to claims and provide a basis or justification for drawing conclusions from the data. Backing is additional information or support that supports a warrant, often in the form of legal basis or other evidence. A disclaimer is a condition or exception that can limit the validity of a claim, and a qualifier is an expression that indicates the strength of the relationship between the data and claim provided by the warrant. Understanding and using these components effectively, stronger and more detailed arguments can be built. The Table 1 describes the elements of student ability in mathematical argumentation [19].

Table 1. Components of mathematical model of student ability in mathematical argumentation (compiled by the authors)

Argumentation Ability	Indicator
Perfection of each element in preparing a mathematical argument	a) Presenting data and claims b) Explain warrants c) Provide trustworthy support d) State the qualifier and disclaimer
The quality of mathematical arguments	a) Uses deductive logic accurately b) Achieve agreement from the audience regarding the truth of the argument c) Make the audience accept and believe the argument without raising doubts

3. Methods

This research method uses Research and Development (R&D), which is directed at developing digital argumentation tools to improve students' mathematical argumentation abilities. This digital device uses the ADDIE model, which consists of analysis, design, development, testing, and evaluation stages (Figure 1).

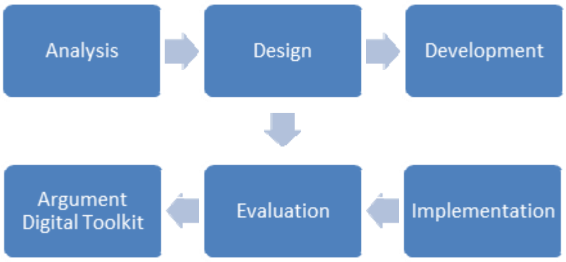

Figure 1. Digital Device development stages using the ADDIE model (developed by the authors)

The analysis stage includes an analysis of digital devices, curricula, and argumentation skills. The purpose of digital device analysis is to determine the use of digital devices developed during learning. Digital device analysis was performed using library research. Curriculum analysis is conducted by examining the curriculum being implemented at the research site. An analysis of argumentation abilities was conducted to determine the students' initial abilities.

The design stage is to produce an initial product (prototype) or product design that is adapted to the analysis of digital devices, curriculum and argumentation skills that have been carried out. The activities carried out included format selection, initial framework, and instrument selection.

The development stage is to elaborate a design for the digital argument tool. Steps in this development stage include: developing a framework design for digital argument tools and instruments that have been created at the design stage, evaluating digital argument tools conducted by experts who are competent in the field of digital tools and mathematical materials, revising digital tools based on suggestions and criticism from experts' revisions to the digital argument tool which was developed based on a questionnaire and observation sheet obtained during the trial.

The implementation stage involved learning assisted by digital argumentation devices on research subjects on a small scale. This stage carried out a pretest-posttest to measure the effectiveness of increasing students' mathematical argumentation skills before and after learning, using digital argumentation tools.

The evaluation stage involves final analysis of digital devices, curriculum and argumentation skills that have been carried out. The activities carried out included format selection, initial framework, and instrument selection.

The research object is a digital argument device. The participants of this research were 120 students in the second semester of the Mathematics Education Study Program at a university in Indonesia. The selected students were from the Mathematics Education Program, which means they had relevant backgrounds and an understanding of the mathematical argumentation material to be explored in this study. Second-semester students were chosen because they are at an early stage in their mathematics education. At this stage, they begin to learn fundamental concepts and principles of mathematical argumentation. This provides an opportunity to evaluate the enhancement of students' abilities in mathematical argumentation using the developed digital tools. The sample size of 120 students aimed to provide a representative analysis and stronger statistics to obtain more varied data and enhance the generalizability of the research results. The students were selected from a university with a similar curriculum to ensure that they had comparable learning experiences. This reduces external variability that could affect research outcomes, such as differences in teaching methods or materials taught.

The research instruments included validation sheets, lecturers, student response questionnaires, and tests. The validation sheet was submitted to digital device experts and material experts to obtain comments, suggestions, or criticism as a basis for product revision in determining the suitability of the digital device argument. The lecturer's response questionnaire sheet was filled out by the lecturer to determine the lecturer's response to the digital argumentation tool being developed. Student response questionnaire sheets were filled out by the students to determine student responses to the digital devices being developed. This test is structured based on the indicators of mathematical argumentation ability. This test was used to measure students' mathematical argumentation abilities before and after using digital argumentation tools.

Data obtained from the validation sheets, lecturers, and student response questionnaires were analyzed using a Likert scale. Each answer chosen by the respondent was multiplied by 100% to obtain the Feasibility Percentage (PK). This percentage was s then converted into the eligibility criteria, according to Table 2.

Table 2. Eligibility criteria for argument digital devices (compiled by the authors based on [20])

	Eligibility Criteria		**Information**
	Validity	**Practicality**	
PK > 80	Very Valid	Very Practical	No Revision Required
60 < PK ≤80	Valid	Practical	No Revision Required
40 < PK ≤ 60	Fairly Valid	Quite Practical	Minor Revisions
20 < PK ≤ 40	Less Valid Invalid	Less Practical	Revision
PK ≤ 20	Fairly Valid	Impractical	Revision

Analysis of student evidentiary ability test data was carried out using the Normalized Gain Score (N-Gain), and the n-Gain value was converted based on the criteria shown in Table 3.

Table 3. N-gain value criteria (Source [21])

Criteria Value	Range Criteria
N-gain > 0.7	High
0.3 ≤ N-gain ≤ 0. 7	Medium
N-gain < 0. 3	Low

4. Results

4.1. Data from Learning Observations, Questionnaires Identifying Aspects and Creativity Indicators

Observations were carried out during the learning process of mathematics instructional media courses, which referred to the Semester Learning Plan, to identify emerging aspects and indicators of student creativity. Next, students completed a questionnaire regarding aspects and indicators to identify their creativity in constructing mathematics instructional media. Table 1 shows the results of the observations and Table 2 shows the student questionnaire on the aspect of creating new ideas.

The following is a description of the results of developing the Digital Argument Tool using ADDIE stages.

4.2. Analysis

Observations were made during n lectures on linear algebra, analysis, abstract algebra, plane geometry, space geometry, calculus, and Functions of Complex Variables. The curriculum in this course solves more evidentiary problems where students not only use deductive logic accurately, but must also have argumentation skills to convince the audience.

Lecturers have applied a blended learning method. More students chose to use Android-based smartphone devices, whereas the rest used computers or laptops. The campus also provides learning facilities using technology, with the availability of digital smart classrooms. This shows that technology development is appropriate for learning applications. Internet users actively use technology for educational purposes, entertainment, and social media [22].

4.3. Design

The design stage produces a product design or initial product that is adjusted to the analysis that has been carried out. The components in the main menu display include the Study of Mathematical Argument Theory, Practice in Compiling Evidence, Practice in Preparing Mathematical Arguments using the Toulmin scheme, as shown in Figure 2.

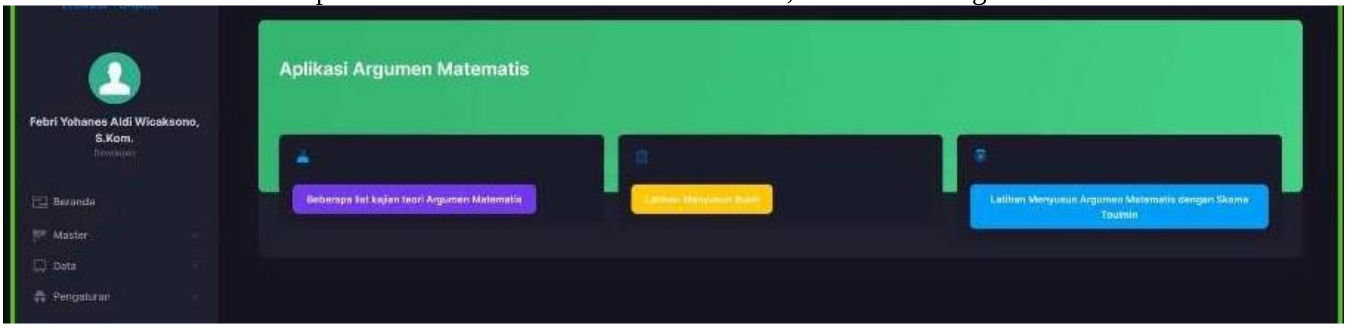

Figure 2. Main menu display (developed by the authors)

Figure 3. Mathematical argument (developed by the authors)

The components in the mathematical argument menu display include statements, strategies and proof steps as in Figure 3. The statement contains the mathematical statement to be proven, students can choose one of the statements presented. The strategy contains the choice of strategy to be used in the proof g and the reasons for using the strategy as shown in Figure 4.

At this stage, research instruments were also designed, consisting of validation sheets, lecturers, and student response questionnaires and tests. The validation sheet was used to assess and provide suggestions for the development of digital argumentation tools by learning material and technology experts. Lecturer and student response questionnaires were used to determine lecturers' and students' responses to the digital argumentation tools developed. The test consisted of a pre-test and post-test based on indicators of students' mathematical argumentation abilities.

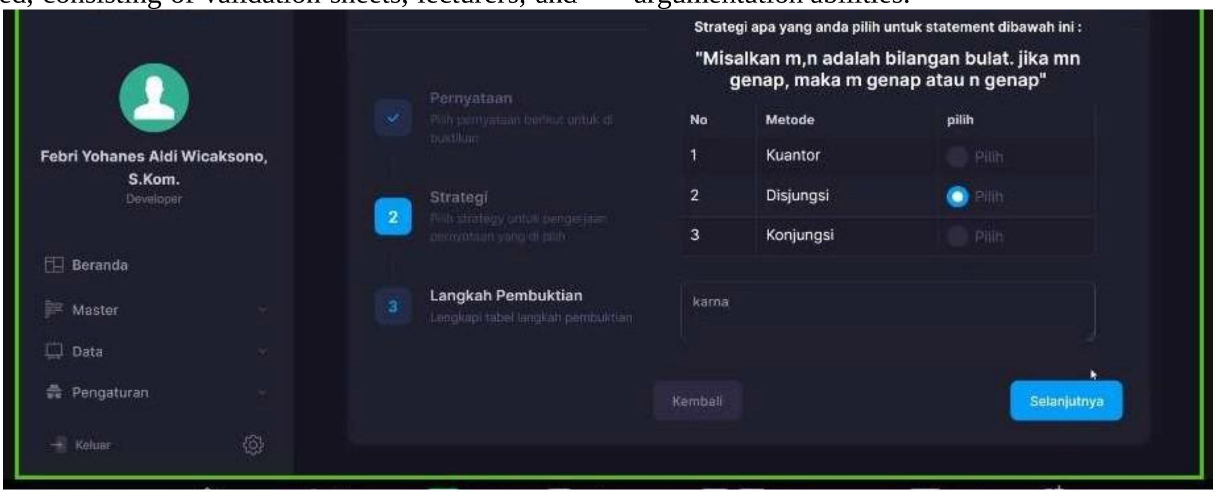

Figure 4. Strategy view (developed by the authors)

4.4. Development

This stage involves the development of a design framework for digital arguments and instrument tools created during the design stage. The developed product was a digital argument device. Furthermore, product validation was developed by material experts and learning technology experts. Material validation was conducted by lecturers in mathematics education. Meanwhile, learning technology validation was conducted by information technology education lecturers. The expert assessment results are given in Tables 4 and 5 as were recapitulated.

Table 4. Material expert validation results (compiled by the authors)

No	Aspect	Criteria	Score	
			Validator 1	Validator 2
1	Format	The material presented is attractive to students	5	4
		The mathematical argument application can be used as an independent learning medium	4	5
2	Contents	Mathematical material and problems are related to mathematical arguments	4	4
		The material and mathematical problems presented are arranged in an orderly manner	3	4
		The material and problems presented are in accordance with student characteristics	4	3
		Images, writing, and animation can be visualized	4	3
		Teaching material is explained in depth and clearly mathematical argument concepts	4	4
		The material and problems presented are sufficient to develop students' mathematical argumentation skills	4	4
		Problems correspond to success indicators	4	3
3	Language	The language used complies with adjusted spelling	3	4
		The language used is easy to understand	4	4
		Total Score	47	49
		Eligibility Percentage (PK)	78%	76%
		Validity Category Information	Valid No revision required	Valid No revision required

Table 4 shows the assessment results from the material experts for the valid criteria. Therefore, a product developed in the form of a mathematical argument application is suitable for use without revision. Expert validators also provided several comments and suggestions for improving product quality. The material expert validator's suggestion is to add various examples of questions so that students' proving and argumentative abilities can be better understood.

Table 5. Media expert validation results (compiled by the authors)

No	Aspect	Criteria	Score	
			Validator 1	Validator 2
1	Simplicity	Ease of operation/Use of mathematical argument applications	4	4
		Mathematical arguments application supports android devices	4	4
		Application of Mathematical Arguments according to student characteristics	3	4
		Videos and animations are understandable and appropriate to teaching materials and mathematical arguments	4	3
		The sentences used are concise, clear and understandable	5	4
2	Integration	The order between pages corresponds to the order of the material and the order of constructing mathematical arguments	4	4
		The instructions used comply with content of the mathematical argument application	4	4
		The symbols used conform to mathematical rules	4	4
		Total Score	32	31
		Eligibility Percentage (PK)	80%	77%
		Validity Category Information	Valid No revision required	Valid No revision required

Table 5 shows the assessment results from the learning technology experts for the valid criteria.

Therefore, a product developed in the form of a digital device is suitable for use without revision. Expert validators also provided several comments and suggestions for improving product quality. The advice from validators who are learning technology experts is that the icon animation is too stiff and can be changed to another animation so that it is not monotonous.

4.5. Implementation

The implementation stage involved testing the product on research subjects at a small scale. Students logged in via the link https://argumentmatematik.com/login, a pretest was carried out, and after implementation, a post-test was carried out to measure the effectiveness of increasing the ability to prove before and after using the digital argument device product.

After product implementation, lecturers and students were asked to complete a questionnaire regarding the use of digital argumentation tools. This measure measures the level of practicality of the product being developed. The recapitulation results of the lecturer and student response assessments are presented in Tables 6 and 7, respectively.

Table 6. Results of recapitulation of lecturer response assessments regarding the level of product practicality (compiled by the authors)

No	Statement	Score
1	The overall appearance of the digital argument device is interesting	4
2	Suitability of digital device component layout arguments	4
3	Suitability of format and display of digital argument devices	4
4	Argument digital devices are easily accessible	5
5	Digital argument tools can be used independently by students	4
6	The content of the material on the digital argument device is in accordance with learning outcomes	4
7	The material on the digital argument device is presented precisely, clearly and coherently	4
8	Correspondence of mathematical symbols and sentences for digital argument devices	4
9	Practice and evaluation questions on digital argument tools are in accordance with learning outcomes	4
10	Digital argumentation tools can train students’ abilities in argumentation	4
	Total Score	41
	Eligibility Percentage (PK)	82%
	Validity Category	Very valid
	Information	No revision required

Table 7. Results of recapitulation of student response assessments regarding the level of product practicability (compiled by the authors)

No	Statement	Score
1	The digital display of the arguments presented made my interested in following the lesson	4
2	The argument digital device makes it easier for me to use	5
3	I have fast access to open argument digital tools	4
4	Digital argumentation tools make me more motivated to learn argumentation	4
5	The display of the digital argument device is systematic	4
6	I can understand the language used in digital argument tools	4
7	The material on the digital argument device is presented precisely, clearly and coherently	5
8	The suitability of mathematical symbols and sentences in digital argumentation devices	4
9	Practice and evaluation questions on digital argument tools according to learning outcomes	4
10	Digital argumentation tools train my abilities in argumentation	4
	Total Score	42
	Eligibility Percentage (PK)	84%
	Validity Category	Very practical
	Information	No revision required

Tables 6 and 7 show the results of the lecturers’ and students’ responses to the level of product practicality in the practical criteria. Therefore, a product developed in the form of a digital device is suitable for use without revision. The results of the pre- and post-test, which show students’ mathematical argumentation abilities, are presented in Table 8.

Table 8. Descriptive statistics of pretest-posttest scores for argumentation ability (compiled by the authors)

	N	Range	Min.	Max.	Sum	Mean	Std. dev.	Variance
Pre-test argumentation ability	60	22.00	43.00	65.00	3180.00	53.00	4.40724	19.424
Post-test argumentation ability	60	15.00	80.00	95.00	5220.00	87.00	3.53194	12.475
Valid N (listwise)	60							

Based on Table 8, the average pretest score is 53, the average posttest score is 87, and the maximum score is 100, so we get an N-gain value = 0.72 in the high criteria.

$$N - Gain = = \frac{Posttest\ scores - Pretest\ scores}{Maximum\ score - Pretest\ scores} =$$
$$= \frac{87 - 53}{100 - 53} = 0.72$$

Based on data analysis of the assessment results from learning material and technology experts, it was concluded that the developed product was suitable for use with revisions in accordance with the suggestions and comments of validators from mathematics material experts and learning technology experts. The results show that digital devices can improve students' mathematical argumentation skills. This shows that digital technology not only facilitates students' learning but also enables them to develop important creative and cognitive skills. The use of digital technology stimulates students to creatively produce new knowledge [23], while product development through digital creativity can improve their creative abilities, visual interactions, and cognitive abilities [24]. Digital products play an important role as intermediaries in online learning. This stimulates students to develop new ideas and adapt to flexibility [25].

The n-Gain calculation result of 0.72 is in the high category. This shows that the increase in students' argumentation skills was high after learning using digital devices. Electronic digital devices can inspire students' creativity and improve their ability to self-regulate [26]. As stated in [25], digital products act as effective mediators in online learning, triggering students to develop new ideas, adapt flexibly, and manage learning independently. Developing digital-based learning products is a new and effective way to attract students' interest in learning so that their' creativity in using these products can develop their abilities [27].

Digital tools can enhance mathematical argumentation skills, because students can visualize abstract concepts. These visualizations make mathematical arguments more concrete and understandable, particularly in the context of proofs and logical reasoning. Visualization can also strengthen students' critical thinking processes when constructing and communicating their arguments. Digital tools provide real-time feedback that can immediately indicate the strengths or weaknesses of the student's argument. Students can make immediate improvements based on this feedback, ultimately strengthening their critical thinking skills. Additionally, analytical data from digital tools help measure students' progress in effectively constructing arguments. Research by [28] highlights the importance of technology in facilitating more engaging and interactive mathematics learning, where technology can transform how students interact with mathematical concepts and arguments. Using digital tools, students can explore various approaches to constructing arguments or proofs, thereby enriching their understanding of the concepts being studied.

Research by [29] focused on cooperative and problem-based learning models, and these findings are highly relevant to the use of digital tools to enhance mathematical argumentation skills. Digital tools can integrate the principles of cooperative and problem-based learning in a more interactive, collaborative, and adaptive manner. Thus, digital tools serve as a means of enriching these learning approaches, providing additional support for students to develop stronger mathematical argumentation skills.

5. Conclusion

The findings of this study indicate that the digital mathematical argumentation tool is valid, practical, and effective in improving students' mathematical argumentation skills. This study contributes new knowledge by introducing a tool based on Toulmin's argument model, which is rarely implemented in the digital form for mathematics education. Previously, some studies focused on developing general mathematical skills or problem-solving abilities; however, this research specifically measured and evaluated mathematical argumentation, an important cognitive aspect that is often overlooked in mathematics education. This study provides a new literature on the use of digital technology to support argumentation-based learning. Teaching mathematical argumentation has rarely used digital tools, making this article a new contribution to mathematics teaching methods.

The development of digital tools to enhance mathematical argumentation skills has significant implications for mathematics education in Indonesia, particularly for improving learning quality, developing critical thinking skills, and incorporating technology in teaching. Lecturers now have a more effective tool for teaching abstract and often difficult-to-understand concepts of mathematical argumentation, providing a more interactive approach than traditional teaching methods.

Based on the results of this study, future research should test this tool at various educational levels, such as secondary or elementary education, to assess its effectiveness in improving mathematical argumentation skills at different learning stages. Further development of this tool may involve additional features, such as collaborative tools that allow students to interact and work together to construct mathematical arguments as a team. This feature expands the potential use of the tool in more dynamic and collaborative learning environments.

Declarations

Author Contributions

Conceptualization, L.B.T.; methodology, L.B.T.; software, T.N.; validation, L.B.T.; formal analysis, L.B.T. and S.M.; investigation, T.N.; resources, L.B.T.; data curation, L.B.T. and T.N.; writing—original draft preparation, all authors contributed equally; writing—review and editing, L.B.T.; visualization, S.M.; supervision, T.N.; project administration, L.B.T. All authors have read and agreed to the published version of the manuscript.

Data Availability Statement

The data presented in this study are available in this article.

Funding

This research was funded by the Directorate General of Higher Education, Research, and Technology (DGHERT) of the Ministry of Education, Culture, Research, and Technology (MOECRT) of the Republic of Indonesia (No. Letter: 0557/E5.5/AL.04/2023).

Acknowledgements

This study is the result of collaborative research funded by The Directorate General of Higher Education, Research, and Technology (DGHERT) of the Ministry of Education, Culture, Research, and Technology (MOECRT) of the Republic of Indonesia (No. Letter: 0557/E5.5/AL.04/2023), dated June 1, 2023, available at https://bima.kemdikbud.go.id/pengumuman, name of the recipient: Toto Nusantara, title: Pengembangan Aplikasi Argumen Matematis dengan Model Infusion Learning Untuk Meningkatkan Kemampuan Argumentasi Mahasiswa (in Indonesian)/Developing a mathematical argumentation application with the infusion learning model to improve students' argumentation ability (in English). We also thank the Head of the State University of Malang and Universitas PGRI Jombang for allowing us to conduct this collaborative research.

Institutional Review Board Statement

The study was conducted in accordance with the Declaration of Helsinki and approved by the Institutional Review Board of the State University of Malang and Universitas PGRI Jombang, Indonesia.

Informed Consent Statement

Informed consent was obtained from all subjects involved in the study.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

References

[1] BOKHOVE C. *Use of ICT for Acquiring, Practicing and Assessing Algebraic Expertise*. Utrecht, The Netherlands: Fisme Scientific Library, Utrecht University, 2011.

[2] JUPRI A., DRIJVERS P., and VAN DEN HEUVEL-PANHUIZEN M. Improving Grade 7 Students' Achievement in Initial Algebra Through a Technology-Based Intervention. *Digital Experiences in Mathematics Education*, 2015, 1: 28–58. https://doi.org/10.1007/s40751-015-0004-2

[3] FUKAWA-CONNELLY T., and SILVERMAN J. The Development of Mathematical Argumentation in an Unmoderated, Asynchronous Multi-User Dynamic Geometry Environment. *Contemporary Issues in Technology and Teacher Education*, 2015, 15(4): 445–488. http://dx.doi.org/10.34944/dspace/6278

[4] NAJJAR L. J. Principles of Educational Multimedia User Interface Design. *Human Factors*, 1998, 40(2): 311–323. https://doi.org/10.1518/001872098779480505

[5] MURTIYASA B. *Pemanfaatan Teknologi Informatika Dan Komunikasi Untuk Meningkatkan Kualitas Pembelajaran Matematika*. Surakarta: FKIP Univ. Muhammadiyah Surakarta, 2012.

[6] JUPRI A. Peran Teknologi Dalam Pembelajaran Matematika Dengan Pendekatan Matematika Realistik. Prosiding Seminar Nasional Matematika Dan Pendidik, 2018.

[7] PRUITT S. L. The Next Generation Science Standards. The features and challenges. *Journal of Science Teacher Education*, 25(2): 145-156.

[8] L. S. NGSS, *Next Generation Science Standards: For States, by States*. Washington, DC: The National Academies Press, 2013.

[9] TRISTANTI L. B. The Process of Thinking by Prospective Teachers of Mathematics in Making Arguments. *Journal of Education and Learning*, 2019, 13(1): 17-24, http://doi.org/10.11591/edulearn.v13i1.6853.

[10] INGLIS M., MEJIA-RAMOS J. P., and SIMPSON A. Modelling mathematical argumentation: The importance of qualification. *Educational Studies in Mathematics*, 2007, 66(1): 3–21, http://doi.org/10.1007/s10649-006-9059-8.

[11] TRISTANTI L. B., and NUSANTARA T. Improving Students' Mathematical Argumentation Skill through

Infusion Learning Strategy. *Journal of Physics: Conference Series*, IOP Publishing, 2021, 1783: 012103. http://doi.org/10.1088/1742- 6596/1783/1/012103.
[12] TRISTANTI L. B., SUTAWIDJAJA A., ASÂ A. R., and MUKSAR M. The Construction of Deductive Warrant Derived from Inductive Warrant in Preservice-Teacher Mathematical Argumentations. *Educational Research and Reviews*, 2016, 11(17): 1696–1708, https://doi.org/10.5897/ERR2016.2872.
[13] HAREL G., and SOWDER L. Students' Proof Schemes: Results from Exploratory Studies. *CBMS Research in Collegiate Mathematics Education*, 1998, III (7): 234–283.
[14] HAREL G. The Development of Mathematical Induction as a Proof Scheme: A Model for DNR-Based Instruction. In CAMPBELL S., & ZAZKIS R. (Eds.) *Learning and Teaching Number Theory*. Ablex Publishing Corporation, New Jersey, 2001, 185–212.
[15] HWANG W. Y., CHEN N. S., and HSU R. L. Development and Evaluation of Multimedia Whiteboard System for Improving Mathematical Problem Solving. *Computers and Education*, 2006, 46(2): 105–121, https://doi.org/10.1016/j.compedu.2004.05.005.
[16] NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM) *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.
[17] HANNA G. Proof, Explanation and Exploration: an Overview. *Educational Studies in Mathematics*, 2000, 44(1): 5–23.
[18] TOULMIN S. *The uses of argument*. UK: Cambridge University Press, 2003. https://doi.org/10.2307/2183556.
[19] TRISTANTI L. B., and NUSANTARA T. The Influence of Infusion Learning Strategy on Students' Mathematical Argumentation Skill. *International Journal of Instruction*, 2022, 15(2): 277–292, https://doi.org/10.29333/iji.2022.15216a.
[20] MAHUDA I., MEILISA R., and NASRULLAH A. Pengembangan Media Pembelajaran Matematika Berbasis Android Berbantuan Smart Apps Creator Dalam Meningkatkan Kemampuan Pemecahan Masalah. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 2021, 10(3): 1745–1756. http://dx.doi.org/10.24127/ajpm.v10i3.3912
[21] NURHAIRUNNISAH N., and SUJARWO S. Bahan ajar interaktif untuk meningkatkan pemahaman konsep Matematika pada siswa SMA kelas X. *Jurnal Inovasi Teknologi Pendidikan*, 2018, 5(2) 192–203. http://dx.doi.org/10.21831/jitp.v5i2.15320
[22] ALIM N., SAAD M. S., MAHMUD H., and GUNAWAN F. Usability and Satisfaction of Google Classroom as an Instructional Teaching and Learning Medium: The Students' Perspective. *World Transactions on Engineering and Technology Education*, 2021, 19(1): 16–20.
[23] YANG H., and ZHANG L. Promoting Creative Computing: Origin, Scope, Research and Applications. *Digital Communications and Networks*, 2016, 2(2): 84–91, https://doi.org/10.1016/j.dcan.2016.02.001
[24] CYBULSKI J. L., KELLER S., NGUYEN L., and SAUNDAGE D. Creative Problem Solving in Digital Space Using Visual Analytics. *Computers in Human Behavior*, 2015, 42: 20–35, https://doi.org/10.1016/J.Chb.2013.10.061.
[25] NOROUZI B., KINNULA M., and IIVARI N. Digital Fabrication and Making with Children: Scrutinizing Adult Actors' Strategies and Challenges in Mediating Young People's Activities. *International Journal of Child-Computer Interaction*, 2021, 28(C): 100267, https://doi.org/10.1016/j.ijcci.2021.100267
[26] ATWOOD-BLAINE D., RULE A. C., and WALKER J. Creative Self-Efficacy of Children Aged 9-14 in a Science Center Using a Situated Mobile Game. *Thinking Skills and Creativity*, 2019, 33: 100580, https://doi.org/10.1016/j.tsc.2019.100580.
[27] PANSKYI T., ROWINSKA Z., and BIEDRON S. Out-of-school assistance in the teaching of visual creative programming in the game-based environment – Case study: Poland. *Thinking Skills and Creativity*, 2019, 34: 100593, https://doi.org/10.1016/j.tsc.2019.100593.
[28] TRISTANTI L. B., and IFFAH J. D. N. Pengembangan Media Pembelajaran Geometri Ruang Berbasis Android Berbantuan Smart Apps Creator dalam Meningkatkan Kemampuan Pembuktian. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 2022, 11(3): 1716–1728. http://dx.doi.org/10.24127/ajpm.v11i3.5103
[29] TRISTANTI L. B., and NUSANTARA T. The Advantage and Impact of CIRC-Typed and Problem-Based Cooperative Learning Models on Students' Mathematical Argument. *Proceedings of the 2nd International Conference on Education and Technology (ICETECH 2021)*. Atlantis Press, 2022, pp. 172-178, https://doi.org/10.2991/assehr.k.220103.026

参考文:

[1] BOKHOVE C. 使用 ICT 获取、练习和评估代数专业知识。荷兰乌得勒支：乌得勒支大学 Fisme 科学图书馆，2011。
[2] JUPRI A.、DRIJVERS P. 和 VAN DEN HEUVEL-PANHUIZEN M. 通过基于技术的干预提高 7 年级学生的初级代数成绩。数学教育中的数字体验，2015，1：28-58。https://doi.org/10.1007/s40751-015-0004-2
[3] FUKAWA-CONNELLY T. 和 SILVERMAN J. 在无主持、异步多用户动态几何环境中发展数学论证。当代技术与教师教育问题，2015，15(4)：445–488。http://dx.doi.org/10.34944/dspace/6278
[4] NAJJAR L. J. 教育多媒体用户界面设计原理。人的因素，1998，40（2）：311-323。https://doi.org/10.1518/001872098779480505
[5] MURTIYASA B. 利用信息和通信技术提高数学学习质量。梭罗：FKIP 大学。穆罕默德迪亚苏拉卡尔塔，2012。
[6] JUPRI A. 技术在用现实数学方法学习数学中的作用。全国数学与教育工作者研讨会论文集，2018
[7] PRUITT S. L. 下一代科学标准。特点和挑战。科学教师教育杂志，25(2)：145-156。
[8] L. S. NGSS.下一代科学标准：为各州，由各州制定。华盛顿特区：国家科学院出版社，2013。
[9] TRISTANTI L. B. 未来数学教师在提出论证时的思考过程。教育与学习杂志，2019，13(1)：17-24，http://doi.org/10.11591/edulearn.v13i1.6853。
[10] INGLIS M.、MEJIA-RAMOS J. P. 和 SIMPSON A.

数学论证建模：
资格的重要性。数学教育研究，2007，66(1)：3–21，http://doi.org/10.1007/s10649-006-9059-8。
[11] TRISTANTI L. B. 和 NUSANTARA T. 通过灌输式学习策略提高学生的数学论证能力。《物理学杂志：会议系列》，IOP 出版，2021，1783：012103。http://doi.org/10.1088/1742-6596/1783/1/012103。
[12] TRISTANTI L. B.、SUTAWIDJAJA A.、ASÂ A. R. 和 MUKSAR M. 职前教师数学论证中从归纳依据衍生的演绎依据的构建。 教育研究与评论，2016，11(17)：1696–1708，https://doi.org/10.5897/ERR2016.2872。
[13] HAREL G. 和 SOWDER L. 学生的证明方案：探索性研究的结果。 CBMS 大学数学教育研究，1998，III (7)：234–283。
[14] HAREL G. 数学归纳法作为证明方案的发展：基于 DNR 的教学模型。在 CAMPBELL S. 和 ZAZKIS R. (Eds.) 学习和教学数论中。Ablex 出版公司，新泽西州，2001，185–212。
[15] HWANG W. Y.、CHEN N. S. 和 HSU R. L. 开发和评估用于提高数学问题解决能力的多媒体白板系统。计算机与教育，2006，46(2)：105–121，https://doi.org/10.1016/j.compedu.2004.05.005
[16] 美国全国数学教师委员会 (NCTM)，《学校数学原则与标准》。弗吉尼亚州雷斯顿：NCTM，2000。
[17] HANNA G. 证明、解释和探索：概述。数学教育研究，2000，44(1)：5-23。
[18] TOULMIN S. 论证的用途。英国：剑桥大学出版社，2003。https://doi.org/10.2307/2183556。
[19] TRISTANTI L. B.，和 NUSANTARA T. 注入学习策略对学生数学论证技能的影响。国际教学杂志，2022，15(2)：277–292，https://doi.org/10.29333/iji.2022.15216a。
[20] MAHUDA I.、MEILISA R. 和 NASRULLAH A. 智能应用程序创建者协助开发基于 Android 的数学学习媒体以提高解决问题的能力。AKSIOMA：数学教育研究计划杂志，2021，10（3）：1745–1756。 http://dx.doi.org/10.24127/ajpm.v10i3.3912
[21] NURHAIRUNNISAH N., 和 SUJARWO S. 交互式教材，以提高 X 级高中生对数学概念的理解。教育技术创新杂志，2018, 5(2) 192–203. http://dx.doi.org/10.21831/jitp.v5i2.15320
[22] ALIM N.、SAAD M.S.、MAHMUD H. 和 GUNAWAN F. 谷歌课堂作为教学媒介的可用性和满意度：
学生的视角。世界工程技术教育学报，2021，19(1)：16–20。
[23] YANG H., 和 ZHANG L. 推动创造性计算：起源、范围、研究与应用。数字通信与网络，2016，2 (2): 84–91，https://doi.org/10.1016/j.dcan.2016.02.001
[24] CYBULSKI J. L.、KELLER S.、NGUYEN L. 和 SAUNDAGE D. 利用数字空间中的创造性问题解决可视化分析。计算机在人类行为中的应用，2015，42：20-35，https://doi.org/10.1016/J.Chb.2013.10.061。
[25] NOROUZI B.、KINNULA M. 和 IIVARI N. 与儿童一起进行数字制造和制作：审视成年人在调解青少年活动中的策略和挑战。《国际儿童计算机交互杂志》，2021，28(C)：100267，https://doi.org/10.1016/j.ijcci.2021.100267
[26] ATWOOD-BLAINE D.、RULE A. C. 和 WALKER J. 科学中心使用情境式教学法对 9-14 岁儿童的创造性自我效能手机游戏。思维技能与创造力，2019，33：100580，https://doi.org/10.1016/j.tsc.2019.100580。
[27] PANSKYI T.、ROWINSKA Z. 和 BIEDRON S. 在基于游戏的环境中进行视觉创意编程教学的校外援助 – 案例研究：波兰。思维能力和创造力，2019，34：100593，https://doi.org/10.1016/j.tsc.2019.100593。
[28] TRISTANTI L. B. 和 IFFAH J. D. N. 智能应用程序创建者协助开发基于 Android 的空间几何学习媒体以提高证明能力。AKSIOMA：数学教育研究计划杂志，2022，11（3）：1716–1728。 http://dx.doi.org/10.24127/ajpm.v11i3.5103
[29] TRISTANTI L. B. 和 NUSANTARA T. CIRC 型和基于问题的合作学习模式对学生数学论证的优势和影响。第二届国际教育与技术会议 (ICETECH 2021)。Atlantis Press，2022，第 172-178 页，https://doi.org/10.2991/assehr.k.220103.026