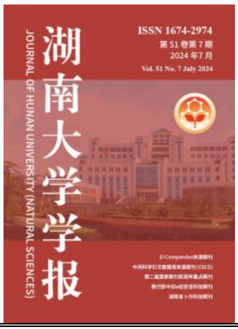

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Didactic Strategy for University Precalculus: An Implementation of Technological Environments and Problem Solving

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Abstract: In initial university semesters, a low academic level in fundamental mathematics is evident. This article describes a didactic strategy focused on the development of the concept of function in the first university mathematics course. The strategy is based on the implementation of a classroom learning environment, guided by resources designed in dynamic software, and focused on problem-solving. The objective of this study is to present a comprehensive approach that not only addresses students' learning difficulties but also enhances professor preparation and the curricular structure. The contribution of this research lies in its focus on the integration of technology and problem solving as key tools for the teaching-learning process. The study was developed in three phases: (i) identification of the learning difficulties of students and professors, (ii) creation of a professor training space that addresses these difficulties, and (iii) implementation of the environment designed to improve student learning. The results indicate the necessity of revisiting the curricular structure, minimum course characteristics, professor preparation, and support material with the aim of proposing guidelines that, if implemented, can have a positive impact on teaching and learning.

Keywords: learning environments, dynamic software, activity sequences, concept of function, problem solving.

大学预科微积分教学策略： 技术环境与问题解决的实施

摘要：在大学最初几个学期，基础数学学科的学术水平较低是显而易见的。本文描述了一种教学策略，重点是在第一门大学数学课程中函数概念的发展。该策略基于课堂学习环境的实施，以动态软件设计的资源为指导，专注于解决问题。这项研究的目的是提出一种全面的方法，不仅可以解决学生的学习困难，还可以增强教授的准备和课程结构。这项研究的贡献在于其重点关注技术与问题解决的整合，作为教学过程的关键工具。该研究分三个阶段进行：（i）识别学生和教授的学习困难，（ii）创建解决这些困难的教授培训空间，以及（iii）实施旨在改善学生学习的环境。结果表明有必要重新审视课程结构、最低课程特征、教授准备和支持材料，目的是提出指导方针，如果实施这些指导方针，可以对教学和学习产生积极影响。

关键词：学习环境、动态软件、活动序列、功能概念、解决问题。

1. Introduction

A significant challenge in the first semesters of higher education is the acquisition of competencies in the basic sciences, particularly mathematics. The deficiencies in the necessary competencies of students who transition to university to study academic programs, especially in engineering, represent a significant challenge for the community of professors and university administrators in terms of improving expected results [1]. Despite educational efforts based on institutional pedagogical models and curricular aspects proposed from national legal frameworks, high levels of failure persist in subjects that address the first semesters, in this case, basic mathematics. Additionally, given the exponential advancement of science and technology, it is imperative that university graduates pursue lifelong education to remain abreast of developments. This context necessitates profound transformations in higher education aimed at training professionals in the competencies demanded by society. Transformations necessitate a reconfiguration of the role of the student, who must become an active protagonist of their own learning, and of the professor, who must provide students with tools to develop their own knowledge and skills.

In this context, the criteria for selecting the object of this research are based on the unsatisfactory results of standard tests and the observation of persistent difficulties in learning the concept of function, which is a fundamental concept in the study of mathematics and other scientific disciplines. This concept is essential not only for success in mathematics, but also for the comprehension of phenomena in science and engineering. Furthermore, this choice is based on the necessity of integrating educational technologies that have been demonstrated to be efficacious in the pedagogical process of abstract concepts through dynamic and interactive presentations.

This study is thus framed within the need to review and propose a didactic approach to the teaching of basic mathematics in the first semester of a university. The objectives of this study are threefold: (i) to identify the learning difficulties of students and professors in the concept of function, (ii) to create a training space that addresses these difficulties through the use of dynamic software, and (iii) to implement and evaluate a learning environment designed to improve student performance in mathematics.

This study was conducted in three stages.

First, the teaching-learning difficulties of both the ptofessor and the students in the basic mathematics course were identified. Second, a training space for professors was designed to address the difficulties identified using technological tools, specifically GeoGebra. Finally, the designed learning environment was implemented using rubrics, GeoGebra resources, and didactic materials for classroom activities and for directed independent work. The above is based on studies and authors who have established relevant aspects, including the consideration of conceptual frameworks, the characterization of mathematical thinking, the development of student learning and learning environments in the classroom [2], and the relevance of problem solving, which posits mathematics as the construction of knowledge and the use of problems and reflection on them as the means to achieve mathematical knowledge. The imparted knowledge is imbued with meaning [3], which generates didactic proposals and promotes practical ways of thinking mathematically in students [4]. Finally, the incorporation of technologies and curricular designs is recognized as influencing the development and understanding of strong and consistent mathematical thinking. Therefore, it has been proposed that curricular designs and learning environments be rethought [5].

The findings of this study underscore the necessity for a review of the curriculum structure, minimum course characteristics, teacher preparation, and support materials. The integration of GeoGebra into the classroom has been demonstrated to enhance student engagement and facilitate the comprehension of mathematical concepts through dynamic and interactive geometry. Furthermore, the utilization of this technological tool has been shown to foster the development of analytical skills and enhance the efficacy of solving mathematical problems [6], [7].

The findings of this study have significant and immediate implications for the improvement of mathematics courses in the first semester of college education. They offer practical recommendations for curricular restructuring and pedagogical approaches that can enhance learning experiences.

Additionally, they provide a robust theoretical and practical foundation for the integration of technology into mathematics education, promoting active and meaningful learning. If implemented, these recommendations have the potential to positively impact the teaching and learning of mathematics in the first semester of a college.

This study offers guidelines that, if implemented, can have a positive impact on the teaching and learning of mathematics during the first semester of university education.

This provides a theoretical and practical basis for the use of technology in mathematics education that promotes active and meaningful learning.

2. Theoretical Framework

Extensive research in the field of mathematics education offers a rich collection of studies that address the issues related to this article. These studies provide a significant and relevant contribution, with special emphasis on the literature on learning environments, connectivism, and problem-solving.

Regarding the concept of a learning environment, some authors posit that it is based on the interaction and transformation between various actors and the environment, considering psychological, pedagogical, and curricular factors [8]. This is a structured environment that seeks to adapt to the needs of each student’s meaningful learning [9].

Moreover, it is crucial to rethink educational environments that include virtual and remote contexts and create hybrid environments that incorporate Information and Communication Technologies (ICT) [10] and [11]. This necessitates the implementation of technological strategies for asynchronous encounters, collaborative work, and autonomous learning as well as socioeconomic factors that facilitate the continuous use of technology [12].

Connectivism is predicated on the notion that learning is the process of integrating disparate sources of specialized information through personal networks and knowledge communities [13]. The advent of the Internet and ease of access to information networks have expanded the scope of learning environments, facilitating remote assistance and collaborative work. This theory highlights eight principles [14], including the importance of diversity of opinions and the capacity to perceive connections between disparate areas of knowledge.

Connectivism posits that learning environments enabled by social networks and collaborative tools can supplement traditional instructional design, transform educational methodologies, and offer novel and more innovative alternatives [15].

Finally, the problem-solving approach in mathematics education has been in use for several decades and has been subjected to extensive analysis to improve the interaction between professors and students. Various methodologies have been proposed, including the five-category hierarchy that highlights application problems for their complexity [17], and the heuristics of [18], which include understanding the problem, proposing a plan, implementing the plan, and analyzing the solution. These methodologies were adapted to the Colombian curriculum [19].

Problem-solving pedagogy is conducive to the principles of mathematical culture in the classroom, which motivates students to engage in discourse and the development of new concepts [20-21].

Furthermore, the integration of ICT in teaching, such as the use of GeoGebra, has transformed mathematics teaching, facilitating the comprehension of concepts through dynamic environments and interactive tools [22].

3. Methodology

The phases implemented in this study are outlined as follows: First, a diagnostic test was administered to students enrolled in the course under investigation, with the objective of characterizing their existing knowledge of the concept of function. Additionally, a focus group was conducted with professors who regularly taught this course, with the aim of identifying their perceptions regarding the difficulties encountered in teaching this concept. Second, the development of the training program, which was conducted as a seminar-workshop, was presented. This was carried out with professors from the Department of Basic Sciences of the university institution Antonio José Camacho UNICAMACHO, with a focus on those who participated in the previous phase of the study. Third, a learning environment was implemented in the classroom.

A diagrammatic representation of the methodological process followed in this study is presented in the flowchart below.

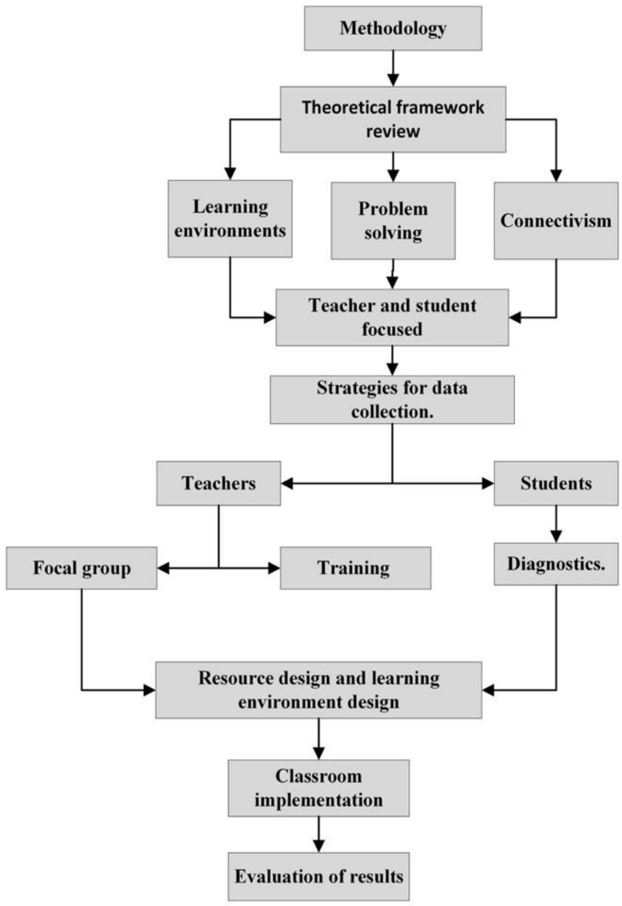

Fig. 1 Diagrammatic representation of the methodological process (The authors' elaboration)

3.1. Diagnostic Test

To understand students’ and professors’ needs and perceptions about the implementation of a didactic strategy based on technology for teaching the concept of function in mathematics, two diagnostics were conducted: one aimed at students and the other at professors.

The diagnostic test was administered to a pilot group of first-semester students from the Faculty of Engineering. This test was conducted remotely using electronic forms during a synchronous meeting, lasted 60 min, and was completely anonymous. It was structured with open-ended questions to evaluate the understanding of concepts, such as the identification of different representations of the same situation, the handling of variable and constant quantities, and the ability to establish relationships between different quantities. Furthermore, a test comprising closed-ended questions was included to identify the most common difficulties encountered in learning and collect opinions on the use of technology in the classroom. This test involved problem solving using GeoGebra [23] and culminated in the professor providing feedback and facilitating socialization of the solutions.

The diagnosis of professors was based on the premise that a teaching team with adequate disciplinary competencies is necessary. Semi-structured interviews and focus groups were conducted, during which an open and objective dialogue was held regarding the concept of function and formulation of problem situations for teaching. Professors shared their perspectives, classroom experiences with and without technology, and individual perceptions, which enriched the overall analysis of the study.

The diagnostics provided a robust foundation for the design and implementation of a didactic strategy that not only addresses the needs identified, but also capitalizes on the opportunities identified to enhance the teaching and learning of the concept of function in university pre-calculus.

3.2. Professor Training

The seminar workshop was conducted in three sessions of two hours each, in a face-to-face modality assisted remotely through videoconferencing. To facilitate participation and independent work, Learning Management System (LMS) spaces were utilized where attendees could engage in discussions in forums and complete activities between meetings. The call allowed the participation of professors involved in the initial phase of the study as well as teachers from other institutions or agencies outside UNICAMACHO. Professionals from disciplines related to the central course of study, including those engaged in the teaching of mathematics, physics, and statistics at the basic, middle, and higher education levels, provided responses.

The training included technological tools and didactic resources to develop learning environments that are useful in the preparation of academic meetings [24]. These resources ranged from theoretical foundations, through recognized authors, to practical implementations using Learning and Knowledge Technologies (LKT) and ICT [25].

The objectives of the event were: (i) to socialize and reflect on the research proposal and preliminary findings of the diagnostic test; (ii) to present learning environments that use appropriate technological resources to enhance didactic alternatives, without neglecting the related literature and the Institutional Pedagogical Model; and (iii) to gather opinions and contributions from the participating professors based on their teaching experience.

These processes are framed within the development of new digital literacy, both for students and teachers [26], and recognize the importance of the disciplines addressed by STEM (Science, Technology, Engineering, Mathematics) by establishing connections between concepts, procedures, and attitudes [27].

For the first objective, a discussion was generated in which the following aspects were highlighted.

- Presentation of the context and purposes of the research development.
- Presentation and analysis of the results of the diagnostic test applied to the students.
- Discussion of perceptions of students who had already passed the subject, focusing on teaching-learning processes, strategies, methodologies, curricular designs, and the use of technological tools in the classroom.

For the second objective, the following points were addressed:

- Socialization of the concept of digital literacy as a fundamental competence in the technological era, covering the technical and cognitive skills necessary for an efficient digital environment.
- Presentation of the concept of "*digital backpack*," including free and paid tools useful for generating didactic resources in the development of subjects.
- Presentation of the concept of activity sequences as a didactic tool, highlighting the GeoGebra platform as a learning community and a repository of didactic resources.
- Disclosure of the referential framework of the research, showing recent references related to the design of learning environments, connectivism, and problem-solving.

Finally, to address the last objective, the following were considered:

- Discussion of the different positions and lines of thought of professors regarding the design of learning activities and digital competencies.
- Proposal for designing collaborative activity sequences among professors socialized at the end of the session.
- The implementation of various tools focuses on classroom activities or autonomous work using physical or online materials.

3.3. Classroom Implementation

In this phase of the study, a learning environment was implemented to facilitate students' understanding of the concept of function in a basic mathematics course. The selected course was developed in a remote technology-assisted face-to-face mode, with synchronous meetings twice a week. This learning environment focused on the development of the concepts of domain, range, and dependent and independent variables, as well as the definition of a function. The methodology employed was problem-based learning (PBL), which fosters profound comprehension and tangible utilization of mathematical principles.

GeoGebra is used as a technological tool in the design of educational resources. Once these resources were designed, two LMS were used during the implementation: Moodle and GeoGebra Classroom. Moodle was used to create activity sequences and evaluation rubrics, whereas GeoGebra Classroom allowed real-time monitoring of student progress and socialization of results in non-gradable activities, guaranteeing the anonymity of the participants.

In this implementation, which was carried out during one academic semester, a total of 32 students participated, of which 24 were first-semester students and eight were repeaters, having failed the course in the previous semester. It should be noted that in the previous semester, the subject was taught in a traditional manner.

Regarding the benefits and disadvantages of the aforementioned implementations, it is evident that a significant advantage was the remote mode course format, which ensured that all students had a device with an internet connection, thus facilitating their participation in the activities. However, a drawback was observed in that due to the complexity of some resources, students occasionally experienced slowness in their execution. This was due to the fact that the students were not only connected to an online platform, but also participated in a simultaneous video call, which represented an additional load on their devices and connections. This aspect should be considered in future developments of this type of learning environment implementation.

Couple of examples of the resources applied are presented below.

First, a situation related to Colombian geography is presented, in which students must move an avatar through different departments of the country. It should be noted that due to its geographical location, Colombia presents a great variety both in altitude above sea level and in average temperature from one region to another. Once the students place the avatar in a given department, the resource shows the altitude above sea level on the left side and the average temperature on the right side, as shown in Fig. 2.

Fig. 2 Colombian geography (The authors' elaboration)

After students interacted with the resource, they were asked to record the data obtained in a table similar to the one shown in Fig 3. This step was designed to familiarize them with the numerical representation of a function.

	Departamento	Temperatura (°C)	Altura (msnm)
1			
2			
3			
4			
5			

Fig. 3 Table excerpt (The authors' elaboration)

Once the table is completed, the students are instructed to locate these points on a Cartesian plane using a specific resource. With this resource, students simply type the point *(x,y)* in the command bar, and the system automatically takes care of locating it. This method was designed to facilitate understanding of the graphical representation of the relationship, as illustrated in Fig. 4.

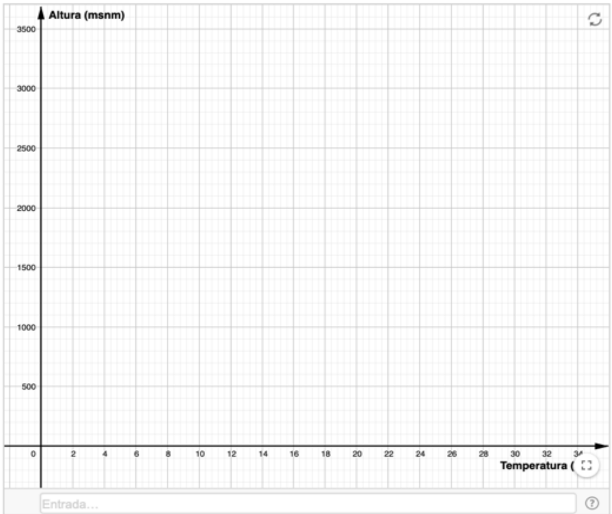

Fig. 4 Cartesian plane (The authors' elaboration)

It is crucial to highlight that the presented figures are integrated into a unified sequence of activities structured in a formal manner and in accordance with academic parameters. These parameters include instructions, objectives, the situation presented, and clear instructions for development. Furthermore, after students commence manipulating resources, an activity is proposed that includes guiding questions. These questions were designed to guide the analysis and provide a deep understanding of the concepts presented.

Second, a resource designed in GeoGebra and implemented in Moodle is presented, which guides students to explore variability in a practical situation that simulates the constant filling of a glass jar with water. With this resource, we sought to bring students closer to the implicit concepts of domain and a restricted range of functions. To do so, students were asked to select one of several available shapes for the pitcher and choose the desired height of the pitcher. Once the filling is started, the resource provides real-time information about the elapsed time and height of the liquid, as shown in Fig 5.

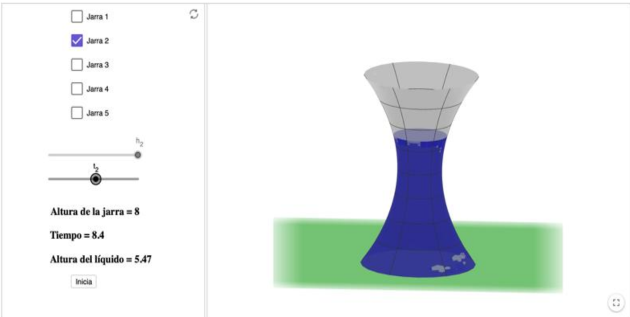

Fig. 5 Time of filling (The authors' elaboration)

At the conclusion of the academic term, a perception survey was administered to students to ascertain the relevance of the instructional resources employed. The primary objective of this survey was to assess students' perceptions of the usefulness and effectiveness of the educational resources utilized in the course, thereby providing direct feedback that could inform the ongoing enhancement of the teaching-learning process.

4. Results

The primary findings of the proposed scenarios are as follows. First, the initial diagnosis findings were presented. Second, conclusions derived from the Seminar-Workshop with teachers, based on the methodological design described, are discussed. Finally, key aspects of the implementation of the classroom learning environment with students are described.

With regard to the development of diagnostic, the following observations merit particular attention:

• The challenge lies in conveying a singular situation across disparate systems of representation for subsequent analyses. A significant proportion of test participants (88.92%) reported difficulties with this process.

• A very low percentage of the population under diagnosis (14.85%) establishes relationships between different quantities, constants, and variables.

• Difficulties were identified in the argumentation process. 92.61 Of the students, 92.61% exhibited this difficulty, which impeded their ability to effectively communicate their ideas.

The findings of this study permit the design of learning environments to be implemented in the development of course content, with the objective of strengthening thinking processes and reducing the difficulties evidenced in the applied test.

Regarding the dialogue with the teachers and subsequent discussions held during the seminar-workshop, the teachers' perceptions regarding the difficulties observed in the students were evident.

The most pertinent of these are:

• It can be argued that professors perceive a lack of motivation on the part of students to succeed in their academic pursuits, particularly in higher education. This was despite the academic preparation of students who graduated from high school.

• Recently, there has been a notable increase in the number of students who have reduced the time they have dedicated to addressing the thematic developments addressed in the course. This has a direct impact on students' performance and autonomous learning processes, as it is becoming increasingly challenging for them to attend classroom meetings.

• The socioeconomic condition of students' families has been found to have a direct relationship with the development of learning processes. This has been revealed through their own unofficial comments and the different spaces of reflection that take place in the middle of the classes. This result agrees with the research previously carried out in the institution by [28] linked to the academic development of students.

• The discussion of new theoretical approaches that are being developed and have gained strength in recent years is of great importance because they mark the direction of the didactic lines focused on the teaching-learning processes of mathematics. In particular, new theoretical developments in terms of educational evolution and the implementation of technological competencies in education should be considered.

• Technological literacy, defined as a set of tools and applications that can be utilized in an educational setting, has emerged as a significant learning opportunity for didactics in the context of remote work facilitated by technology. However, there is still a need to improve the ability to effectively integrate technological approaches in a timely manner because of the rapid evolution of these technologies and the lack of emphasis on this type of technological conditioning.

• The utilization of didactic instruments, such as those proposed through sequences of activities and learning environments focused on problem-solving using GeoGebra software, enables us to transcend the limitations of a traditional classroom. Furthermore, when we discuss the digital backpack and present tools, environments, and resources, technological developments are expanded to independent, directed work, encouraging the individual, internal, and personal learning processes that each student undergoes.

Finally, it is important to consider the interdisciplinary nature of the factors that influence the curricula of each program. This allows for the development of approaches to enhance the discussion of a given topic. One potential approach is to facilitate group discussions among peers to expand the scope of the discussion. In this manner, individual domains of knowledge can be interwoven, thereby furnishing students with the capacity to establish relationships and connections between the various structural elements of their entire curriculum.

In terms of the implementation of the learning environment, positive results were identified in several key aspects.

- The implementation of interactive resources resulted in a notable increase in student motivation and participation compared to other traditionally oriented subjects. This observation aligns with the existing literature, which indicates that interactive and technology-based learning environments can significantly enhance student motivation [29] [30].
- The implementation of interactive resources, such as GeoGebra and Moodle, has led to an improvement in the development of technological skills, particularly in the use of these tools. These resources not only facilitated the appropriation of mathematical concepts, but also developed valuable technological skills for their university careers in mathematics and related subjects. Previous studies have shown that the integration of educational technologies can enhance learning and development of technological skills [31].
- The implementation of dynamic resources has advantages and disadvantages. However, these resources allow for continuous validation and refinement based on experience and observations gathered, which is crucial for the continuous improvement of the teaching-learning process. According to [32], constant feedback and resource adaptation are essential for optimizing educational outcomes and ensuring effective teaching.
- The dropout of the course object of this study was lower than that of other semesters in which the same subject has been oriented in a traditional way, which represents an important contribution to the phenomenon of university dropout since academic performance is constituted as a factor that positively impacts dropout [33].

5. Conclusions

In the context of higher education, particularly in first-semester mathematics courses that address foundational concepts such as relations and functions, it is imperative to rethink learning environments to optimize the teaching-learning process. A preliminary step is to challenge traditionalist imagination and prioritize the incorporation of diverse didactic strategies that facilitate more active and participatory learning. In accordance with the tenets of constructivist learning, whereby students construct their knowledge through interaction with the environment and the resolution of real problems, learning environments must be dynamic and flexible, allowing for the integration of educational technologies and active methodologies such as project-based learning and collaborative learning.

It is essential to consider the creation of learning spaces outside traditional classrooms such as mathematics laboratories and collaborative virtual environments. These spaces should be designed to promote exploration, experimentation, and exchange of ideas, contributing to the development of critical and creative thinking around mathematical notions and objects.

Another essential aspect is the obligatory articulation with the technological and digital environment available for the teaching-learning process. The use of specialized software such as GeoGebra facilitates the graphical visualization of abstract concepts and promotes a deeper understanding of complex topics such as relationships and functions. Additionally, this software fosters autonomy, curiosity, and self-learning, which are essential skills in higher education. The community created around GeoGebra resources promotes networked learning, facilitating connections between students, professors, and subject matter experts, and providing access to a wide range of resources and up-to-date knowledge.

Online platforms create dynamic and participatory learning environments. Teachers can foster critical thinking and creativity by exposing students to multiple perspectives and approaches to mathematical problems. Forum discussions and group project collaborations allow students to explore different methods and solutions, and develop a more holistic understanding of concepts.

During the course of this study, several coincidences were identified among the participating professors. These coincidences include the need to update teaching methods to adapt to new technologies, such as specialized software and online platforms that facilitate the organization of activities and student follow-up. Additionally, the importance of continuous training in technological tools to improve educational practices has been emphasized. However, the integration of technology faces significant barriers due to the lack of accessible resources and insufficient institutional support in the form of training.

The implementation of innovative teaching strategies and the use of educational technologies not only optimizes the learning of the concept of function in mathematics, but also prepares students to face academic and professional challenges more effectively. The adaptation and continuous improvement of teaching methods, supported by robust teacher training and adequate technological resources, are essential for the success of mathematics education in the 21st century.

This study makes a substantial contribution to the field of mathematics education by addressing the critical need to modernize learning environments in first-semester courses in higher education. Furthermore, it highlights the value of constructivist and participatory learning and offers insights into how technological tools such as GeoGebra can enhance students' comprehension of intricate mathematical concepts. Consequently, this study contributes to the existing literature by illustrating how technology and active methodologies can be effectively integrated to enhance the efficacy of the teaching-learning process in mathematics.

The limitations of this research pertain to the necessity of having sufficient technological resources and appropriate infrastructure for the proposed learning environments. Otherwise, the implementation would be ineffective. Not all students are familiar with interactive educational software; therefore, prior support and digital literacy are required. Finally, a deficiency was identified in the preparation and willingness of teachers to adopt new technologies and didactic strategies, which should be reinforced to obtain optimal results in this line of research.

To inform future work, it is recommended that studies be conducted to evaluate the long-term impact of didactic strategies and technologies on learning mathematical concepts. It is also crucial to research and develop new technological tools and learning platforms, including artificial intelligence and augmented reality, to improve the visualization and understanding of abstract concepts. In addition, designing continuous training programs for professors using advanced educational technologies can significantly improve teaching practices and student performance.

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