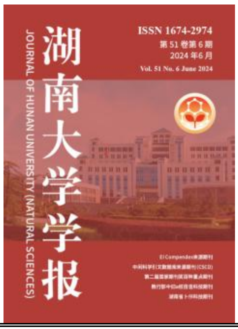

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Variable Analysis for Supporting Reservoir Impoundment Modelling

R. Harimukti Rosita[1]*, Pitojo Tri Juwono[2]*, Lily Montarcih Limantara[2], Emma Yuliani[2]

[1] Doctoral Student, Department of Water Resources, Faculty of Engineering, University of Brawijaya, Malang, Indonesia

[2] Department of Water Resources, University of Brawijaya, Malang, Indonesia

* Corresponding authors: alwayssosita@student.ub.ac.id, pitojo_tj@ub.ac.id

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Abstract: This research investigates accurate predictions of increases in reservoir levels, and it was conducted for 10 different reservoirs. The methodology consists of developing a series of symbolic regression models and evaluating the performance based on the root mean square error. Dams are constructed primarily to impound and store a large body of water. However, when dam construction is complete, the flow to the dam site resumes and the reservoir begins to fill with water. The first filling of a reservoir can be defined as the increase in the water level behind the dam from the time the construction is complete until it reaches the desired operating level. The construction works below were undertaken within a restricted timeframe owing to the rising water levels during reservoir filling. For a multipurpose dam constructed in the upstream reach of a river, various water users acquire water, and the dam must release outflow for supplying water to users engaged in irrigation, water supply, and maintenance flow. Regardless of whether it takes several months or years to occur naturally or with the aid of pumping units, the first filling of a reservoir should be planned, controlled, and closely monitored to reduce the risk of failure. The results show that the symbolic regression model provides accurate predictions with an RMSE of less than one day, producing a complex non-linear relationship between reservoir volume variables and average inflow for determining the duration of reservoir filling.

Keywords: reservoir impoundment, filling duration, regression.

支持水库蓄水建模的变量分析

摘要：本研究调查了水库水位上升的准确预测，研究对象为10个不同的水库。该方法包括开发一系列符号回归模型，并根据均方根误差评估其性能。大坝的主要建造目的是拦蓄和储存大量水。然而，当大坝建成后，水流恢复，水库开始蓄水。水库的首次蓄水可以定义为从建造完成到达到所需运行水位期间大坝后水位的上升。由于水库蓄水期间水位上升，下面的施工工作是在有限的时间内完成的。对于在河流上游建造的多用途大坝，各种用水者都需要用水，大坝必须释放出水，为从事灌溉、供水和维护流量的用户供水。无论是自然蓄水还是借助泵送装置需要几个月或几年的时间，水库的首次蓄水都应经过规划、控制和密切监测，以降低故障风险。结果表明，符号回归模型提供了准确的预测，均方根误差小于一天，并在水库体积变量和平均流入量之间建立了复杂的非线性关系，从而确定了水库蓄水持续时间。

关键词：水库蓄水，蓄水历时，水位下降。

1. Introduction

The initial filling of a reservoir is the first test that the dam will perform the function for which it was designed. A carefully managed initial filling is crucial for the future success of the dam [1, 2]. According to a study completed by the Bureau of Reclamation on internal erosion failure modes, "approximately two-thirds of all failures and one-half of all dam incidents occur on first filling or in the first 5 years of reservoir operation" [3].

Dams are constructed primarily to impound and store a large body of water. However, when dam construction is complete, the flow to the dam site resumes and the reservoir begins to fill with water [4]. The first filling of a reservoir can be defined as the increase in the water level behind the dam from the time the construction is complete until it reaches the desired operating level [5]. Depending on the location, type, size, and intended purpose of the dam, the duration and rate of its first filling can vary.

While carrying out reservoir filling, certain construction tasks were accomplished within a restricted timeframe due to the rising water level, such as the plugging of diversion tunnels, the execution of hydromechanical operations, and the installation of dam monitoring instruments. For multi-purpose dams constructed on upstream reaches of rivers, there are water users who acquire water from these structures. Consequently, the dam must release an outflow to provide water for irrigation, water supply, and maintenance flows to these water users [6]. Regardless of whether it takes several months or years and occurs naturally or with the aid of pumping units [7], the first filling of a reservoir should be planned, controlled, and closely monitored to reduce the risk of failure.

To determine the accuracy of predictions related to the increase in reservoir water level elevation, a statistical analysis approach is employed in the model. Seasonal variations, rainfall patterns due to climate change, and fluctuating water demand present challenges in optimizing reservoir filling [1]. Predicting the duration of reservoir filling accurately is important for the design and management of water resources. The symbolic regression analysis is used to model the relationship between the total volume of the reservoir, average inflow, and duration of reservoir filling to identify the model that can predict the duration of filling with high accuracy. This research employs data from ten distinct reservoirs to establish a range of symbolic regression models with varying levels of complexity, with the objective of assessing their performance based on the root mean square error (RMSE).

The use of symbolic regression for modeling complex relationships between variables through the employment of mathematical equations is a promising approach for comprehending and predicting reservoir filling dynamics. This research demonstrates the efficacy of symbolic regression in the intricate relationship modeling between hydrological variables [8].

The variables of inflow, storage volume, and duration have the most significant impact on the duration required to fill the reservoir. Undertaking the statistical analysis approach, a model will be constructed to accurately predict the increase in reservoir water levels.

2. Materials and Methods

2.1. Research Data

This research utilizes secondary data from various reservoirs located in Indonesia. The data obtained includes the total volume (V), average inflow (I), and duration (D) of each reservoir. The selected reservoirs for this study are Jatigede, Kuningan, Gongseng, Bendo, Pidekso, Gondang, Rotiklot, Titab, Tukul, and Bringin Sila. The data collected consist of three variables: total volume (V), average inflow (I), and duration (D). The information on these variables is presented in Table 1.

Table 1 Model simulation data (The authors)

Reservoir	V (million m^3)	I (m^3/s)	D (day)
Jatigede	727.08	72.67	472
Kuningan	24.65	1.03	287
Gongseng	22.43	2.32	114
Bendo	43.46	2.52	203
Pidekso	24.55	0.93	123
Gondang	7.09	0.80	157
Rotiklot	3.30	3.09	85
Titab	12.80	8.13	66
Tukul	8.63	1.03	371
Bringin Sila	24.75	3.03	86

Impoundment is illustrated below as a monitoring graphic and photographs of daily monitoring.

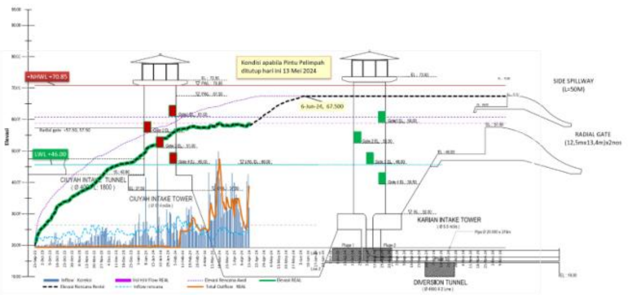

Fig. 1 Graphics of reservoir elevation during the impoundment (reservoir filling) period at Karian Dam (The authors)

Fig. 2 River bed condition during initial impoundment (reservoir filling) on Day 1 (The authors)

Fig. 3 Reservoir condition after initial filling (impoundment) on Day 21 (The authors)

Fig. 4 Reservoir condition after initial filling (impoundment) on Day 39 (The authors)

2.2. Nonlinear Regression

If the quadrature of errors or deviation of linear regression is substantially large, the regression model is deemed insignificant. In such a case, it is imperative to conduct an incompatibility test of the linear model. To perform this test, the middle square of errors is computed to assess the observations within the group of x-variables, which is referred to as the pure error square. This value is then compared with the middle square of the entire error. The discrepancy between these two error squares is attributed to model incompatibility.

If there is a k-observed group, the middle square of error in the k-I group (I = 1 to k) is as follows:

$$S_i^2 = \frac{\sum_{j=1}^{n_i}\left(y_{ij} - \bar{y}_i\right)}{n_i - 1} \quad (1)$$

The combined middle square due to the incompatibility is as follows:

$$S_i^2 = \frac{\sum_{i=1}^{k}(n_i - 1)S_i^2}{n = k} = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(y_{ij} - \bar{y}_i\right)}{n - k} \quad (2)$$

Analyzing the middle square enables us to assess whether the incompatibility is substantial enough to warrant an alternative choice. If the incompatibility proves to be substantial, non-linear regression may be chosen as an alternative.

Two primary techniques are commonly utilized: the transformation method and the construction of a polynomial function that incorporates the variable squared or higher. The most common transformations are exponential, inverse, hyperbolic, and exponential functions.

The polynomial regression model for a variable or variable square is as follows:

$$y = a_0 + a_1 x + a_2 x^2 \quad (3)$$

Therefore, each observed data point satisfies the following equation:

$$y_i = a_0 + a_1 x_i + a_2 x_i^2 + e_i \quad (4)$$

or

$$e_i = y_i - a_0 - a_1 x_i - a_2 x_i^2 \quad (5)$$

The square of the error is as follows:

$$JKG = \sum_{i=1}^{n}(y_i - a_0 - a_1 x_i - a_2 x_i^2)^2 \quad (6)$$

To minimize the error square, differentiation is performed as follows:

$$\frac{\partial(JKG)}{\partial(a_0)} = 0 \quad (7)$$

$$\frac{\partial(JKG)}{\partial(a_1)} = 0 \quad (8)$$

$$\frac{\partial(JKG)}{\partial(a_2)} = 0 \quad (9)$$

Therefore, the simultaneous equation can be written as follows:

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 \end{bmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i^2 y_i \end{bmatrix} \quad (10)$$

2.3. Theory of Symbolic Regression

This study employs the technique of symbolic regression, which involves the use of a genetic algorithm to identify the optimal mathematical model for reservoir filling data. The method of symbolic regression involves the application of a genetic algorithm to automatically determine the structure of the equation, a process that is distinct from identifying the model that best captures the relationship between the independent variables [9].

The traditional regression method has a distinguishing characteristic that it necessitates assumptions about the form of the relationship between variables. The symbolic regression can automatically find the nonlinear relation and complex interactions between variables. In symbolic regression, no significant tests, such as simultaneous influence (ANOVA) or partial influence (t-test), are conducted as in traditional regression models. The model produced from symbolic regression is a mathematical equation, which facilitates interpretation and understanding of how independent variables influence dependent variables.

Symbolic regression's potential to enhance the accuracy of mathematical models through the identification of the optimal model with data remains a significant advantage in terms of prediction accuracy. This study employs the Turing Bot's symbolic regression model to achieve superior results in the dynamic modeling of reservoir filling. By doing so, it offers novel insights and enhances efficiency.

The symbolic regression analysis in this study incorporates the functions of addition, multiplication, division, exponential log, log-2, log-10, square root, power, and absolute value. The model solution's limitation is achieving an RMSE of less than 1, which signifies that the prediction error for reservoir filling duration is less than one day. This may be due to one or two observations having an error greater than one day.

3. Results

The mathematical operations used are division, reduction, and the usage of abs function (absolute value). The model indicates that the time required for reservoir filling is affected by a combination of linear and non-linear factors, including reservoir volume and average inflow, with some interaction between the two variables. Some interpretation points of the model are as follows:

1) *Influence of volume and inflow:* The model shows that reservoir volume (V) and average inflow (I) affect the filling duration [10] due to the complex interaction between both variables. This indicates that changes in one of the two variables can affect the duration of reservoir filling.

2) *Complexity of the model:* The complexity of the model (with a size of 46) suggests that the relationship between the volume, inflow, and duration of reservoir filling is intricate and involves intricate non-linear interactions. The model can reflect the physical reality of the reservoir filling process, which is influenced by numerous factors and conditions that can affect the outcome.

3) *High accuracy:* The very low RMSE shows that this model can make very accurate predictions. In the research context and reservoir management, having an accurate predictive model is important for designing and optimizing reservoir operation.

3.1. RMSE

The RMSE is a method of measuring the average error or difference between a predicted value and an actual value. It is calculated by squaring each error, taking the average of the squared errors, and then taking the square root of this average.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x_i})^2}$$

where:
N - number of data
xi - real value
ˆxi - predicted value

Based on Table 2, the model with a size of 46 gives RMSE < 1, i.e., 0.0895361. It marks that the model has an average prediction error of less than one day in predicting the duration of reservoir filling, which indicates a very high accuracy level. This model uses a complex combination of the reservoir volume (V) and average inflow (I) variables to predict the duration of reservoir filling.

$$\begin{aligned} D = \left(\frac{-0.2408803586340865}{I - 3.017224475004751}\right) + 34.52857618128883 \\ + \left|\left(\frac{-9.287586119653277}{-1.056127758519905 + (-0.009481335194391519 + I)}\right) \right. \\ \left. + \left(21.11591576935355 + \left(\left(\frac{-7.391032670121236}{0.3351748091742818 + I}\right) - 0.4515167850146117\right) \cdot (-10.06242841269821 + V)\right)\right| \\ + \left(\frac{11.62334517808248 + V}{0.1598586335377158 \cdot (1.723636895626801 + I)}\right) \end{aligned}$$

Table 2 Symbolic regression solution (The authors)

Size	RMSE	Function
1	130.393	196.4000003773866
3	117.438	186.8470276325101+I
5	92.3881	0.4326201594049471*V+157.5187838802518
7	86.7739	V-5.902729437467862*I+162.9148112930038
8	81.9286	(209.8049828780447/I)+0.6526682525668467*V

Size	RMSE	Function
9	76.6674	I*(0.4455955147252915*(-65.33443546161224+I))+234.7362394895957
10	72.9268	(504.6455332455504/(1.428690559887087+I))+0.6398485549285554*V
12	33.4148	105.7771810587326+(((-1.934139423863681)/(-1.037420795878003+I))-(-0.5071462646167876*V))
14	27.0473	V-5.088059050029177*I+((-1.545971932605875)/(I-1.036055538304567))+112.5721620853624
16	24.5918	1.487003407342499*V-9.995952763222219*I+((-1.255412903958011)/(I-1.034990956620233))+117.7718645418866
18	20.865	abs(((5.189756810746371/I)+0.5766646083762963)*V+(87.57163304570625/(I-1.235712780332954)))
19	19.3179	((-1.810258287538998)/(-1.036622858359758+I))-((6.578814251506152/(-2.595731200144379+I))+(-93.74726996823534+(-0.5194813544041322*V)))
22	15.2409	abs(((5.098996618266567/I)+0.5133129438082655)*(-14.26435328371769+V)+(55.19327479404188+(64.61618426393518/(I-1.191377282848368))))
24	11.0943	abs((2.436521028787195/(I-1.039859752400481))-(((-38.93564431728164)/(3.716231164606522-0.0969227844781012*V))+(-8.534724400003158*I+147.4753802813173)))
28	7.16469	abs((3.134533045829507/(-0.9983913623989698*I+1.041135343697051))+(119.2757320675261-(6.570297889983751/(0.05472451341554586*V-(-0.1636874285141897+I))))+(-8.12522748554537*I))
30	5.48807	4.17288691503046*(abs(-11.50769375514865+1.511647872310845*I)-((-1.373669018837775)/(0.05122269583612836-(I-1.001733394820702))))+(10.96297215266328/(I-0.05575805511797421*V))+61.71708649055628
33	5.29476	36.91440296546499+abs(((-7.076424507479584)/(-1.057373899668908+I))+(19.16218511769667+((((-6.149665925707793)/I)-0.4205536566704033)*(-13.18987647994363+V))))+(V/(0.1041708441604894*(1.970994466450075+I)))
34	4.74545	42.05612426298923+abs(((-9.814397317370799)/(-1.066750226002719+I))+((((-10.9466696628841)/(0.756810536989068+I))-0.4395067879525296)*(-10.8991842969937+V)+22.06455364949668))+((V/0.3298020137063624)/I)
35	4.48775	38.20075343044072+abs(((-9.758840097340052)/(-1.066807302890034+I))+(23.9553850162988+((((-7.637472412873317)/(0.1825534418379288+I))-0.4523534218560504)*(-11.13177964168296+V))))+(V/(0.1685473739788387*(0.863862922451 2714+I)))
37	4.34012	38.00250492958005+abs(((-10.49871401756279)/(-1.057864652944814+(-0.01147792879032783+I)))+(25.6140666121862+((((-7.661218047433147)/(0.171194337817276+I))-0.4567008877246435)*(-10.60480089605172+V))))+(V/(0.174269074130853*(0.7956801288563012+I)))
38	3.12641	abs(((V-18.07525064547213)/(0.8876100988639742-0.4191930607724529*I))+((10.42984373609476/(log10(V-3.299826244720884)*log2(I)))+0.6152613788474742*V)-17.80775925365826)+65.77174982198645
39	2.51204	12.38170718487556+abs(((-14.62936951806192)/(-1.059601698429831+(-0.0238810486033712+I)))+(48.2299959810718+((((-8.195942221027424)/(0.3292427996053519+I))-0.5025390731900627)*(V-4.192232775507088))))+((17.84618136243971+V)/(0.1589892793814443*(1.494496426158953+I)))
41	1.50859	((-9.855775737153825)/((I*(I-1.189623275397775)-3.453664675949679)*log(-0.00668898213444 2161+I)))+(log2(-0.4189516181390001+I)*(1.172972760438738*I-29.09158041630781))-((-8.254866757680848)/(8.706676766933045-V))+125.171448836964
45	1.36779	((-9.812548443606588)/((I*(I-1.182404699121273)-3.471371561528321)*log(-0.00679242837404275+abs(I))))+(log2(-0.4150236567093035+I)*(1.173961941125599*abs(I)-29.21783841729425))-((-7.677380071816537)/(8.701170487223594-V))+125.4762810086323
46	0.0895361	((-0.2408803586340865)/(I-3.017224475004751))+34.52857618128883+abs(((-9.287586119653277)/(-1.056127758519905+(-0.009481335194391519+I)))+(21.11591576935355+((((-7.391032670121236)/(0.3351748091742818+I))-0.4515167850146117)*(-10.06242841269821+V))))+((11.62334517808248+V)/(0.1598586335377158*(1.723636895626801+I)))

4. Discussion

The outcomes of the symbolic regression analysis, particularly the model with a size of 46, exhibit exceptional proficiency in forecasting the reservoir filling duration using data comprising the total volume and average inflow. This model reached very low RMSE (< 1) that marks minimal prediction error level. This finding indicates a significant relationship between reservoir volume, average inflow, and filling duration, as shown in the simplified equation below.

$$D = \left(\frac{-0.241}{I - 3.017}\right) + 34.529 + \left|\left(\frac{-9.287}{-1.056 + (-0.00948 + I)}\right) + \left(21.116 + \left(\left(\frac{-7.391}{0.335 + I}\right) - 0.451\right) \cdot (-10.062 + V)\right)\right| + \left(\frac{11.623 + V}{0.160 \cdot (1.723 + I)}\right)$$

The model suggests that not only the total volume and average inflow are significant factors, but the relationship between them also plays a role in determining the duration of filling. This shows that the process of reservoir filling is a multi-factorial phenomenon that cannot be fully explained by a simple linear relation. This result answers the research aims by providing a deeper understanding of the factors that influence the duration of reservoir filling and highlights the complexity of the relationships between the variables.

5. Conclusion

The initial filling of a reservoir is the first test that the dam will perform the function for which it was designed. Dams are constructed primarily to impound and store a large body of water. However, when dam construction is complete, the flow to the dam site resumes and the reservoir begins to fill with water. The first filling of a reservoir can be defined as the increase in the water level behind the dam from the time the construction is complete until it reaches the desired operating level. Depending on the location, type, size, and intended purpose of the dam, the duration and rate of its first filling can vary. During reservoir filling, certain construction activities were carried out within a restricted timeframe due to the rising water level, such as plugging diversion tunnels, performing hydromechanical work, and installing and monitoring dam instruments. Regardless of whether it takes several months or years to occur naturally or with the aid of pumping units, the first filling of a reservoir should be planned, controlled, and closely monitored to reduce the risk of failure. Thus, the variables of inflow, outflow, and storage volume are the most significant factors that determine the duration of filling a reservoir.

This research uses symbolic regression analysis to model and predict the duration of reservoir filling based on the total volume of the reservoir and average inflow. The analysis results, particularly those involving models with a size of 46, indicate a remarkable achievement in attaining a high level of accuracy. The models exhibited low RMSE of less than one day, which suggests a strong and intricate association between reservoir volume, average inflow, and the duration of filling. This relationship reveals the interplay between variables in comprehending the reservoir's filling dynamics.

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