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Mathematical Modeling in Teaching-Learning Ordinary Differential Equations: A Case Study of Series Electric Circuits

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Abstract: The objective of this study is to establish the relationship between mathematical modeling and practices developed in the teaching-learning of ordinary differential equations through a strategy based on the determination and solution of first-order differential equations that mathematically model the potential differential in a capacitor connected to an electric circuit in series with a resistor. This electrical situation is a physical phenomenon that exhibits a logarithmic curve during the charging period. This study is based on the context of a differential equations course at a Colombian university using three methodological alternatives to achieve the objective: 1) theoretical model, 2) simulated model, 3) mathematical modeling through numerical differentiation of the 1st order and with the use of technology. The analysis of the results was based on a comparison between models using the coefficient of determination R^2 .

Keywords: electrical circuits, mathematical modeling, numerical differentiation, ordinary differential equation.

数学建模在常微分方程教学中的应用：以串联电路为例

摘要：本研究的目的是通过基于一阶微分方程的确定和求解的策略建立数学建模与在常微分方程教学中发展起来的实践之间的关系，该策略以数学方式模拟与电阻串联的电路中的电容器中的电位差。这种电气情况是一种物理现象，在充电期间呈现对数曲线。这项研究以哥伦比亚一所大学的微分方程课程为基础，使用三种方法来实现目标：1) 理论模型，2) 模拟模型，3) 通过一阶数值微分和使用技术进行数学建模。结果分析基于使用判定系数 R^2 对模型进行的比较。

关键词：电子电路、数学建模、数值微分、常微分方程。

1. Introduction

In recent years, science and technology have exhibited a remarkable growth trajectory, which has had an impact on lifestyles, information dissemination, production methods, and modes of thinking.

In light of this situation, education plays a transcendental role in the integral formation of the human being, which requires a general transformation of individuals capable of satisfying the demands of society. In particular, university education is fundamental because it is responsible for training

professionals with the abilities to create, analyze, research, innovate, and adopt new technologies necessary for the development of a nation.

To contribute to the realization of this principle, mathematics and its teaching-learning process play a transcendental role because of the contribution of this science to the development of human logical thinking. Therefore, mathematics education has become a field of vital importance and, at the same time, of great scientific interest, which requires the development of new technologies.

In particular, mathematical education, especially for students in the field of engineering, has been approached from different points of view that have evolved over the years. Originally, it was thought that students should master all the formal structures of mathematics to solve problems and apply them. However, through experience, it has become clearer that the type of mathematics used in engineering education, especially the numerous modeling techniques that arise in the classroom, plays a significant role in fostering the development of new knowledge [1].

These concepts are incorporated into the curriculum for differential equations in engineering programs as they contribute to a more thorough understanding of mathematics and improve students' ability to grasp the diverse phenomena and processes that are modeled. Furthermore, they can serve as valuable pedagogical tools for future engineers [2].

Furthermore, differential equations facilitate the transition from basic and applied mathematics, enabling the investigation of various problems in the physical, social, and biological sciences. This is achieved through the formulation of a mathematical model that involves the conversion of imprecise hypotheses that approximately describe a real situation.

The implemented strategy articulates both material and symbolic tools that facilitate the learning of differential equations. These tools act as mediators between students and knowledge, facilitating understanding and problem-solving. This instrumental mediation highlights the importance of using adequate resources to support learning [3].

An effective method to enhance students' understanding of mathematical concepts and promote deep comprehension is through the utilization of case studies involving electrical circuits in both theoretical and experimental laboratory settings. This approach provides a robust foundation for developing analytical and practical skills [4].

2. Theoretical Framework

A number of studies have explored the potential of mathematical modeling to enhance the teaching and learning of differential equations. Rodríguez and Quiroz [5] employ mathematical modeling to facilitate the teaching of differential equations within the context

of electrical circuits, specifically those comprising a resistor and a capacitor (RC).

Lagomarsino and Abdel [6] employ a Mathematica simulator to compare theoretical results with reality through the study of ordinary differential equations, thereby providing a fundamental tool in the teaching-learning process.

In [7], mathematical models for fuel cells that facilitate the autonomy of electric vehicles were investigated. These models are based on first- and second-order differential equations.

In Colombia, Plaza [8] modelled a series of physical and natural phenomena that require the use of mass and/or energy in conjunction with auxiliary processes and employed tools such as Excel and numerical differentiation, which generate first-order ordinary differential equations.

The experimental phenomena in question yield data that, when two of them are equally spaced with respect to an independent variable, can be expressed as follows:

$$f'(t_i) = \frac{-f(t_{i+2}) + 8f(t_{i+1}) - 8f(t_{i-1}) + f(t_{i-2})}{12h} \quad (1)$$

where h represents the difference between two consecutive data points or step size, that is, $h = t_{i+1} - t_i$, $f(t_i)$ denotes the evaluation of f at time t_i .

A differential equation was established based on [8], particularly Newton's cooling and/or heating law, and using (1) demonstrated in [9]. This equation models the potential difference in the capacitor at any instant.

3. Materials and Methods

3.1. Materials Used

The materials used in the experimental assembly of the RC series circuit are as follows:

- 1) Resistors
- 2) Capacitors
- 3) Protoboard
- 4) Mounting brackets
- 5) USB memory stick

3.2. Equipment Used

The following is the list of equipment utilized to ascertain the potential differences in the capacitor at any given time and process the information subsequently:

- 1) Computer
- 2) Simulation Multisim 14.3 Education®
- 3) Xplorer GLX-PS-2002
- 4) Power Supply 0-24 V AC/DC 3630.00 - Frederiksen
- 5) Multimeter 2706B BK Precision – Tool Kit

3.3. Teaching Strategies

This concept encompasses the mathematical

depiction of everyday situations that significantly impact teaching and learning differential equations. This is achieved through the development of a theoretical model and the generation of ad hoc solutions based on simulations and experiments, resulting in the formulation of first-order differential equations that model the RC series electric circuit.

These alternatives are implemented in the study of first-order differential equations and their applications. These strategies are based on a master's thesis [10] and the work of the mass-spring system modeled with second-order differential equations shown in [11].

In the first part, students are confronted with the transition between a real situation→pseudo-concrete model→mathematical model, using Kirchhoff's laws as a theoretical model.

A theoretical model of the potential differential in the capacitor can be constructed using a first-order ordinary differential equation.

$$\begin{cases} Rq' + \frac{1}{C}q = E \\ q(0) = 0 \end{cases} \quad (2)$$

In the second part of the experiment (Alternative 1) students face the transition from a real situation→pseudo-concrete model→physical model→mathematical model. The simulator Multisim® was employed to observe the potential difference in the capacitor of the RC circuit compared to the previous part of the experiment.

The following steps are to be taken to simulate the capacitor load:

- Open Multisim 14.3 Education® Simulator;
- Place a source DC, capacitor, resistor, and ground, which are located in the component selection space;
- Place measuring probes in differential configuration (one in positive pin and other in the negative pin);
- Configure the simulation: Simulate→Analysis and simulation→Transient and configurate the following parameters:
 - ✓ Initial conditions
 - ✓ Start time
 - ✓ End time
 - ✓ Maximum time step (TMAX): Determine automatically.
 - ✓ Outputs: Select the positive probe.
 - ✓ Save and run.

In the third part (Alternative 2), the students encounter the same transitions as in the second part, utilizing the physical-technological resources of the physics and electronics laboratory to perform the experimental assembly of the RC electric circuit. With the data obtained through numerical differentiation, as described in (1), the mathematical model can be determined.

Finally, a comparison is made between the theoretical, simulated, and numerical models and the experimentally obtained data.

4. Case Study

The next step was to address the issue of an RC series electric circuit. To do so, we apply the three alternatives to obtain a mathematical expression that models the potential difference in a capacitor as a function of time. In this case, we designed an RC series electric circuit, as illustrated in Fig. 1, with $R = 470000 \Omega$, capacitance $C = 0.022 F$, and a voltage input $E = 5V DC$ (direct current). The capacitor is initially uncharged ($q(0) = 0$).

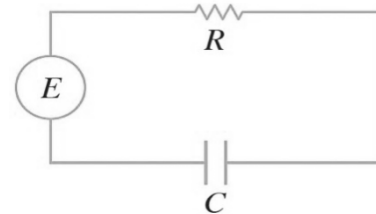


Fig. 1 RC circuit [12]

4.1. Theoretical Model

The establishment of the initial value problem, which incorporates a differential equation and an initial condition that characterizes the charge on the capacitor at any given moment, is grounded in Kirchhoff's laws, as evidenced in [13].

The initial value problem is established and solved as a first-order linear differential equation using the numerical data.

$$470000q' + \frac{1}{0,022}q = 5 \quad , \quad q(0) = 0$$

$$470000q' + \frac{1}{0.022}q = 5$$

$$q' + \frac{1}{10340}q = \frac{1}{94000}$$

$$q(t) = \frac{\int e^{\int \frac{1}{10340} dt} \cdot \frac{1}{94000} dt + k}{e^{\int \frac{1}{10340} dt}}$$

$$q(t) = \frac{\frac{1}{94000} \int e^{\frac{1}{10340} t} dt + k}{e^{\frac{1}{10340} t}}$$

$$= \frac{\frac{10340}{94000} e^{\frac{1}{10340} t} + k}{e^{\frac{1}{10340} t}}$$

The constant of integration, k , can be calculated by assuming that $q(0) = 0$.

$$q(0) = \frac{\frac{10340}{94000} e^{\frac{1}{10340}(0)} + k}{e^{\frac{1}{10340}(0)}} = 0$$

$$\frac{10340}{94000} + k = 0$$

$$k = -\frac{10340}{94000}$$

$$q(t) = \frac{0.11e^{\frac{1}{10340} t} - 0.11}{e^{\frac{1}{10340} t}} = 0.11 - 0.11e^{-\frac{1}{10340} t}$$

To calculate the potential difference in the capacitor,

it is necessary to multiply $q(t)$ by the constant $1/C$. This yields

$$V_C = \frac{1}{C} q(t) = \frac{0.11 - 0.11e^{-\frac{1}{10340}t}}{0.022}$$

$$= 5 - 5e^{-\frac{1}{10340}t} \quad (3)$$

4.2. Alternative 1

The Multisim 14.3 Education® simulator, as shown in Fig. 2, was employed to construct an RC series electrical circuit with the following specification: $R = 470000 \Omega$, capacitance $C = 0.022 F$, and a voltage input $E = 5V DC$ (direct current).

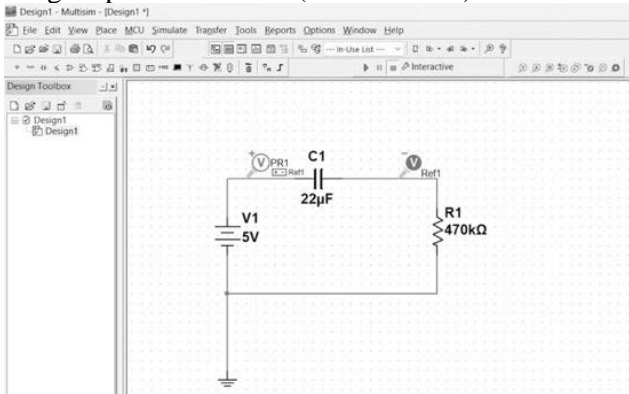


Fig. 2 RC circuit simulation to implement Alternative 1 (Own work)

Subsequently, the data obtained from the simulator are analyzed. A polynomial regression of degree five has been obtained, as indicated by expression (4), with a coefficient of determination $R^2 = 1$, as illustrated in Fig. 3.

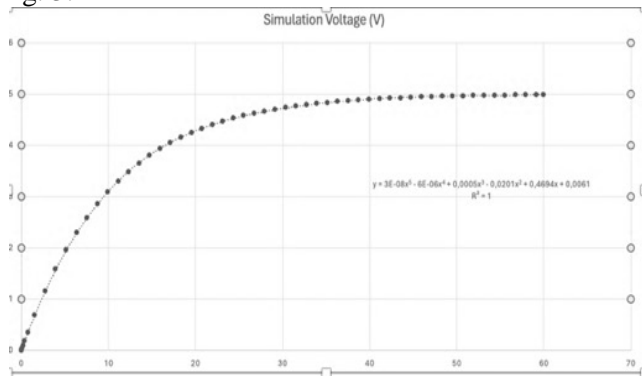


Fig. 3 Data obtained using the Multisim simulator with polynomial regression (Own work)

$$V_C(t) = 0.00000003t^5 - 0.000006t^4 + 0.0005t^3 - 0.0201t^2 + 0.4694t + 0.0061. \quad (4)$$

4.3. Alternative 2

Using identical data from the preceding strategies, an experimental configuration of an RC series electrical circuit was executed, as illustrated in Fig. 4.

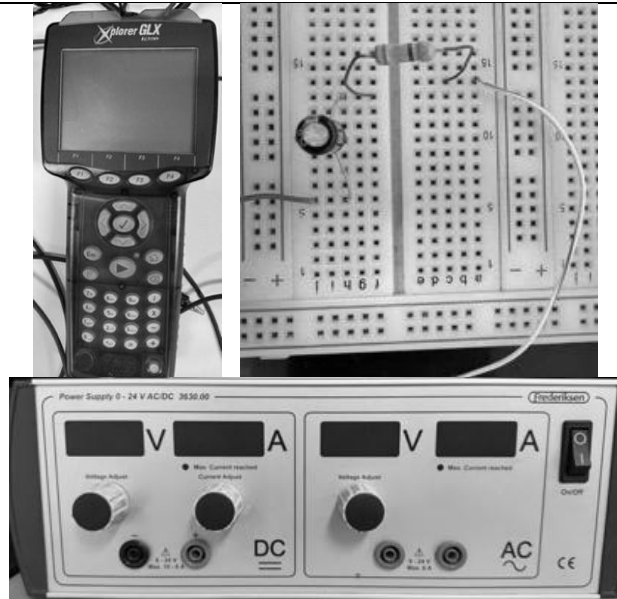


Fig. 4 Experimental setup for Alternative 2 in the laboratory: materials and equipment used (Own work)

The application of numerical differentiation using centered divided differences of order $o(h^4)$ to five steps yields the first derivative in every second and for a period of one minute, as demonstrated in Fig. 5, using the expression provided in (1).

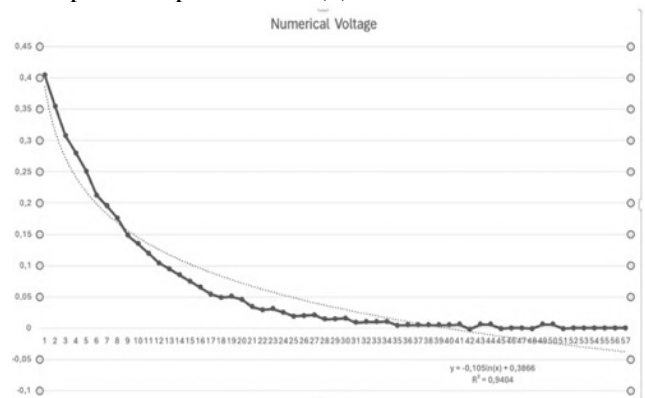


Fig. 5 Logarithmic regression of the first numerical derivative (Own work)

$$V'_c(t) = -0,105 \ln t + 0,3866,$$

$$V_c(2) = 0,92$$

$$\int V'_c(t) dt = \int (-0,105 \ln t + 0,3866) dt$$

$$V_c(t) = -0,105(t \ln t - t) + 0,3866t + k$$

$$V_c(2) = -0,105(2 \ln(2) - 2) + 0,3866(2) + k = 0,92$$

$$k = 0.08236090792$$

$$V_c(t) = -0,105(t \ln t - t) + 0,3866t + 0.08236090792. \quad (5)$$

5. Results

The results were analyzed using mathematical expressions (3), (4), and (5), as shown in Table 1.

Table 1 Mathematical models obtained theoretically and with the two alternatives (Own work)

Model 1: Theoretical Model
$470000q' + \frac{1}{0,022}q = 5$
$V_C = 5 - 5e^{\frac{-1}{10340}t}$
Alternative 1: Simulation (Using the Multisim® Simulator)
No differential equation generated
$V_C(t) = 0.00000003t^5 - 0.000006t^4 + 0.0005t^3 - 0.0201t^2 + 0.4694t + 0.0061$
Alternative 2: Numerical Differentiation 1 st Order
$V_C'(t) = -0,105 \ln t + 0,3866$
$V_C(t) = -0,105(t \ln t - t) + 0,3866t + 0.08236090792$

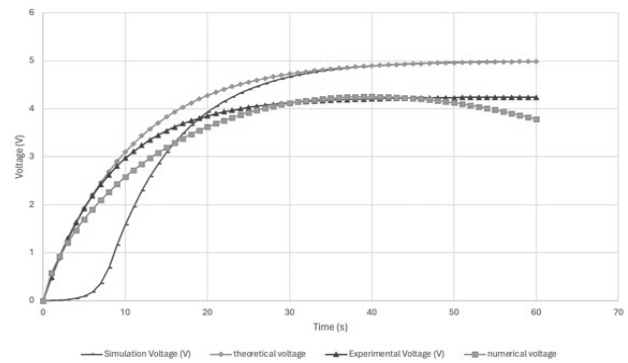


Fig. 6 Graphical comparison of the experimental data, theoretical model, and the two alternatives (Own work)

Finally, a comparative analysis was performed using the coefficient of determination R^2 of the theoretical model and two alternatives, which were then compared with the experimentally obtained data, as shown in Table 2.

Table 2 Coefficients of determination of theoretical, simulated, and numerical models, compared with experimental data (Own work)

Representations Compared to Recorded Data	Theoretical Model	Alternative 1: Simulation	Alternative 2: ND Order 1
R^2	0.990	0.883712	0.97597

The analysis in Table 2 indicates that the theoretical model provides a more accurate representation of the data, with a coefficient of determination (R^2) of 0.99, compared to the experimentally obtained data.

This outcome is expected, given that the theoretical model operates under the assumption of ideal components as there are numerous variables within electrical circuits that can impact behavior, including additional impedances, operational time, and temperature, among others.

6. Conclusion

The development of a didactic strategy for teaching-learning differential equations through mathematical modeling involves activities that move between different domains. These activities begin with a theoretical model and two alternatives, which are then compared with experimentally obtained data.

This strategy can be implemented to solve a series electrical circuit comprising an inductor (coil) with inductance L , resistance R , and a capacitor with capacitance C , as illustrated in Fig. 7. The three models permit the calculation of the potential differential across the capacitor as a function of time.

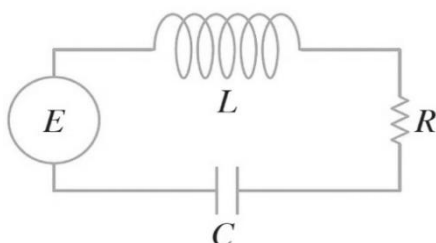


Fig. 7 RLC circuit [12]

Furthermore, a system of differential equations can be employed to theoretically model the currents i_2 and

i_3 in the electrical circuit shown in Fig. 8.

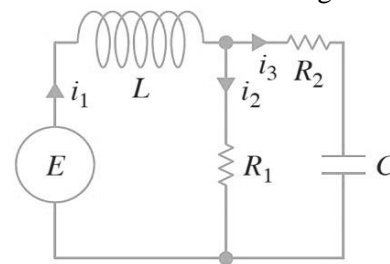


Fig. 8 RLC circuit: two-mesh configuration [12]

A number of practical applications of this research can be identified, including the use of capacitor banks to correct the power factor in AC circuits (Fig. 9) and the incorporation of protective devices within computer chargers (Fig. 10).



Fig. 9 Capacitor bank [14]



Fig. 10 Laptop charger [15]

A solid mathematical foundation is an indispensable prerequisite for developing such a strategy because it allows for the formulation of sophisticated solutions. The solution of differential equations with their techniques, accompanied by mathematical modeling using first-order numerical differentiation, must be within the scope of the student's knowledge.

The instructional approach implemented in the differential equations course emphasizes simulation and experimentation. To enhance the teaching process, it is necessary to provide technical assistance for the effective use and implementation of electronic components in the laboratory setting. In light of this study's findings, the suggested methodology can be replicated in diverse contexts where teaching-learning differential equations is concerned. For example, it may be applied to situations involving exponential growth, Newton's cooling model, emptying a tank, a simple pendulum, or a mass-spring system, among others.

The transition between the theoretical models and two proposed alternatives, which are novel in teaching and learning differential equations in professional training, represents a significant academic contribution. This approach contributes to the field of mathematics didactics by introducing an educational strategy that diverges from the traditional approach.

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