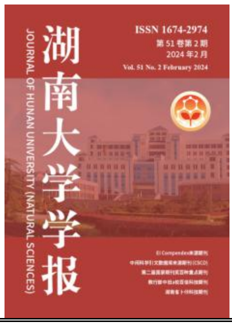

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The Literary Genre of Tales in Teaching Irrational Numbers

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Abstract: The school mathematical discourse of teachers in class occurs to communicate abstract concepts to students. In mathematics classes, when teaching some concepts, due to their level of abstraction, metaphors or literary figures of speech are used as a mental tool in the cognitive dimension to establish connections between concepts. Mathematical concepts, such as negative integers and rational numbers, can be taught using metaphors to relate them to ordinary concepts acquired through experience in particular contexts. However, there are concepts, such as infinity or irrational numbers, whose level of abstraction is so low that it is complicated to find concrete concepts to build a metaphor to explain them. This article shows the results of qualitative research aimed at determining the effectiveness of using a little story instead of a metaphor to teach the topic of irrational numbers. This strategy using the literary genre of fairy tales to explain the topic of irrational numbers at the university education level and its positive results constituted the main novelty of this study. This strategy generated a-didactic situation as part of a didactic strategy based on the theory of didactic situations for teaching irrational numbers to students in a first-semester fundamental mathematics course for engineering careers. To determine the effectiveness, a test was designed and applied before and after taking the students to participate in the teaching situations supported by the tale, a teaching guide, and a kit of didactics material. There was a positive evolution in the level of knowledge about the characteristics of the mathematical concept under study. The results contribute to the teaching of irrational numbers and the research field of mathematics education. The tale strategy constitutes a way of generating a-didactic situation required when the theoretical framework of research is the theory of didactic situations.

Keywords: a-didactic situation, didactic situation, irrational numbers, tale’s literary genre, theory of didactic situations.

講授無理數的故事文學體裁

摘要：老師在課堂上的學校數學話語是為了向學生傳達抽象概念。在數學課堂上，在教授一些概念時，由於其抽象程度，會使用隱喻或文學修辭作為認知維度的心理工具，建立概念之間的連結。數學概念，例如負整數和有理數，可以使用隱喻來教授，將它們與透過特定背景下的經驗獲得的普通概念聯繫起來。然而，有些概念，例如無限大或無理數，其抽象程度很低，以至於很難找到具體的概念來建構隱喻來解釋它們。本文展示了定性研究的結果，旨在確定使用小故事而不是隱喻來教導無理數主題的有效性。這種利用童話文學體裁在大學教育層面解釋無理數主題的策略及其正面成果構成了本研究的主要新穎之處。該策略產生了一種教學情境，作為基於教學情境理論的教學策略的一部分，用於在工程職業第一學期基礎

數學課程中向學生教授無理數。為了確定有效性，在讓學生參與由故事、教學指南和教學材料套件支援的教學情境之前和之後設計並應用了測驗。關於所研究的數學概念的特徵的知識水平有了積極的發展。這項成果對無理數教學和數學教育研究領域做出了貢獻。故事策略構成了當研究的理論架構是教學情境理論時所需的一種生成教學情境的方式。

关键词：a-教學情境、教學情境、無理數、故事的文學體裁、教學情境理論。

1. Introduction

The tale's literary genre as a didactic resource in teaching mathematical concepts is generally used at primary education levels, as corroborated in [1], which studies this resource based on a series of works as essential background information on the use of tales to teach mathematics, mainly at the primary education level. Among the findings of this research is that the tale is a motivating and integrating element between mathematics and language.

Similarly, [2] uses tales to teach school mathematics at the primary education level. This author found a close relationship between the emotional and intellectual of students when they are learning mathematical concepts with the help of tales, manifesting multiple feelings that motivate participation in the activities that the teacher proposes.

The relationship between the affective and the cognitive is also highlighted in [3], which considers that the tale favors the student–teacher relationship and interdisciplinarity. It is a motivational factor that generates interest in the subject in a fun way and brings imagination and fantasy into play, two key aspects that awaken creativity. In short, the tale allows "the connection between the affective and the cognitive, a necessary condition for the significance of learning concepts."

Vera and Soto [4] developed research in which they applied the tale strategy in teaching French as a second language, also at the primary education level, to generate competence in reading and writing in that language. Although it is not in the field of mathematics teaching, it is a proposal applicable to the mathematics classroom in the sense that they found that the tales "appeal to the senses of our students, to the imagination, participation, and communication producing much more significant learning" something desirable also in the case of teaching mathematical concepts.

At the secondary education level, [5] developed research in the eleventh grade to teach the concept of infinity using a tale of his creation. This author considers that using a tale to teach mathematics transcends convention because it favors interaction and debate, both teacher–student and student–student. The author states, "In the proposed strategy, feelings and emotions are awakened for meaningful learning." This research was carried out under the direction of the Research Group in Mathematical Thought and Communication (GIPEMAC) of the Technological University of Pereira, in which the interest lay in determining the effectiveness of a didactic proposal that involved the use of tales, such as resources for teaching mathematical concepts that are a little more complicated than those of primary education, where they are regularly used.

The use of the tale's literary genre to teach a complicated and abstract mathematics topic like irrational numbers was chosen as a research object because it has proven to be a very effective teaching strategy to teach the infinite at the secondary education level, as demonstrated in [5].

In this way, we can say that the favorable aspects found in the works at the primary education level and the positive findings shown when applying the strategy at the secondary education level motivated investigating to determine the possibilities of involving the tale's literary genre in the teaching of irrational numbers, in higher education, with engineering students who are studying first-semester mathematics at the Technological University of Pereira.

The findings found here could apply as an alternative strategy to teach mathematics not only with irrational numbers but also with other topics as abstract as irrational numbers. This strategy was helpful without specific concepts to create learning metaphors.

2. Theoretical Framework

Here, the definition that [6] provides for the tale is assumed. According to [6], the tale is a short story with imaginary incidents and simple plot development, in which the characters perform a series of acts with an unforeseen ending, although appropriate to the outcome of the events. This author considers that, at least in early childhood education, the tale performs three functions: one of a psychological nature, another of a didactic nature, and a third of a linguistic nature. These functions are desirable in first-semester students at the university level, given possible feelings of anxiety and fear observed in some of them in initial courses that involve topics with a level of abstraction, such as that of irrational numbers. The first function related to the search is to satisfy the need for knowledge, capture and maintain attention, and make students feel identified with the characters. Second, she realizes the tale is a motivating resource that supports the didactic units. Through the told events and the characters develop, creativity emerges, which is an essential aspect of learning mathematical concepts. In addition, if the tale content interests the students, they will be attentive and concentrated, which is desirable in mathematics class. Regarding the linguistic function, a tale is a tool that achieves the phases of comprehension, namely, retention, organization, and assessment, a desirable aspect in teaching mathematical concepts that require, for example, using mathematical modeling processes [6].

Structurally, according to [7], a tale must contain at least three main parts: a specific beginning, a conflict to develop related to the said beginning, and a solution to the conflict raised. Additional aspects, but not less important, are the characters that participate in the tale, the context of these characters, and the time. Through the conflict or problem in which the characters get involved, human values come into play that, as expected, generate emotions in those who read or listen to the tale. This aspect, according to [7], is available through a simple structure, as in the case of traditional tales. Grimm Brothers' tales are examples of tales with a simple structure with a narrative based on a meditation on binary qualities in opposition, such as "courage/cowardice, security/fear, wealth/poverty, vanity/modesty, and the most powerful, good/bad."

Egan [7] states that, through these opposites, on which the construction of the narrative is based, it is possible to emotionally lead students to connect with the content intended to be taught. In this regard, [7] states that binary opposites "are not simple logical concepts; they are affective concepts–their meaning lies in our emotional sense" and, therefore, the concepts that underlie these opposites are understood from the emotions that they generate.

According to [8], the tale, as a didactic resource, makes it possible for the learning of mathematical concepts to be meaningful to transcend the rote if it promotes the use of imagination, awakens fantasy, and leads the student to relate new knowledge with specific concepts learned through previous experiences, which encourages interest in learning, given the usefulness of the concepts involved in the tale. In other words, through the tale they connect what they learn with what they know. All these advantages highlighted by the authors focus on primary and preschool students.

Egan [7] highlights the essential role that imagination plays in the student's learning process and states that it is through this that "teachers begin to forge new ways of making classes coherent and engaging." He considers that the imagination staged in classes, not only in mathematics but also in any science, constitutes a fundamental pillar for learning, which is often not considered. In the interest of achieving a specific type of efficiency that turns out to be very limited and, in this sense, "sometimes we forget that involving the imagination can produce better results in the true learning that we want to achieve in children, than many of the supposedly efficient techniques that displaced it."

Apostol [9] defines irrational numbers, simply, as "real numbers that are not rational." This definition can leave students a little perplexed regarding the understanding and handling of this type of number. To delve a little deeper into the conceptualization of these numbers, it is necessary to go to Euclid, who defines commensurable and incommensurable magnitudes as follows:

Definition 1: Commensurable magnitudes are measured with the same measure, and incommensurable magnitudes are those for which it is impossible to have a general measurement [10].

It is a definition referring to the magnitudes of segments. The latter, in the author's time, were called straight lines. Regarding what is referred to in the previous definition, when considering an initial segment as a base segment, in the Elements, a universe of magnitudes is composed of two disjoint sets: one of commensurable magnitudes and another of incommensurable magnitudes. Here, the latter is of interest, to which Propositions 1 and 2 refer, as can be read below:

Proposition 1: Given two unequal magnitudes, if a magnitude greater than its half is removed from the larger one and, from what remains, a magnitude greater than its half removed, and so on, there will remain a magnitude that will be less than the given smaller magnitude [10].

Proposition 2: If, when continuously and successively subtracting the smaller from the larger of two unequal magnitudes, the remaining magnitude never measures the previous magnitude, the magnitudes will be incommensurable [10].

Recalde [11] interprets Euclid's Definition 1. The author states that "two magnitudes A and B are commensurable if there are two numbers n and m, such that $n\text{A} = m\text{B}$." That is, for magnitudes A and B in which the numbers m and n do not exist, they are considered incommensurable or irrational in current terms, and they will be rational if they are commensurable, also in current terms.

Two rational numbers in an arithmetic sense a and b are distinct if the difference $a–b$ gives a positive or negative value. If this difference is positive, a is greater than b, i.e., $a > b$. In the other case, a is less than b, i.e., $a < b$. With these two possibilities present in the way a and b are different. Dedekind [12] considers that the following properties are met:

Property 1: If $a > b$ and $b > c$, then $a > c$. Whenever a and c are two different (or unequal) numbers and b is greater than one of them and less than the other, we want to express it without fear of being reminiscent of geometric representations, i.e., b is between the numbers *a* and *c*.

Property 2: If *a* and *c* are different numbers, there are always infinitely many numbers *b* between *a* and *c*.

Property 3: If *a* is a given number, all numbers in the system R decompose into two classes, A_1 and A_2, each of which contains infinitely many individuals; the first class A_1 covers all numbers a_1 that are less than *a*, and the second class A_2 covers all numbers a_2 that are greater than *a*. The number *a* can be arbitrarily assigned to the first or second class and is accordingly either the largest number of the first class or the smallest of the second class. In each case, the division of the system R into two classes A_1 and A_2 is such that every number in the first class A_1 is less than every number in the second class A_2.

As can be seen, Dedekind uses the letter R instead of Q to symbolize the system of rational numbers.

Recalde [13] considers the following warning from Dedekind about "the geometric line L, the prototype of the continuum, is nothing more than an aggregate of infinite points distributed in a special way, whose fundamental property is established through the operation of *cut*," establishing a connection between the elements of Q and the points of a line L.

According to [12], every rational number *a* generates a cut (A_1, A_2), with the property that in A_1, there is a maximum value or in A_2, there is a minimum value. In contrast, if a given cut (B_1, B_2) has the property that in B_1 there is a maximum value or in B_2 there is a minimum value, that value is a rational number. However, Dedekind says that rational numbers do not produce infinitely many cuts. These define a new type of number that he calls an irrational number.

Once the irrational numbers are known, the question arises: Are all irrational numbers of the same type?

To answer this question, consider that, for example, the number root of 2 is the root of the quadratic polynomial $x_2 – 2 = 0$, and something similar happens with the root of 3, the root of 5, the root of 7, etc. These cases that have just been shown make up a specific set of irrational numbers [14]. This book states that when a number is the root of a polynomial with rational coefficients, it is said to be an algebraic irrational number. Those that do not meet this condition are said to be transcendent irrational numbers in the sense that they transcend algebraically.

Two well-known cases of transcendent irrational numbers are *e* and π. The French mathematician Charles Hermite (1822-1901) first demonstrated the transcendence in 1873. Using methods similar to those of Hermite, Lindemann (1852-1939) established in 1882 that π was also transcendental.

Regarding the theory of didactic situations (TDS), Brousseau's work in the French school of mathematics didactics gave rise to it. Brousseau [15] considers that this theory is a significant contribution to the teaching of mathematics proposed as a construction that helps to understand how the teacher, the student, and mathematical knowledge interact during class and allows us to understand how the condition of the form what is taught and how that knowledge is learned. It was in 1970 that the first version of this theory was published in the magazine of the Association of Public Mathematics Teachers of France. Here, the author shows the results of the analyses on his own experience with children at the primary education level, with whom he worked as a teacher in a small rural school, some knowledge that he had acquired as a student of mathematics and psychology.

Regarding the notion of situation, [15] states that it is a model of interaction between a subject and a specific medium. The resource available to the subject to achieve or maintain a favorable state in this environment is a range of decisions that depend on precise knowledge.

The author considers the medium to be an autonomous subsystem that opposes the student. The media have to do with teaching resources, such as teaching materials, textbooks, work guides, and technological resources.

This author says that a didactic situation is related to those activities carried out in the classroom by the teacher when teaching the student in a situation of learning a concept. Brousseau [15] classified didactic situations into situations of action, formulation, validation, and institutionalization.

The activities begin with the action situation in which the student chooses the elements of the environment that question him based on the aspects of the said environment that are familiar to him, thanks to which he will be able to act in a search to elucidate relationships between the environment and information that he already possesses about the knowledge in question. The student acquires new information through his decisions and reactions to the situation posed by the teacher, and from observations about the student's behavior in the situation, it is rescued as "a response pattern" evidenced through "an implicit model of action" [15].

In the action situation, the author warns that initially, the student is exposed to a situation independent of the teacher's intervention, which he calls a-didactic situation. Regarding this notion, [15] states that "canonically constituted knowledge is that which is intelligible to others, shared, following the didactic will of society, whose importance is guaranteed by history and culture." According to this concept, the author has in mind the knowledge that the student should possess previously as a subject immersed in a particular social and cultural context. The TDS proposes that it is the teacher who manages to identify, helped by his didactic strategy, that knowledge in the actions that the student follows to resolve the situation, through the conceptions that he has of it, and that he does so in an autonomous manner. Each situation when the student is exposed must be designed so that the didactic intention with which he has conceived and constructed it is not explicit. Brousseau [15] considers that, although in a-didactic situations students are expected to act independently of the teacher, it should not be forgotten that it is the latter who designs them. However, this should not intervene until the student accepts that the problem posed for the a-didactic situation is a problem that he must solve and that he also provides a solution to the said problem until he is “able to use it in situations that he encounters outside of any teaching context.” Brousseau [15] states that the student will certainly not have acquired the knowledge at stake.

After the students have carried out the activities designed by the teacher for the action situation, it is necessary to continue with the formulation of knowledge to involve other students. This part of the process is what the author calls the formulation situation since it is at this moment that the teacher must expect the student to be able to coherently model the components of the knowledge that he or she has managed to recognize, identify, and relate in a linguistic system, to communicate it to others and that he or she can use it later. At this point, [15] considers that to control what is communicated, "it is also necessary that the two interlocutors cooperate in the control of an external environment," so that the control is not individual but rather it is only through collaboration between students in the formulation of knowledge that the problem posed in the situation is resolved.

In the initial action situations and subsequent joint formulation, students glimpse aspects of the desired knowledge that require continuous revision. To do this, students must resort to the knowledge they have previously acquired, either through the educational system or in the cultural context in which they are immersed. Here, [15] says that there is a transformation of the role of students as communicative subjects if the one who issues the formulation as an issuer of knowledge transcends his condition as a simple issuer to become a proponent and the one who receives the formulation transcends his condition as a simple recipient, to be an opponent, since both are in very similar conditions concerning the knowledge involved in the problem. The author calls it a validation situation since the students seek to ensure that the knowledge they are building cooperatively fits the already established conceptual domain. The group can demand demonstrations on those points where disagreements or lack of clarity arise.

In the constitution of this theory, Brousseau and his collaborators initially thought that these three situations, action, formulation, and validation, exhausted the possible types of situations. However, from careful observation of the actions of the teachers who participated in their investigations, they deduced that to move from one topic to the next, they needed to review what had been done in the lesson in which each topic was discussed. They expressed disagreement with only the three didactic situations mentioned, and they argued that the learning process could not be reduced only to these three situations. This is how the researchers found it difficult to accept and be aware that the teachers who participated in the research had to account for what their students did in each lesson and that they could describe the events so that the lesson could be structured to state the events evidenced in the action and formulation situations. They also required identifying the teaching object involved in constructing the underlying mathematical knowledge in said didactic situations. Teachers should ensure students’ productions regarding the knowledge at stake are closer to the knowledge established in the cultural context of both students and teachers. The above means that this knowledge can be used again. Brousseau [15] called this the last situation of institutionalization, and it includes a theoretical elaboration of the teachers, which arises through reasoning that considers the existing institutional knowledge regarding both the discipline and the culture that seeks to eliminate contradictions that could be in students’ formulations. These elaborations are “phases of institutionalization,” in which the theoretical elaborations of the teachers will allow the knowledge constructed by the students “the indispensable cultural state of knowledge” [15].

3. Materials and Methods

According to [16], a qualitative approach was followed because it sought to investigate the evolution of knowledge about an abstract mathematical concept that a group of students expressed on aspects related to five categories identified on irrational numbers from epistemological aspects of the concept. Regarding the qualitative, [16] states that: “Cultural models are at the center of the qualitative since they are flexible and malleable entities that constitute frames of reference for the social actor and are constructed by the unconscious.”

According to these same authors, qualitative research is local in the sense that the results are valid for the participating population and are usually not generalizable. However, in the field of mathematics education, the results of research of this type are validated in different contexts but in similar circumstances, that is, in situations of teaching mathematical concepts where a teacher and a group interact with students.

The experience was applied to two groups of first-semester students in engineering careers who were studying fundamental mathematics during the second academic period of 2022. For the analysis, they were called groups 1 and 2. The first consisted of 28 students and the second consisted of 30 students.

The strategy of involving a tale to teach irrational numbers arises from the results generated with a project developed in [5] on the use of the literary genre of the tale to address the concept of infinity within the Research Group in Mathematical Thought and Communication – GIPEMAC.

This research was conducted with students of the last secondary level, in which the entire topic was addressed through the story. In the case shown here, the participating students were at the initial level at the university, and the story is used initially to support the creation of the a-didactic situation, an aspect that is part of the theory that frames the present investigation.

Previous research carried out in the GIPEMAC group identified some restrictions, such as the difficulty in finding metaphors that would allow connecting the topic of irrational numbers with concrete experiences. Thus, it was considered appropriate to take advantage of the elements offered in [15], given that a characteristic element of this theory is the a-didactic situation. For the latter, a tale was designed and written based on everyday social situations in the context of the Coffee Axis region in Colombia, which, in its content, involves the theme of irrational numbers. For the conception and writing of the tale, the suggestions in [6] and [7] were followed, as well as the contributions on the golden number, an algebraic irrational number whose properties are studied in [17].

With these brief clarifications, it is intended to justify the variation that the use of the tale implies for the theory to elucidate the possibilities of this literary resource, in the approach of a proposal for a didactic sequence for the teaching of irrational numbers, a complex topic both educationally and cognitively.

TDS supported the didactic sequence, designing a didactic work guide composed of 6 items. For the design of the guide, the five categories and didactic situations of the TDS were considered: action, formulation, validation, and institutionalization situations. To support the guide, a didactics material kit was built consisting of 13 elements: a compass, three wooden wheels of different radii, three wires with diameter measurements, three wires with circumference measurements, a 45° square and one of 30-60°, and a plastic model with a conical pinion.

To show visually the research methodology, we elaborated the following flowchart:

Start
¦
V
Define objectives and learning outcomes
¦
V
Assessing initial knowledge with a pretest on irrational numbers
¦
V
Consideration of the TDS categories
- Action situations
- Formulation situations
- Validation situations
- Institutionalization situations

¦
V
Design of didactic sequences based on TDS categories
¦
V
Design Didactic Work Guide with 6 items to support the development of didactic sequences
¦
V
Design and construct a Didactics Material Kit (to support the Didactic Work Guide) with 13 elements:
- Compass
- Three wooden wheels of different radii
- Three wires with diameter measurements
- Three wires with circumference measurements
- 45º square
- 30-60º square
- Plastic model with a conical pinion

¦
V
Teach two groups of students about subject irrational numbers using the designed sequence, guide, and material kit
¦
V
Evaluate the effectiveness of the teaching strategy
¦
V
Assess knowledge improvement using a post-test on irrational number characteristics
¦
V
Compare pre-test and post-test results to measure improvement
¦
V
Analysis of the results of the comparison of the two tests
¦
V
Conclusions
¦
V
End

This flowchart outlines the steps involved in the methodology, from defining objectives to assessing initial knowledge with a pre-test, designing and developing the didactic sequence, teaching using the sequence and material kit, evaluating effectiveness through post-testing, and finally presenting the results and conclusions.

4. Results and Discussion

The categories of analysis that were considered for the development of the research are as follows:

1. Recognition of irrational numbers' characteristics.
2. Recognition of algebraic irrational numbers' characteristics.
3. Use of theoretical and geometric tools to draw algebraic irrational numbers on a number line.
4. Recognition of transcendent irrational numbers' characteristics.
5. Contextualization of the concept of irrational numbers (both algebraic and transcendental) to the resolution of some problem situations.

These categories arise from the research within the GIPEMAC group carried out in [18] on the meaning of irrational numbers from their historical-epistemological development and didactic possibilities.

To analyze the evolution of knowledge about irrational numbers, expressed by the students who participated in the experience, in accordance with these categories, a test was designed and applied before the experience, corresponding to the pretest, and after the students were exposed to the teaching situations, with the help of the following four teaching aids: the story, tests, the teaching work guide, and didactics material kit. The scale in Table 1 was designed to analyze the results of the test. In this scale, the initial letter of the rating is considered, and letter B is taken as the last one, given that letter A was already assigned for the approximate rating.

Table 1 Rating scale for student responses in the pretest and posttest (The authors)

Letter	Evaluation
C	Correct
A	Approximate
N	Does not answer
I	Incorrect
B	Absurd

Fig. 1 shows the case of the pretest responses of student 7 from group 2. It is observed that he answers incorrectly to question 1 regarding the number root of 5. He writes that it is a rational number with a periodic and finite decimal expression. According to these issues, this student shows difficulties in recognizing the classification and characteristics of this irrational number. Likewise, he writes that the number π is rational with a periodic decimal expression. Regarding the extension of the decimal expression, he equates the value with the finite decimal expression of a decimal fractional number and then writes that the extension is infinite, which is inconsistent. Regarding the number *e*, he similarly equates its value with the finite decimal expression of a decimal fraction and answers that it is periodic and then says that it is infinite, which is incoherent. For point 2, the student has the quadratic formula in mind; however, the application presents algebraic procedural errors and ends with an incorrect answer.

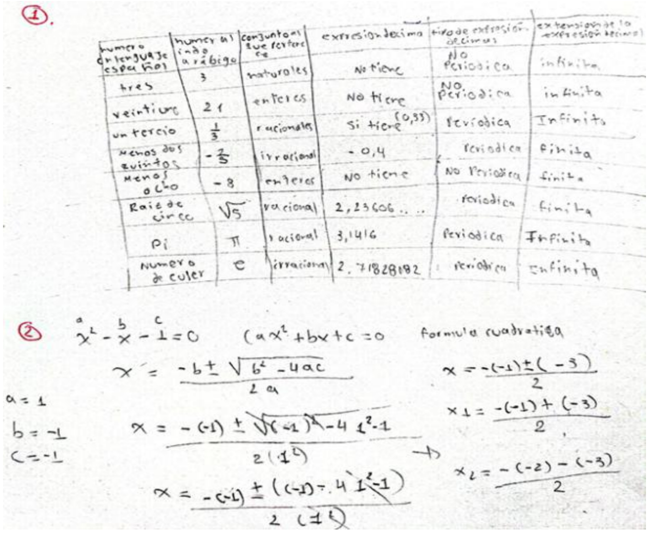


Fig. 1 Answers to Questions 1 and 2 of the pretest, Student 7, Group 2 (The authors)

Fig. 2 shows the answers of the same student 7 to questions 3 to 7. He states that he does not know or does not remember how to solve the problem situations posed in each question.

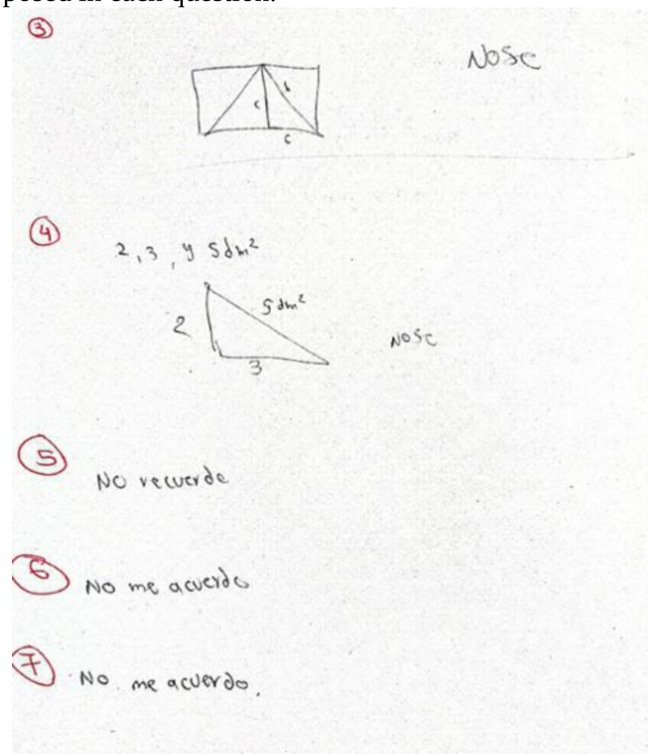


Fig. 2 Answers to Questions 3 to 7 of the pretest, Student 7, Group 2 (The authors)

A similar reaction occurred with the rest of the students in both groups. The results for group 1 are shown in Fig. 3, 68% of the students left the question unanswered or answered incorrectly. Exactly 33% write that they do not know the answer or leave the question unanswered, 28% give an incorrect answer, and 7% answer absurdly. Only 19% gave a correct answer, and 13% of students answered approximately.

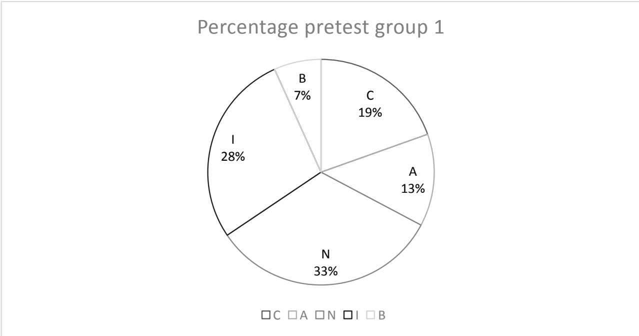


Fig. 3 Percentage of frequencies of results obtained when applying the pretest to Group 1 (The authors)

Fig. 4 shows the results for group 2. Here also, 68% either do not respond or give a wrong answer. Exactly 49% write that they do not know or do not answer, 15% write an incorrect answer, and 4% write an absurd answer. In this case, only 17% responded correctly and 15% responded approximately.

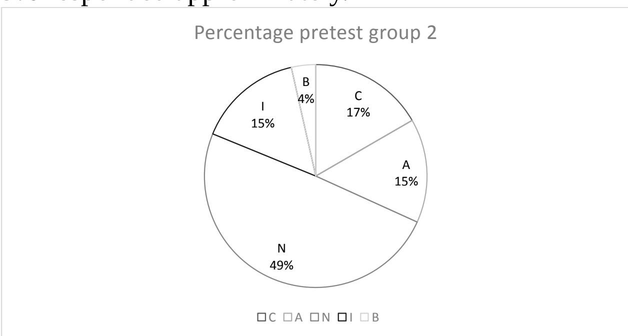
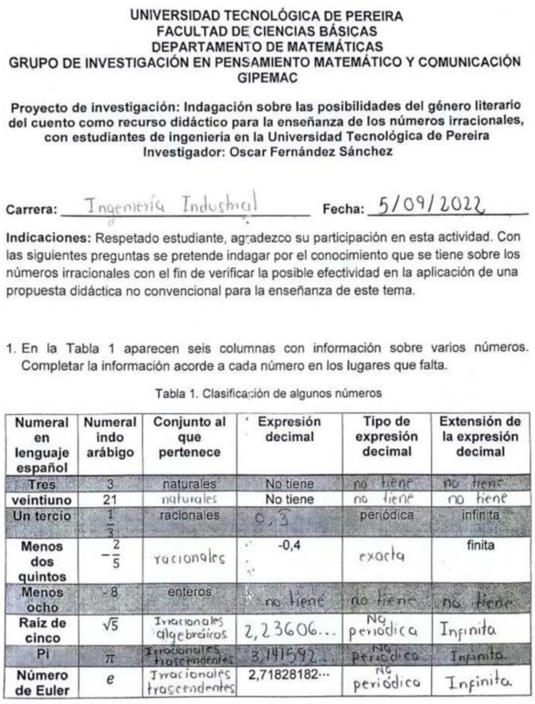


Fig. 4 Percentage of frequencies of results obtained when applying the pretest to Group 2 (The authors)

Fig. 5 shows that student 25 from group 1 correctly answers all the questions about the root of 5, π and e in point 1. He classifies the first as an algebraic irrational number and the other two as transcendent irrational numbers.



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GIPEMAC

Proyecto de investigación: Indagación sobre las posibilidades del género literario del cuento como recurso didáctico para la enseñanza de los números irracionales, con estudiantes de ingeniería en la Universidad Tecnológica de Pereira
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Carrera: Ingeniería Industrial Fecha: 5/09/2022

Indicaciones: Respetado estudiante, agradezco su participación en esta actividad. Con las siguientes preguntas se pretende indagar por el conocimiento que se tiene sobre los números irracionales con el fin de verificar la posible efectividad en la aplicación de una propuesta didáctica no convencional para la enseñanza de este tema.

1. En la Tabla 1 aparecen seis columnas con información sobre varios números. Completar la información acorde a cada número en los lugares que falta.

Tabla 1. Clasificación de algunos números

Numeral en lenguaje español	Numeral indo arábigo	Conjunto al que pertenece	Expresión decimal	Tipo de expresión decimal	Extensión de la expresión decimal
Tres	3	naturales	No tiene	no tiene	no tiene
veintiuno	21	naturales	No tiene	no tiene	no tiene
Un tercio	$\frac{1}{3}$	racionales	$0,\overline{3}$	periódica	infinita
Menos dos quintos	$-\frac{2}{5}$	racionales	-0,4	exacta	finita
Menos ocho	-8	enteros	no tiene	no tiene	no tiene
Raíz de cinco	$\sqrt{5}$	Irracionales algebráicos	2,23606...	No periódica	Infinita
Pi	π	Irracionales trascendentes	3,141592...	No periódico	Infinita
Número de Euler	e	Irracionales trascendentes	2,71828182···	No periódica	Infinita

Fig. 5 Answers about irrational numbers in Question 1 of the post-test from Student 25, Group 1 (The authors)

Fig. 6 shows that the student solves the quadratic equation posed in point 2, approximately, following the steps of the quadratic formula algorithm. This solution is considered approximate because it correctly finds only one of the two roots; the other does not coincide with the solution because of a calculation error in the radical of the root. For point 3, he makes correct use of the compass as a concrete geometric tool and the Pythagorean theorem as a theoretical geometric tool for the location of the two irrational numbers root of 2 and root of 3 on the number line. For the situation posed in point 4, he finds the values of the sides of the tiles using the square root. Given the problem situation posed in point 5, he manages to correctly approximate the value of the largest circumference of the pinion. For the problem in point 6, he makes a calculation error when he substitutes the interest value into the expression to calculate the final capital, which leads to an incorrect answer.

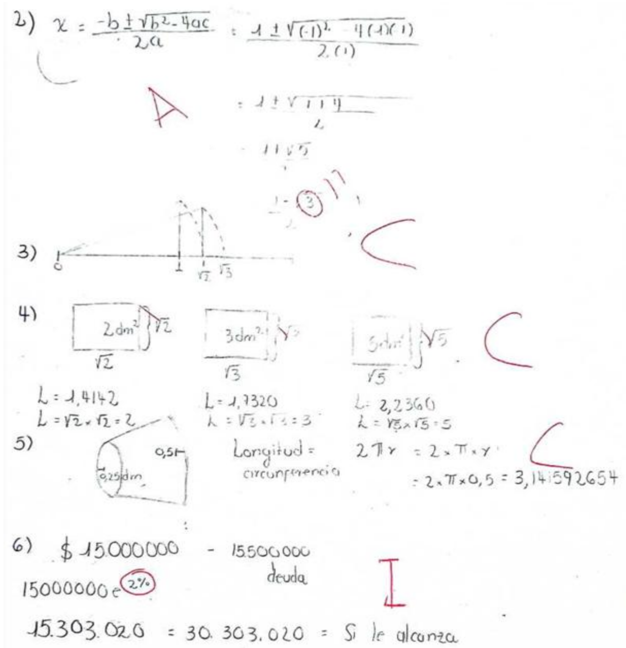

Fig. 6 Answers to Questions 2, 3, 4, 5, and 6 of the post-test from Student 25, Group 1 (The authors)

Fig. 7 shows that this same student responds correctly to the way the measurements of the golden rectangle are related.

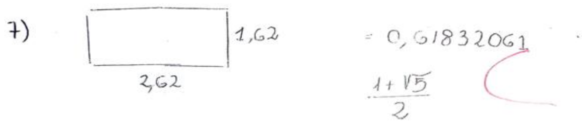

Fig. 7 The answer to Question 7 of the post-test from Student 25, Group 1 (The authors)

Fig. 8 shows the positive evolution of knowledge about the evaluated aspects of irrational numbers expressed by group 1 through the percentages of the evaluations of the post-test solution compared with the results of the pre-test. The percentage of positive reviews in this case was 74%, much higher than the 32% reported the first time. A favorable difference of

42% can be observed. Likewise, negative evaluations increased from 68% to 26%. Although this unfavorable percentage remains to be worked on, improvements in the results were achieved.

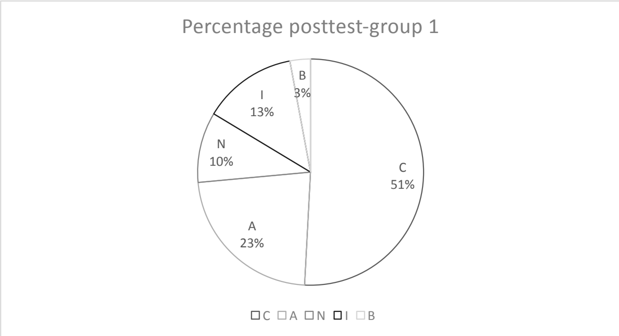

Fig. 8 Percentage of frequencies of the post-test results for Group 1 (The authors)

Fig. 9 shows the positive evolution in knowledge about the evaluated aspects of irrational numbers expressed by group 2. According to this, this group had better results than group 1. It should be considered that the difference in favor of group 2 could have been generated because the students in group 1 answered the post-test individually and group 2 as a group. This is how this group shows a percentage of positive evaluations of 98% compared with the 32% obtained in the pretest. A favorable difference of 66% is observed. Regarding negative evaluations, they were reduced from 68% to 2%.

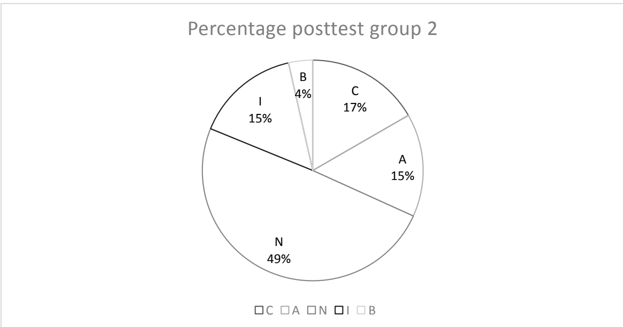

Fig. 9 Percentage of frequencies of the post-test results for Group 2 (The authors)

5. Conclusion

It was possible to demonstrate, with the comparison of the pretest results and the post-test, the evolution in the students' conceptions of irrational numbers, mainly in what has to do with their classification, the characteristics of their representation, and some possible applications in solving related problem situations in engineering.

Regarding the question about the possibilities of the tale's literary genre as a didactic resource for teaching irrational numbers, with engineering students at the Technological University of Pereira, the tale has possibilities as a didactic resource in the development of research from the theoretical framework of TDS, in the part when it is necessary to expose students to an a-didactic situation. In the tale, the concept appears as part of a problem related to the collective imagination, related to the structures of knowledge to be studied outside the classrooms or the educational system in general.

The tale offers teachers and students motivational elements that enable them to delve into the study of the topic and learn more about the characteristics of these fascinating numbers.

Regarding the categories of analysis, regarding the recognition of the irrational numbers' characteristics, with the didactic situations, it was observed in the results of the post-test that, compared to the results of the pretest, the students showed a significant evolution. They could recognize the characteristics of irrational numbers. Furthermore, they recognized, in a large percentage, that the numerals that represent irrational numbers have infinite and non-periodic decimal expressions and use six or seven figures for their representation, when before they only used two and without suspense points.

The category recognition of the characteristics of algebraic irrational numbers represented a higher level of complexity. When asked questions related to this category in the pretest, everyone responded that they did not know about this type of number.

After going through didactic situations supported by teaching aids, students, according to their answers in the post-test, learned that algebraic irrational numbers are the solution to an algebraic equation.

Regarding the category management of theoretical and geometric tools to plot algebraic irrational numbers on the number line, a general difficulty was detected in the pretest since the participants expressed ignorance of this procedure for plotting the points on the number line, whose coordinates are algebraic irrational numbers.

After participating in the experience to solve the problem situations in this sense, the positive results of the post-test demonstrated improvement in the knowledge and ability to plot points with irrational numbers as coordinates. The evidence shows that students adequately use the concrete geometric tool of the compass and the theoretical one of the Pythagorean theorem to plot algebraic irrational numbers on the number line.

For category recognition of transcendent irrational numbers' characteristics, all students expressed ignorance of these numbers in the pretest. After experiencing the didactic situations, in the post-test, students recognize some transcendent irrational numbers, such as irrational numbers that are not the solution of an algebraic equation. Given the lack of time, we only worked with numbers π and e. Likewise, due to time constraints, it was impossible to address the more complex epistemological issues of this type of irrational number, such as the demonstration of transcendence or the approach to other transcendent irrational numbers.

This study offers an innovative and original perspective to the field of mathematical education research by exploring the possibilities of the tale's literary genre as a teaching resource at the university level when the use of metaphors to teach subjects as complicated as irrational numbers is not possible because the teacher cannot find a concrete example to build a metaphor that allows a connection between irrational numbers and some concrete context.

Finally, regarding the category contextualization of the concept of irrational numbers (algebraic and transcendent) to the resolution of some problem situations, in the pretest, it became evident the difficulty in using the irrational numbers to solve problem situations where knowledge required the characteristics of these numbers. In the post-test, by validating the students' work in the didactic situations, it was evident that they managed to adequately use the theoretical geometric tool of the Pythagorean theorem to solve problems involving algebraic irrational numbers. Likewise, the correct resolution of problem situations involves transcendent irrational numbers.

Acknowledgments

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