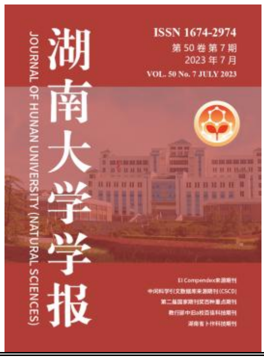

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Incidence of Metaphorical Language in Learning Irrational Numbers: A Case Study

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Abstract: This study delves into the crucial role of metaphors in classroom communication and their impact on student learning. It followed three goals. The first was to analyze the presence of metaphorical language in teaching abstract mathematical concepts within the context of seventh-grade classes at the Pablo Sexto Educational Institution in Dosquebradas, Colombia. This objective aims to identify the type of metaphors used in the classroom language of the teacher to facilitate comprehension of abstract concepts and to understand how these metaphors influence the students' understanding of such concepts. The second objective was to evaluate the impact of using metaphorical language on the students' understanding of the irrational number concept. This involves examining how the teacher's metaphoric phrases affect the understanding and retention of abstract mathematical concepts, as well as identifying their strengths and weaknesses. Third goal seeks to establish links between the presence or absence of metaphors and students' perceptions of their own understanding, considering whether metaphorical language contributes or hinders the conceptual clarity of mathematical content. Employing qualitative research tools, the study revealed a noteworthy presence of metaphors in the teachers' discourse. This study stands as a promising starting point for future research endeavors in this field, while concurrently serving as a valuable tool to enhance teaching practices in mathematics education. Armed with an awareness of how metaphors can shape students' comprehension, teachers can optimize their educational approaches and thereby contribute to their students' academic accomplishments. In conclusion, this study emphasizes the paramount importance of metaphors in educational communication and their pivotal role in shaping the teaching and learning processes. By diligently considering the influence of metaphors on students' understanding, educators can refine their pedagogical practices and make a profound contribution to their students' academic achievements, not only in mathematics but also in various academic disciplines.

Keywords: conceptual metaphor, incidence of language, metaphorical language, teaching irrational numbers.

隐喻语言在学习无理数中的发生率：案例研究

摘要：本研究探讨了隐喻在课堂沟通中的关键作用及其对学生学习的影响。它遵循了三个目标。首先是在哥伦比亚多斯奎布拉达斯的巴勃罗第六教育机构七年级课程背景下，分析隐喻语言在教授抽象数学概念时的存在。这一目标旨在确定教师课堂语言中使用的隐喻类型，以促进对抽象概念的理解，并了解这些隐喻如何影响学生对这些概念的理解。第二个目的是评估使用隐喻语言对学生理解无理数概念的影响。这包括检查教师的隐喻短语如何影响抽象数学概念的理解和保留，以及确定它们的优点和缺点。第三个目标试图在隐喻的存在或不

存在与学生对自己理解的感知之间建立联系，考虑隐喻语言是否有助于或阻碍数学内容的概念清晰度。采用定性研究工具，该研究揭示了教师话语中隐喻的显着存在。这项研究是未来这一领域研究工作的一个有希望的起点，同时也是加强数学教育教学实践的宝贵工具。有了隐喻如何影响学生理解的意识，教师可以优化他们的教育方法，从而为学生的学术成就做出贡献。总之，本研究强调了隐喻在教育传播中的至关重要性及其在教学过程中形成的关键作用。通过认真考虑隐喻对学生理解的影响，教育工作者可以完善他们的教学实践，并为学生的学术成就做出深刻的贡献，不仅在数学方面，而且在各个学科.

关键词：概念隐喻，语言的发生率，隐喻语言，教学无理数.

1. Introduction

The conceptual metaphor theory proposed by Soriano in [1] maintains that the metaphor is a linguistic resource with which a likelihood relationship is established between a conceptual field, generally abstract, and a known concrete conceptual field, through which characteristics of the specific domain that need to be emphasized and communicated in the abstract domain are extrapolated to institutionalize students' personal knowledge [2]. That is, the metaphor allows the identification of aspects that are difficult to know about an abstract domain through the identification of those aspects with similar characteristics in a specific known domain. This is how humanity has understood (internalized) unknown concepts, relating them to known concepts, that is, establishing metaphors unconsciously.

The math teacher is not aloof from this communication phenomenon. He is forced to use metaphorical language in his classes when he attempts to teach and expects his students to understand the concepts of the formal language of mathematics. For example, concepts such as number, variable, equality, function, and set, or verbs such as adding, dividing, square or cubic root, order, and derive, that is, abstract entities that are in the imaginary of a conceptual field not recognizable by the student. Therefore, the teacher seeks, through metaphorical language (usually unconscious), to teach these abstract concepts by highlighting some characteristics of these with related phenomena in a specific conceptual field that are familiar to the collective social imaginary in which the students are, such as the collective social imaginary of the region of the Coffee Axis considered in this investigation.

This article presents the results obtained in the development of an investigation whose main purpose was to determine how it influenced (how it impacted) the learning of the concept of irrational numbers, the metaphorical language used by a teacher with seventh graders from the Pablo Sexto educational institution of the municipality of Dosquebradas in the Department of Risaralda in Colombia. For the numerical assignment, a Likert scale [3] was used to generate a numerical qualification of the teacher discourse–student understanding and teacher discourse–mathematical knowledge relationships.

2. Theoretical Framework

Metaphors are considered mental phenomena with which experience is categorized [4]. According to Black in [5], cited in Fernandez and Angulo in [6], there are three approaches to metaphor. The first is the comparison approach, according to which the metaphor consists of the presentation of the analogy or similarity that underlies the proposition between the metaphorical and literal expressions. A representative of this approach is Aristotle, who in [7], cited in Fernandez and Angulo [6], states, "metaphor is the translation of a foreign name, or from gender to species, or by analogy," or in [8], cited in Fernandez and Angulo in [6] "the simile is also a metaphor". The second is the substitution approach, which according to Black in [5] its definition is: "any approach that holds that a metaphorical expression is used instead of some equivalent literal expression." A representative of this approach is Cicero, who in [9], cited in Fernandez and Angulo [6], states that "metaphors are those words that change their meaning thanks to their similarity with another notion, either for reasons of charm or because the word does not have its own meaning." The third approach is the interaction approach, proposed by Black in [5] and cited in Fernandez and Angulo [6], who states that "in the simple formulation when we use a metaphor, we have two thoughts of different things that are activated together and supported in a simple word or phrase, whose meaning results from their interaction".

The term conceptual metaphor appears in [10], who draw attention to the way people perform the process of categorizing their experience. According to these authors, to be able to talk about abstract concepts, concrete concepts taken from people's experiences are used. However, an appropriate distinction between conceptual metaphor and metaphorical linguistic expression is of vital importance. As Soriano in [1] puts forward, "Conceptual metaphors are abstract schemes of thought that are expressed in many ways, including language." Metaphors are relationships between two conceptual fields anchored in the imaginary of the individual and the community, and the linguistic phrases that result from these metaphors are "metaphorical phrases," that is, phrases (usually unconscious) that lead to a metaphor (unconscious thinking of a relationship) [10]. Therefore, the "metaphorical phrases" that the teacher says in class when trying to teach a concept affect the understanding of the concept by the student.

It is evident, then, that the math classroom does not stand aloof from the presence of metaphorical language, which implies that, even before class, in the preparation process, when the didactic transposition occurs [11], metaphors have been incorporated into the teacher's thought, expressed through his written language, which, during the class dynamics, the teacher transforms into oral language. The latter is part of the *sender–message–receiver* relationship, connected with the metaphorical language used to facilitate the understanding of the subject taught. However, metaphors can fulfill their purpose and make it easier for the student to understand, or they can hinder the learning process by transferring incorrect knowledge due to their inappropriate and unconscious use. Besides, there is the possibility, among others, that the coincidence between what the teacher meant by the metaphorical phrase and what the student understood is positive, but with a misleading interpretation of the mathematical concept by the issuer, which would reproduce the conceptual error in the students, and this is what this research also considers as a negative incidence.

The didactic triangle proposed by Chevallard in [11] and cited in [12] was used to determine the criteria of the teacher's metaphorical language incidence in teaching the concept of irrational numbers, as shown in Fig. 1.

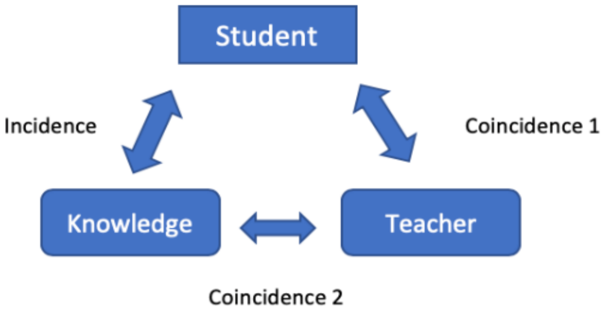

Fig. 1 Relationships between the factors in the didactic triangle [13]

It was assumed as a criterion that the incidence (student - knowledge) is a function of coincidence 1 (student - teacher) and coincidence 2 (teacher - teacher). It was determined that the incidence is positive if both coincidences are positive; otherwise, it is negative. There are several scenarios: one of them, for example, is the case where the student has not understood what the teacher tried to explain with a certain metaphorical phrase (coincidence 1) regardless of whether the teacher's explanation with that metaphorical phrase was consistent or not with the relevant literature (coincidence 2). In this case, the incidence would be negative because the student did not understand the teacher's speech. Another scenario would be when the student did understand what the teacher wanted to explain with a certain metaphorical phrase (positive-coincidence 1), but the teacher's explanation with the metaphorical phrase was not consistent, according to the literature or primary conceptual references (negative-coincidence 2). In this case, the incidence is considered negative because the student would be assimilated (through the teacher's speech) into a mathematical error. Here we present a case of the generation of epistemological obstacles [14] by teaching a mathematical concept incorrectly.

Regarding the mathematical object irrational numbers, [15] states in a simple way that "real numbers that are not rational are called irrational". An important fact about irrational numbers is found in [16] when he speaks of immeasurable magnitudes. In Proposition 8 of Book X, he writes, "If two magnitudes do not have to one another the ratio which a number has to a number, then the magnitudes are incommensurable."

Regarding commensurability, Recalde in [17], also cited in [13], claims that "two magnitudes A and B are commensurable if there are two numbers n and m, such that nA = mB (the concept of rational number)". Regarding irrational numbers, [18], cited in [13], quote the following:

Richard Dedekind (1815-1897) makes a separation of the set of rational numbers into two disjoint classes A1 and A2, which he notes as (A1, A2) which he calls "cuts", so that all elements of A1 are Less than the elements of A2. However, there are cuts in which A1 does not have a maximum and A2 does not have a minimum; therefore, this cut could not be generated by a rational number and created with this analysis a new number that he calls irrational. Irrational numbers include algebraic and transcendent irrational numbers. Regarding this fact, [18], cited in [13], makes the following historical quotation: French mathematician Adrien-Marie Legendre (1752-1833) conjectured that π could not be the root of a polynomial equation of degree n with rational coefficients. Such observations led to a distinction between types of irrational numbers; on the one hand, algebraic numbers, roots of algebraic equation with rational coefficients, and on the other hand, transcendent numbers, named in this way by Euler in 1744, for "transcending the period of Algebraic methods."

3. Method

To determine the incidence of metaphorical language in teaching the concept of irrational numbers, we analyzed the coincidence between the stated intentions of the teacher and the understanding expressed by the students regarding each metaphorical phrase present in the teacher's speech (coincidence 1). In turn, the intention expressed in the teacher's explanation was compared with the formal concept stated in the corresponding scientific literature (coincidence 2) to determine, with the result of these comparisons, the level of final incidence in the understanding of the concept of irrational numbers by the students. To do that, a Likert scale was used [3], according to which a grade of 5 is equivalent to "strongly agree", 4 to "Agree", 3 to "indifferent", 2 to "disagree" and 1 to "strongly disagree". For this purpose, three classes of two teachers were recorded. Audio recordings of the three classes in which the purpose was to explain the concept of irrational numbers, with which the metaphorical discourse was identified around the mathematical concept in question. Subsequently, a questionnaire was administered to the students to determine what they understood with each metaphoric phrase identified and thus determine the incidence of metaphorical language in teaching the concept of irrational numbers. This was done in the three classes analyzed: two classes for the same teacher, teacher 1 (initial class and feedback class) and one class for another teacher, teacher 2. Teacher 1 has a Mathematics and Physics Teacher Degree and an Educational Technology Management Master's Degree. Teacher 2 has a Mechanical Engineering Degree and a Mathematics Education Master's Degree.

4. Results

Table 1, Table 2, and Table 3 show the results of coincidence 1. In the final column of the mentioned tables, this coincidence follows a simple shading code: very soft shading corresponds to a high coincidence, soft shading corresponds to an indifferent coincidence level, and strong shading corresponds to a low coincidence level. In these tables, the code of the metaphorical phrases is registered; for example, metaphorical phrase 6 of teacher 2 class 2 is coded as FM62D1 in Table 2.

Table 1 Coincidence 1 – class 1 – teacher 1 (Developed by the authors)

	Strongly agree	Agree	Indifferent	Disagree	Strongly disagree	Coincidence 1
FM11D1	2	4	4	16	4	2,47
FM21D1	1	14	0	8	7	2,8
FM31D1	0	7	11	8	4	2,7
FM41D1	0	3	19	5	3	2,73
FM51D1	2	4	17	0	7	2,8
FM61D1						
FM71D1	0	4	24	1	1	3,03
FM81D1	1	2	16	1	10	2,43
FM91D1	3	5	18	1	3	3,13
Sum	9	43	109	40	39	2,76
Frec	3,75%	17,92%	45,42%	16,67%	16,25%	

Table 2 Coincidence 1 – class 2 – teacher 1 (Developed by the authors)

	Strongly agree	Agree	Indifferent	Disagree	Strongly disagree	Coincidence 1
FM12D1	6	7	3	3	9	2,93
FM22D1	1	13	1	5	8	2,78
FM32D1	6	9	1	8	4	3,18
FM42D1	2	6	7	5	8	2,61
FM52D1	0	4	4	5	15	1,89
FM62D1	0	2	11	6	9	2,21
FM72D1	0	14	5	2	7	2,93
FM82D1	0	12	8	2	6	2,93
FM92D1	1	10	13	0	4	3,14
Sum	16	77	59	34	70	2,73
Frec	6,35%	30,56%	21,03%	14,29%	27,78%	

Table 3 Coincidence 1 – teacher 2's class (Developed by the authors)

	Strongly agree	Agree	Indifferent	Disagree	Strongly disagree	Coincidence 1
FM11D2	1	12	4	6	13	2,5
FM21D2	8	1	5	8	14	2,47
FM31D2	17	6	7	3	3	3,86
FM41D2	13	3	11	3	6	3,39
FM51D2	3	1	10	2	20	2,03
FM61D2	12	6	6	7	5	3,36
FM71D2	23	1	5	3	4	4
FM81D2	14	6	9	1	6	3,58
FM91D2	8	14	7	4	3	3,55
FM101D2	12	5	10	0	9	3,3
FM111D2	0	1	11	23	1	2,33

Continuation of Table 3						
FM121D2	3	6	15	5	7	2,8
Sum	114	62	100	65	91	3,1
Frec	26,39%	14,35%	23,15%	15,05%	21,06%	

Table 4 shows some answers that the students gave to the question concerning the understanding of the metaphorical phrases identified in the transcriptions of the audio recordings of the classes to the professors participating in this research.

Table 4 The students' answers to the question concerning the understanding of certain metaphorical phrases (Developed by the authors)

Metaphor	**Metaphorical phrase**	**Some of the students' answers**
M22D1: A rational number is a state	FM22D1: How do we convert a number to decimal? What is the process? I mean, the division helps me get the rational number	►I understood that through a division, you can go from rational to decimal ►I understood that division helps me reach the rational number ►I understand that if we split it thanks to the numerator, a rational number will be found
M72D1: Irrational numbers are manipulated objects that do not allow one to be placed on top of another.	FM72D1: that's why they are called irrational because they can't be put in the form a / b	►I understood that decimal numbers must have characteristics to be passed to the form of a/b ►I understood that in no way can an irrational number be passed to decimal ►You can't place a/b because they are infinite
M41D2: The square root of two is something impossible	FM41D2: " yep (root of two) nobody knows, that is incredible for anyone, that happened to them (Pythagoreans), they discovered something that was impossible to think, impossible, impossible"	►I understood that this discovery was so rare for them that they doubted the discovery that Hipaso made ►That Hipaso discovered something so crazy that the Pythagoreans said that idea was impossible and decided to change to another idea ►That they discovered something new that nobody could have imagined
M71D2: Irrational numbers are numbers that are not logical	FM71D2: "Irrational, that's why their name, because it was not logical for them (Pythagoreans) that there was a number like that (root of two)"	►They are called irrational because they have no logic and are not rational ►They called it that because they didn't understand or understood it, then it's called irrational ►That the irrational name was illogical for the Pythagoreans

Subsequently, coincidence 2 was determined, which is the comparison between what the teacher intended to explain with a specific metaphorical phrase and the formal definition of the mathematical concept. Table 5 shows the value of coincidence 2 for the three classes taught by the two teachers.

Table 5 Coincidence 2: both teachers (Developed by the authors)

Teacher 1–Class 1	**Coincidence 2**	**Teacher 1–Class 2**	**Coincidence 2**	**Teacher 2**	**Coincidence 2**
FM11D1	4	FM12D1	1	FM11D2	5
FM21D1	4	FM22D1	4	FM21D2	2
FM31D1	4	FM32D1	5	FM31D2	5
FM41D1	2	FM42D1	4	FM41D2	5
FM51D1	4	FM52D1	4	FM51D2	2
FM61D1		FM62D1	5	FM61D2	5
FM71D1	5	FM72D1	1	FM71D2	5
FM81D1	2	FM82D1	5	FM81D2	5
FM91D1	5	FM92D1	5	FM91D2	5
				FM101D2	5
				FM111D2	5
				FM121D2	5
Average	3,75		3,78		4,5

Based on the data shown in the tables above, the following assessments are presented:

• FM81D1 is the phrase "When the residue is zero, it is finite because I reached an end; at this point, we can go as far as we want and we will never reach an end, and that's why it is infinite". This phrase obtained a numerical value of 2.43, being the one with the lowest coincidence 1 in class 1 for teacher 1. In the answers given by the students, they understood this metaphorical phrase as if the teacher's main concern were to provide them with an explanation of the division process (especially when it is finite and when it is infinite), adding that 16 out of 30 students did not respond or did not know what to say about this metaphorical phrase.

• The expression FM52D1 "I also got 1, 414213562… is there any period over there that has been repeated? It is called the root of 2, which means that there are roots and there are rational numbers that also behave like these gentlemen and are called irrational numbers" was rated with a numerical value of 1.89. It was found that, regardless of the teacher's intention with this phrase, a negative impact on the students' understanding was generated because a conceptual error is presented here because no rational number behaves like an irrational number, as the teacher poses.

In the discourse of both teachers, there is a marked difference. Teacher 1 presented his classes by proposing that the numbers that do not meet the conditions established to be rational (representation a/b), would, therefore, be irrational (cannot be represented as a/b). On the other hand, teacher 2 presented his class from a historical – etymological point of view concerning the concept of irrational numbers (it is evidenced in FM71D2 that it was the only one with a high coincidence for both teachers). The latter initially addressed the historical part of the immeasurable for the Pythagoreans, attending to their ontology; therefore, initially, the numbers that did not follow this ontology were not considered true, logical, or rational, and therefore, they were irrational (in the etymological sense).

Now, with the assigned value of both coincidences to each metaphorical phrase, the incidence of each one is determined (Table 6, Table 7 and Table 8), and finally, the incidence of the metaphorical language of these two teachers in the concept of irrational numbers. For this, it is necessary to consider that, in order to achieve successful communication between the teacher and the student, it is not only enough to guarantee that the message reaches the student as the teacher plans (coincidence 1), but it is also necessary that the content of the message has the level of veracity sufficient with respect to institutionalized mathematical knowledge (coincidence 2) so that the incidence is considered positive because if any of these cases are not met, it cannot be guaranteed that the comprehension of the mathematical concept that the teacher wants the student to learn can be achieved. If one of the two coincidences does not occur, it is considered that the teacher's speech did not affect the student positively, so his learning process would surely be hindered.

Table 6 Incidence of metaphorical language teacher 1 in his first class (Developed by the authors)

Teacher 1-Class 1			
	Coincidence 1	**Coincidence 2**	**Incidence**
FM11D1	2,47	4	Negative
FM21D1	2,8	4	Negative
FM31D1	2,7	4	Negative
FM41D1	2,73	2	Negative
FM51D1	2,8	4	Negative
FM61D1			
FM71D1	3,03	5	Negative
FM81D1	2,43	2	Negative
FM91D1	3,13	5	Negative

Table 7 Incidence of metaphorical language teacher 1 in his second class (Developed by the authors)

Teacher 1–Class 2			
	Coincidence 1	**Coincidence 2**	**Incidence**
FM12D1	2,93	1	Negative
FM22D1	2,78	4	Negative
FM32D1	3,18	5	Negative
FM42D1	2,61	4	Negative
FM52D1	1,89	4	Negative
FM62D1	2,21	5	Negative
FM72D1	2,93	1	Negative
FM82D1	2,93	5	Negative
FM92D1	3,14	5	Negative

Table 8 Incidence of metaphorical language teacher 2 in his class (Developed by the authors)

Teacher 2-Class 1			
	Coincidence 1	**Coincidence 2**	**Incidence**
FM11D2	2,5	5	Negative
FM21D2	2,47	2	Negative
FM31D2	3,86	5	Negative
FM41D2	3,39	5	Negative
FM51D2	2,03	2	Negative
FM61D2	3,36	5	Negative
FM71D2	4	5	Positive
FM81D2	3,58	5	Negative
FM91D2	3,55	5	Negative
FM101D2	3,3	5	Negative
FM111D2	2,33	5	Negative
FM121D2	2,8	5	Negative

5. Conclusions

- The analysis of the coincidence between the teachers' intention and the students' comprehension (coincidence 1) was carried out, which allowed the conclusion that they did not understand 96.6% of the metaphorical phrases that the teachers used. Only one of 29 sentences had a high coincidence.
- An analysis of the coincidence between the teacher's intention and the metaphorical phrases was made when comparing said intention with the formal definition of the respective mathematical concept (coincidence 2). In general, in the discourse of both teachers, a high coincidence (or close to it) was found between these two comparison factors. From the differentiation between coincidence 1 and coincidence 2, it can be concluded that both teachers have adequate knowledge of irrational numbers; however, the metaphorical phrases used by them hindered their students' learning of the subject.
- The incidence of classroom metaphorical language in the learning of irrational numbers in the seventh grade of the educational institution Pablo Sexto de Dosquebradas was negative for both teachers because what they tried to explain was different from what the students understood, regardless of whether the mathematical concept intended to be explained was correct. This implies that the incidence of the unaware metaphorical phrases used in the teachers' discourse was negative, which hindered the students' understanding of the issue of irrational numbers. Of the 29 metaphorical phrases analyzed by both teachers, only one sentence (3.4%) had a positive incidence.
- This study offers an innovative and original perspective to the field of mathematics education research by exploring the interactions between metaphorical language and the teaching of abstract mathematical concepts. Likewise, the results presented here highlight the cognitive drawbacks generated by

the unaware use of metaphorical phrases in mathematics classes. This perspective not only enriches the understanding of educators' beliefs about mathematical concepts through the metaphors they use to convey complex ideas but also offers the possibility of improving didactic communication from a novel point of view, such as through the aware use of metaphors to teach abstract mathematical concepts.

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References

[1] SORIANO C. The conceptual metaphor. In: ANTUÑANO R. I., & J. V. MANZANARES (eds.) *Cognitive Linguistics.* Anthropos, Barcelona, 2012: 87-109.
[2] GODINO J., & BATANERO C. Institutional and personal significance of mathematical objects. *Recherches Magazine in Didactique des Mathématiques*, 1994, 3(14): 325-355. https://www.ugr.es/~jgodino/funciones-emioticas/03_SignificadosIP_RDM94.pdf
[3] HERNÁNDEZ R., FERNÁNDEZ C., and BAPTISTA P. *Research methodology*. McGraw-Hill, Mexico, 2006.
[4] DE BUSTOS, E. *The metaphor. Transdisciplinary essays*. National University of Distance Education, Madrid, 2000.
[5] BLACK, M. Metaphor. In: JOHNSON M. (ed.) *Philosophical Perspectives on Metaphor*. Universidad de Minessota, Minnesota, 1985: 63 – 82.
[6] FERNÁNDEZ O., & ANGULO M. *Lenguaje metafórico en el abordaje de conceptos matemáticos. El caso de algunos profesores en el Eje Cafetero*. Universidad Tecnológica de Pereira, Pereira, 2019.
[7] ARISTOTLE. *Poetics*. Alianza, Madrid, 2006.
[8] ARISTOTLE. *Rhetoric*. Alianza, Madrid, 2007.
[9] CICERO. *The Orator*. Alianza, Madrid, 2013.
[10] LAKOFF G., & JOHNSON M. *Metaphors We Live By*. Cathedra, Madrid, 2019.
[11] CHEVALLARD Y. *The didactic transposition*. Aique, Buenos Aires, 1988.
[12] D'AMORE B., & FANDIÑO M. An analytical approach to the "didactic triangle". *Mathematics Education Magazine*, 2002, 1(14): 48-61.
[13] ARANGO-GONZÁLEZ M. *Incidence of the teacher's metaphorical language in the teaching of irrational numbers in seventh grade of Pablo Sexto of Dosquebradas Educational Institution*. Master's degree Thesis, Technological University of Pereira, 2019.
[14] BACHELARD, G. *The formation of the scientific spirit. Contribution to a psychoanalysis of objective knowledge*. Siglo 21, Mexico City, 2000.
[15] APOSTOL, T. M. *Análisis matemático*. Reverté, Barcelona, 1982.
[16] EUCLIDES. *Elements*. *Books VIII-XIII*. Gredos, Madrid, 2007.
[17] RECALDE L. *Lecturas de historia de las matemáticas*. Universidad del Valle, Cali, 2018.
[18] MORA C., & TORRES J. *Conceptions of math students in real numbers*. National Pedagogical University, Bogotá, 2007.

参考文:

[1] SORIANO C. 的概念隐喻。在: ANTUÑANO R. I., 和 J. V. MANZANARES（编辑。）认知语言学。人类,巴塞罗那, 2012: 87-109.
[2] GODINO J., 和 BATANERO C. 数学对象的制度和个人意义。数学教学法研究杂志, 1994, 3(14): 325-355. https://www.ugr.es/~jgodino/funciones-emioticas/03_SignificadosIP_RDM94.pdf
[3] HERNÁNDEZ R., FERNÁNDEZ C., 和 BAPTISTA P. 研究方法。麦格劳-希尔,墨西哥, 2006.
[4] DE BUSTOS, E. 的比喻。跨学科论文。马德里国立远程教育大学, 2000.
[5] BLACK, M. 比喻。在: JOHNSON M.（编辑）关于隐喻的哲学观点。明尼苏达大学, 1985: 63 – 82.
[6] FERNÁNDEZ O., 和 ANGULO M. 数学概念的方法中的隐喻语言。咖啡轴部分教师的案例。佩雷拉科技大学,佩雷拉, 2019.
[7] ARISTOTLE. 诗学。联盟,马德里, 2006.
[8] ARISTOTLE. 修辞。联盟,马德里, 2007.
[9] CICERO. 演说家。阿利安扎,马德里, 2013.
[10] LAKOFF G., 和 JOHNSON M. 我们赖以生存的隐喻。 马德里大教堂, 2019.
[11] CHEVALLARD Y. 说教换位。有那个,布宜诺斯艾利斯, 1988.
[12] D'AMORE B., 和 FANDIÑO M. "说教三角"的分析方法。数学教育杂志, 2002, 1(14): 48-61.
[13] ARANGO-GONZÁLEZ M. 多斯奎布拉达斯教育机构保罗六世七年级无理数教学中教师隐喻语言的发生率。佩雷拉科技大学硕士学位论文, 2019.
[14] BACHELARD, G. 科学精神的形成。对客观知识的精神分析的贡献。西格洛 21, 墨西哥城, 2000.
[15] APOSTOL, T. M. 分析数学家。恢复了,巴塞罗那, 1982.
[16] EUCLIDES. 元素。书籍 VIII-XIII. 格雷多斯，马德里.
[17] RECALDE L. 数学阅读史。山谷大学， 卡利, 2018.
[18] MORA C., 和 TORRES J. 数学学生在实数的概念。波哥大国立教育学院, 2007.