第 48 卷 第 5 期
2021 年 5 月

湖 南 大 学 学 报（自 然 科学版）
Journal of Hunan University（Natural Sciences）

Vol. 48. No. 5.
May 2021

Open Access Article

Determining Ship’s Position by the Celestial Altitude Difference Using the Least Squares Method

Nguyen Thai Duong*, Luong Tu Nam

Faculty of Navigation, Vietnam Maritime University, Haiphong, Vietnam

Abstract: Although obtaining the position of a ship by a celestial method is not very accurate by today’s standards, it is an independent method, especially for offshore navigation. In modern navigation, a ship will typically be equipped with two electronic chart display and information systems (ECDISs). Thus, celestial ship positioning will be a primary backup method. In some special situations, the requirement for the accuracy of the ship’s position by a celestial method is not as high as when modern navigational methods are being used. By traditional methods, a ship’s officers would observe, measure, calculate, and plot the lines of position on a nautical chart to determine the ship’s position. Recently, some studies have proposed frameworks to solve the celestial problem in determining the longitude and latitude of the ship using appropriate algorithms. Each method has its own advantages, disadvantages, and certain applicability in practice. This paper proposes a new method to fix a ship’s position by calculating the least squares of the altitude variations of celestial bodies. A program for calculating a ship’s position with high reliability and applicability based on the new algorithm was also constructed, demonstrating its effectiveness in practice.

Keywords: ship positioning, celestial navigation, altitude variation, least squares method.

使用最小二乘法通过天高差确定船的位置

摘要：尽管按照当今的标准，通过天体方法获取船的位置并不十分准确，但它是独立的方法，尤其是对于海上航行而言。在现代航行中，船舶通常将配备两个电子海图显示和信息系统（ECDIS）。因此，天船定位将是主要的后备方法。在某些特殊情况下，通过天体方法对船舶位置的准确性的要求不如使用现代导航方法时高。按照传统方法，船员会观察，测量，计算和绘制海图上的位置线，以确定船舶的位置。近来，一些研究提出了使用适当算法来解决在确定船舶的经度和纬度方面的天体问题的框架。每种方法都有其自身的优点，缺点和在实践中的某些适用性。本文提出了一种通过计算天体高度变化的最小二乘来确定船舶位置的新方法。还构造了一个基于新算法的具有高可靠性和适用性的船舶位置计算程序，证明了其在实践中的有效性。

关键词：船舶定位，天体导航，高度变化，最小二乘法。

1. Introduction

Celestial navigation is an important ship positioning and navigation method due to its low cost, high reliability, and strong independence. It is also one of the traditional navigational skills required of ship officers by the International Maritime Organization (IMO). A 2010 amendment to the Standards of Training, Certification, and Watchkeeping (STCW) Code placed a continuing emphasis on celestial navigation-related education and training. The amendment describes e-Navigation as the focus of the development of navigation technology and the application of information technology as a key direction of development. In addition, there is explicit encouragement in the use of electronic nautical almanacs and celestial navigation calculation software.

Received: February 18, 2021 / Revised: March 16, 2021 / Accepted: April 6, 2021 / Published: May 28, 2021
About the authors: Nguyen Thai Duong, Luong Tu Nam, Faculty of Navigation, Vietnam Maritime University, Haiphong, Vietnam
Corresponding author Nguyen Thai Duong, nguyenthaiduong@vimaru.edu.vn

Navigation safety is endangered when satellite signals are cut off or disrupted, or when a satellite receiver system is broken; both situations can render the positioning of a vessel impossible with systems that rely on those satellite signals. Some devices, such as Radar or Loran-C, only determine a ship's position in certain areas. During open-sea navigation, the ship's offices mainly use Global Navigation Satellite Systems for determining the ship's position. Therefore, when satellite systems have problems, it is not easy to achieve a ship's position. Even today, when navigation is dominated by GPS, a celestial fix still serves as an important backup measure [1].

Celestial navigation is practiced on a daily basis on training vessels. Standard practice relies on quartermasters skilled in the use of handheld marine sextants and paper-and-pencil sight-reduction techniques. The basic method has not changed much in one hundred years, although almanacs and other sight-reduction tools have become more convenient to use [2].

The basic theory behind the celestial navigation method is to use the measured altitudes of celestial bodies associated with the estimated ship position in order to determine the parameters of the line of position (LOP) equation; the equation system for the ship's position is then solved by computer software or by plotting on a nautical chart [3]. Recently, some new methods have been proposed to specifically solve the equations for a ship's position. Chen et al.'s [4] simultaneous equal-altitude equations method (SEEM) describes a complicated procedure for determining a ship's position. Tsou [1] used evolution algorithms in artificial intelligence to apply a genetic algorithm and particle swarm optimization to fix the optimal ship's position. Zhang [5] proposed a self-contained interactive iteration positioning and orientation coupled navigation method based on skylight polarization, but it was too complicated to apply on board a ship. A stand-alone celestial navigation positioning method was presented by Pierros [6] for both stationary and moving observers. Lusic [7] used the positioning equations through spherical trigonometry. Nguyen [8] fixed a ship's position by observing the azimuth of a celestial body using an iterative method. Although the above-mentioned studies have contributed significantly to celestial positioning fixing, they require the measurement of the altitude or azimuth of either at least two celestial bodies or a single celestial body at two different times. Moreover, the existing methods of celestial navigation that require more than two celestial bodies are time-consuming to use because there are many steps involved.

This paper proposes an advanced celestial method to determine a ship's position by the celestial altitude difference based on the least squares method. This is a new approach to celestial ship positioning. The ship's position is fixed by determining the most probable ship's position by the least squares method. The ship's position is the position with the least squares of the altitude variations from the simultaneous observation of celestial bodies. It overcomes the limitations of previous methods, quickly fixes a ship's position, and is not influenced much by the error of the estimated position. Based on the new method, a new algorithm is introduced, and a program to automatically calculate a ship's position with high accuracy was created. This automatic program can improve the quality of the celestial ship positioning method, satisfies the requirements of a backup navigational method, and meets the standards for safe modern navigation [9].

The main advantages of this method in comparison with previous methods can be summarized as described in the following.

First, the proposed method can be applied with one or more celestial bodies at one moment in time. In the case of only one celestial body observed, it still can be applied, while the traditional method requires the measurement of the altitude or azimuth of either at least two celestial bodies or a single celestial body at two different times. In the proposed method, with two or more celestial bodies observed, the accuracy of the ship's position can be significantly improved.

Second, the traditional method depends on the error of the estimated position, but our method is independent of this error.

Third, the construction of the search domain leads to a reduction in the time required for the algorithm.

Applying the proposed technique to fix a ship's position can be seen as one of the new approaches of advanced mathematics in celestial navigation. This technique can be a useful reference for both navigators and marine students.

To verify the effectiveness of the proposed program, a shipboard experiment was performed. The results showed that this program had fast performance for accurately determining the ship's position. This article consists of the following parts: the principle of celestial ship positioning and the suggested new method are presented in Section 1; Section 2 proposes the celestial ship-positioning program construction by a new technique and its application on board a ship. Discussion is described in Section 3. The conclusion is summarized in Section 4.

2. Materials and Methods

2.1. The Principle of Celestial Ship Positioning

One of the most important tasks of a ship officer in navigation is to determine the ship's true position at sea and plot it in a nautical chart. Ship positioning is the determination of the observer's geographic latitude and longitude (φ, λ) at a certain time. In celestial navigation, the observer's position is fixed by calculating the position of the zenith on the celestial sphere and converting it into geographic coordinates and then plotting them directly in a nautical chart. Therefore, the first step is to consider the relation between the geographic coordinates and the position of the zenith on the celestial sphere.

2.1.1. The Relation between the Geographic Coordinates and the Position of Ship's Zenith on the Celestial Sphere

The celestial sphere is shown in Fig. 1. From the ship's position M(φ_M, λ_M) on the Earth's surface, project M to the sphere to obtain the zenith Z_M.

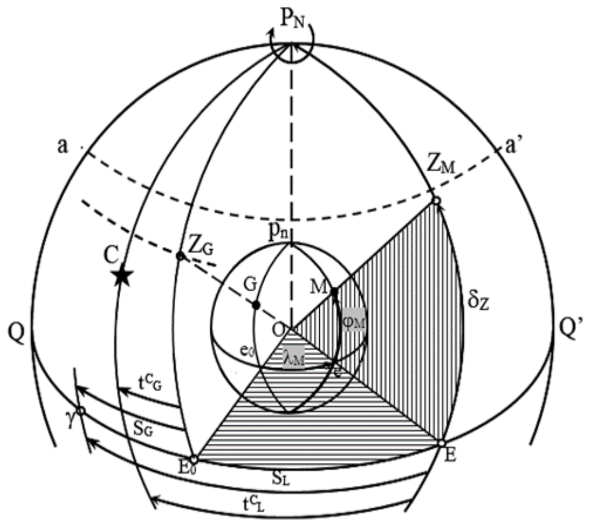

Fig. 1 The position of the ship's zenith on the celestial sphere

Similarly, a point G in the prime meridian will have its own zenith Z_G. The relation between the ship's position on Earth and her zenith in the celestial sphere is presented by the following formulas [10]:

$$EZ_M = eM \text{ and } E_O E = e_O e \tag{1}$$

or

$$\varphi_M = \delta_Z \text{ and } \lambda_M = t_L^C - t_G^C \tag{2}$$

where:

e is the intersection of the observer's meridian and the equator;

e_O is the intersection of the prime meridian and the equator;

E is the intersection of the observer's celestial meridian and the celestial;

E_O is the intersection of the prime celestial meridian and the celestial equator;

δ_Z is the declination of the observer's zenith;

t_L^C is the local hour angle of celestial body C;

t_G^C is the Greenwich hour angle of celestial body C.

2.1.2. Methodology of Determination of Observer's Position on Earth

On board, the officer carries out the observation of altitude (h) and azimuth (A) of the celestial body; the time T_{TK} is also noted. When solving the celestial triangle, the formula of equal altitude can be achieved:

$$\sinh = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos(t_G^\gamma \pm \lambda_W^E) \tag{3}$$

where:

δ: is the declination of the celestial body (can be checked in the Nautical Almanac by T_{TK}),

t_G^γ : is the Greenwich hour angle of the Aries γ (can be checked in the Nautical Almanac by T_{TK}),

t_W^E : is the local hour angle of the estimated position.

To theoretically obtain the ship's position, the simultaneous observation of two or more celestial bodies is required to get the system of the celestial equation as follows:

$$\begin{cases} \sinh_1 = \sin\varphi \sin\delta_1 + \cos\varphi \cos\delta_1 \cos(t_G^\gamma \pm \lambda_W^E) \\ \sinh_2 = \sin\varphi \sin\delta_2 + \cos\varphi \cos\delta_2 \cos(t_G^\gamma \pm \lambda_W^E) \\ \ldots \\ \sinh_n = \sin\varphi \sin\delta_n + \cos\varphi \cos\delta_n \cos(t_G^\gamma \pm \lambda_W^E) \end{cases} \tag{4}$$

The celestial equation (4) system could be solved by different methods to determine the ship's position.

2.2. Plotting Method

By observing two celestial bodies, C_1 and C_2 (as shown in Fig. 2), the altitudes h_1 and h_2 and zenith distances z_1 and z_2 can by calculated as follows [11]:

$$Z_1 = 90^0 - h_1 \text{ and } Z_2 = 90^0 - h_2 \tag{5}$$

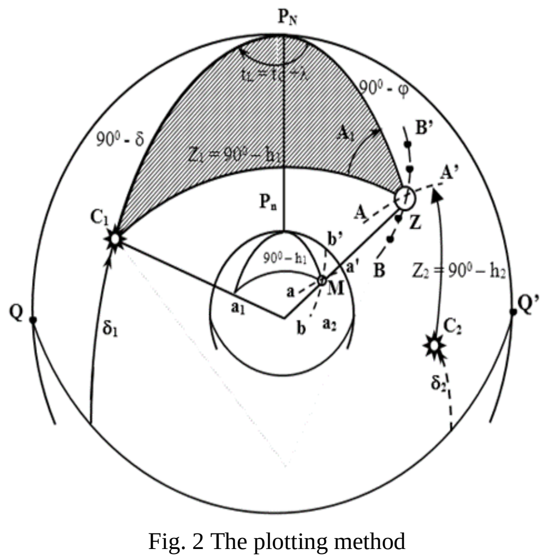

Fig. 2 The plotting method

The sphere with center c_1 and radius z_1 will intersect the celestial sphere at the line of position (LOP) BB'. Similarly, the sphere with center c_2 and radius z_2 will intersect the celestial sphere at the line of position AA'. The intersection of two lines of position AA' and BB' is the observer's zenith z_M .

Projecting the center of the celestial sphere to Earth surface, we have:

- The sub-stellar of the celestial bodies c_1 and c_2 are a_1 and a_2;
- The sub-stellar of the observer's zenith z_M is the observer's position M;
- The side c_1z of the triangle of position is arc a_1M;
- The projection of LOP AA' on Earth surface is the arc aa' with center a_1 and radius $a_1M = Z_1 = 90^0 - h_1$;
- The projection of LOP BB' on Earth surface is the arc bb' with center b_1 and radius $b_1M = Z_2 = 90^0 - h_2$.

The intersection of two arcs is the ship's position: $aa' \cap bb' = M$.

The plotting method is difficult to proceed in practice due to the complexity of the constructions of sphere to determine position. With the ratio 1 mm on the sphere as 1' (or 1 nautical mile), the diameter of the sphere will be 360x60/3.14 = 6878.98 mm ≈7m, which is not applicable onboard a ship [12].

2.3. Circle of Position Method

Nowadays, the circle of position (COP) method is the Saint–Hilaire method (the altitude variation method). The core of this method is to proceed with the COP according to its moving elements in comparison with the estimated position [13, 14]. The elements of the COP are:

Moving direction: A_C

Altitude variation: $\Delta h = h_S - h_C$

where:

A_C : estimated azimuth (calculated azimuth);

Δh : altitude variation;

h_C : calculated altitude;

h_S : true altitude (measured altitude);

Z_C : calculated zenith distance;

Z_S : true zenith distance.

At the time of ship positioning, the ship's estimated position is $M_C(\varphi_C, \lambda_C)$, the observer's zenith and the calculated altitude of the celestial body are Z_C and h_C, respectively (Fig. 3). The zenith Z_C will be at the circle of equal altitude $h_C h_C^{'}$ with the center as the position of celestial body C, radius as the zenith distance $Z_{ZC} = 90^0 - h_C$.

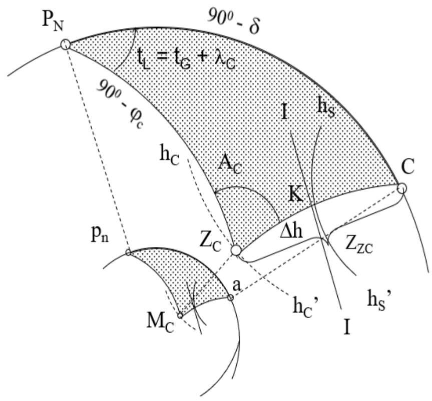

Fig. 3 Circle of position method

Assumed that the ship's true position is $M_0(\varphi_0, \lambda_0)$ at the same time, then measure and adjust the true altitude h_S to the celestial body C and the true zenith distance $Z_S = 90^0 - h_S$. The ship's true position will be at the circle of equal altitude $h_S h_S^{'}$ with the center as the position of celestial body C, radius as the zenith distance $Z_S = 90^0 - h_S$.

Construct a vertical circle through the celestial body C. The arc between this plane and the observer's celestial meridian is the azimuth A_C to the body. The vertical circle ($Z_C C$) will cross the circle of equal altitude $h_S h_S^{'}$ at point K near the zenith Z_C :

$$\begin{aligned} \Delta h &= Z_C K = Z_C - Z_S = (90^0 - h_c) - (90^0 - h_S) \\ \Delta h &= h_S - \mathrm{h}_C \end{aligned} \tag{6}$$

Line I-I, the tangent line to the circle of equal altitude $h_S h_S$' (I-I is perpendicular to $Z_C C$), is the position circle. Project this COP from the celestial sphere to Earth and plot on the nautical chart as Fig. 4:

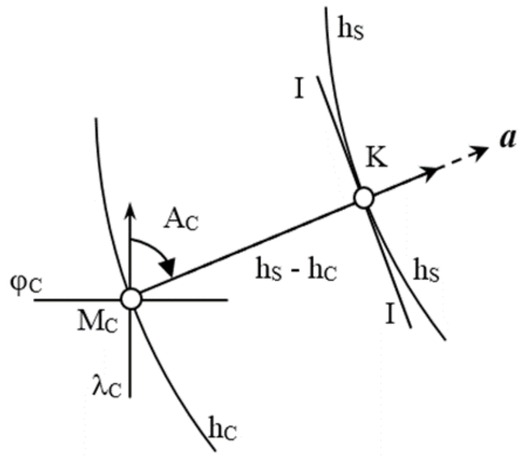

Fig. 4 The circle of position on the nautical chart

Hence, to plot the circle of equal altitude on a nautical chart, some factors, such as the estimated position M_C, the estimated azimuth A_C, and the altitude variation Δh. According to the number of an observed celestial body, the ship's position will be the intersection of two or more circles of position (Fig. 5).

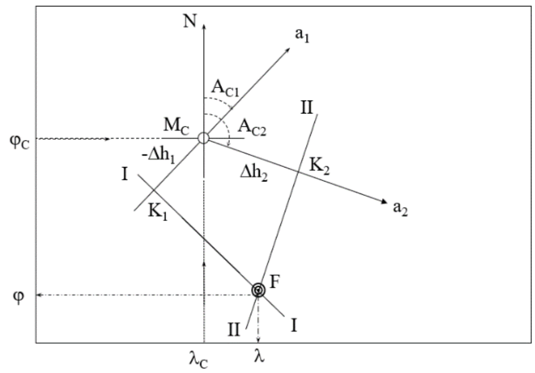

Fig. 5 Ship positioning by plotting the COP on nautical chart

2.4. Ship's Position Determination by the Celestial Altitude Difference Based on the Least Squares Method

Based on the principle of calculating the most probable ship's position in the searching domain around the estimated Mc of each circle of equal altitude, the most probable ship's position is determined by the Least Squares Method of single calculating results. The process of the calculation is as follows:

Step 1: Determine the searching domain.

Latitude limitation: $\varphi_{\min} \div \varphi_{\max}$ (Southern limitation $\varphi_0 = \varphi_{\min}$ and Nothern limitation $\varphi_a = \varphi_{\max}$)

Longitude limitation: $\lambda_{\min} \div \lambda_{\max}$ (Western limitation $\lambda_0 = \lambda_{\min}$ and Eastern limitation $\lambda_b = \lambda_{\max}$)

where:

$\varphi_{\min} = \varphi_C \mp |\Delta\varphi_C|$, $\varphi_{\max} = \varphi_C \pm |\Delta\varphi_C|$, $\Delta\varphi_C$ the error of the estimated latitude φ_C , take (-) when ship's position is in Northern Hemisphere, take (+) when ship's position is in Southern Hemisphere.

$\lambda_{\min} = \lambda_C \mp |\Delta\lambda_C|$, $\lambda_{\max} = \lambda_C \pm |\Delta\lambda_C|$, $\Delta\lambda_C$ the error of the estimated longitude λ_C, take (-) when ship's position is in Eastern Hemisphere, take (+) when ship's position is in Western Hemisphere.

The error of the ship's estimated position is determined by the Least Squares Method in each particular navigation situation such that the probability of the ship's true position inside the set $\{A\}$ is greater than 95%. Actually, in some special circumstances, when the celestial method is used as the back-up to determine the ship's position, there is only an estimated position of a ship. Then, the radius of mean square error (R) is calculated as [15]:

$$M = \sqrt{(S\varepsilon_L)^2 + (S\varepsilon_{TK})^2} \tag{7}$$

where:

S: the distance the ship traveled,

ε_L : the error in compass adjustment,

ε_{TK} : the error in tachometer adjustment.

Step 2: Establish the set of assumed ship positions in the searching domain.

Construct the longitude-latitude net to establish the set of points $A = \{F_{xy}\}$ with $x = \{1, 2, ..., b\}$ *and* $y = \{1, 2, ..., a\}$.

Southern latitude limitation $\varphi_0 = \varphi_{\min}$ and Northern latitude limitation $\varphi_a = \varphi_{\max}$

Western longitude limitation $\lambda_0 = \lambda_{\min}$ and Eastern longitude limitation $\lambda_b = \lambda_{\max}$

The guaranteed difference of longitude and latitude: $\varphi_{1+1} - \varphi_i = 0^0000001$ and $\lambda_{1+1} - \lambda_i = 0^0000001$.

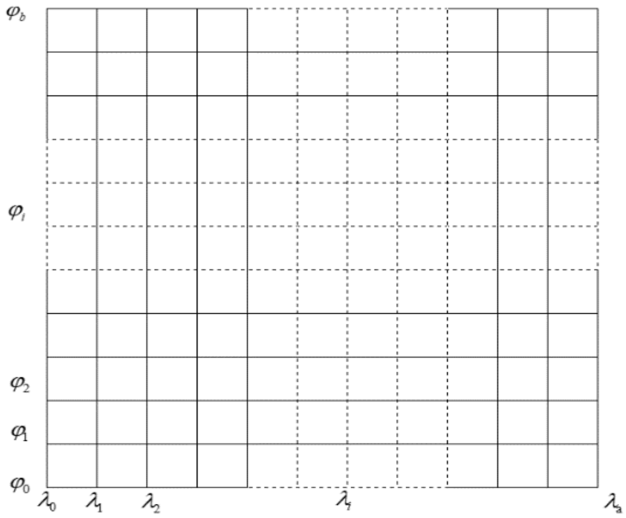

Fig. 6 The set of points in the searching domain

Step 3: Find the most probable ship's position corresponding to a measured altitude (single probable position).

The celestial body C_1 is observed from the ship with the adjusted altitude h_{O1} .

Consider a ship's estimated position $F_{xy}(\varphi_x, \lambda_y)$. The calculated altitude is as follows:

$$h_{Cxy} = \sin^{-1}[\sin\varphi_x \sin\delta + \cos\varphi_x \cos\delta \cos(t_G \pm \lambda_{yW}^{yE}] \tag{8}$$

The altitude variation can be calculated as follows:

$$\Delta h_{xy} = h_{Cxy} - h_{O1} \tag{9}$$

The most probable position $F_k(\varphi_k, \lambda_k)$ corresponding to the altitude of the body C_1 satisfies the following condition:

$$\begin{aligned}(\Delta h_{m_1 n_1})^2 &= \min\{(\Delta h_{xy})^2\} \\ &= \min\{(h_{Cxy} - h_{O1})^2\} \\ &= (h_{m_1 n_1} - h_{01})^2\end{aligned} \tag{10}$$

where:

$m_1 = \{1, 2, ..., b\}$ *và* $n_1 = \{1, 2, ..., a\}$

Therefore, with the measured altitude of C_1, we can have a single probable position $F_{m_1 n_1}(\varphi_{m_1 n_1}, \lambda_{m_1 n_1})$.

Step 4: Determining the most probable ship's position

For achieving the ship's position, at least two altitudes of celestial bodies are required.

Generally, with multi-altitudes of multi-bodies $\{C_1, C_2, ..., C_k\}$, the respective probable positions are: $\{F_{m_1 n_1}, F_{m_2 n_2}, ..., F_{m_k n_k}\}$

The most probable ship's position $F(\varphi, \lambda)$ with the coordinates calculated by the Least Squares Method as follows [9]:

$$\varphi = \sqrt{\frac{\sum_1^k (\varphi_{mi})^2}{k}} \; và \; \lambda = \sqrt{\frac{\sum_1^k (\lambda_{ni})^2}{k}} \tag{11}$$

φ *and* λ are defined according to φ_C *and* λ_C the general situation. In special cases, the searching domain covers both hemispheres, φ *and* λ should be separately defined in each hemisphere.

3. Results

3.1. Celestial Ship Positioning Program Construction

3.1.1. Flow Chart of the Program

With the above mathematical methodology, the flow chart of ship positioning by the celestial altitude difference based on the Least Squares Method is proposed in Fig. 7.

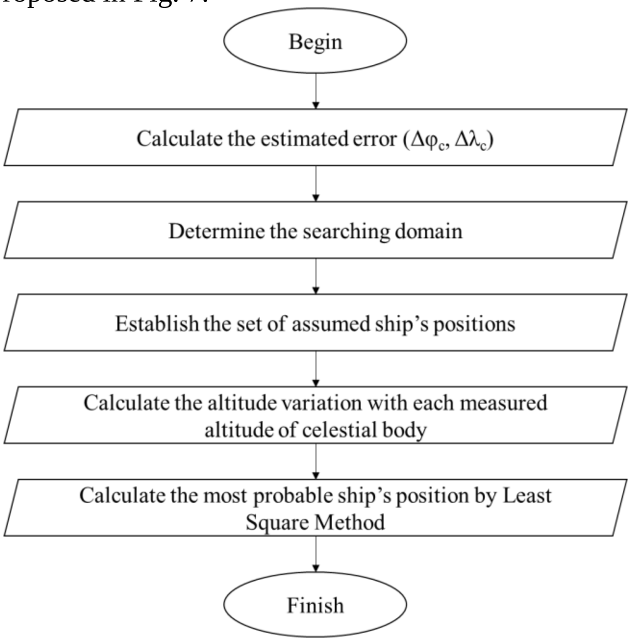

Fig. 7 Flow chart of ship positioning by the celestial altitude difference based on the Least Squares Method

3.1.2. Procedure of the Program

Step 1: Calculate the estimated error, then determine the radius of the circle of the position probability, according to the requirements of navigation accuracy (> 95%). When some ship positioning methods such as terrestrial method, GPS, etc. are not applicable, the celestial method could be used in some special situations. These situations occur during offshore navigation when the estimated position is determined based on the error in the tachometer correction and the error in the compass correction. These two errors are estimates and cumulative. On the other hand, due to offshore navigation, the determination of wind and current effects during offshore navigation is not very accurate. Therefore, it is necessary to accept the estimated position with a large error tolerance. To reduce the impact of the error of estimated position, it can be overcome by expanding the search domain in Step 2.

Step 2: Determine the searching domain on the basis of the estimated position to ensure the probability of containing ship's position meeting the requirements of the International Maritime Organization (> 95%). In order to increase the probability of containing the ship's location of the searching domain, it is necessary to consider the effects of wind and currents to increase the safety factor.

Step 3: Establish the set of assumed ship's positions in the searching domain with coordinates and reasonable order. According to the particular situation, the distance between estimated positions should be reasonably selected to make the calculation in step 4 more convenient.

Step 4: Calculate the altitude variation corresponding to each estimated position in the searching domain. Determine the position with the least square of altitude variation to a celestial body. In special cases with multiple results, we can reduce the number of results by changing the distance between estimated positions in the searching domain.

Step 5: Determine the most probable ship's position by Least Squares Method. The ship's position is the position with the least square of altitude variation when there are simultaneous observations of two or more celestial bodies.

3.1.3. Ship Positioning Program

Based on the proposed methodology, the automatic program to calculate the ship's position is constructed. The program is simply designed on the base of Excel, suitable to the condition and ability of the mariner. The Main parameters are shown in Table 1.

Table 1 Ship positioning program

Least Square Difference Altitude Celestial Body Method				
Celestial Body	**GMT**	**Ho**	**GHA**	**Dec**
Star 1	h – m - s	deg	deg	deg
Star 2	h – m - s	deg	deg	deg
Star 3	h – m - s	deg	deg	deg
Star 4	h – m - s	deg	deg	deg
Dead Reckoning	Lat Long	Ship Positions	Saint – Hilaire	Lat Long

Compass errors	deg	Genetic Algorithm	Lat
Speed log errors	%		Long
Course	deg	Least Square	Lat
Speed	knots		Long

The proposed program is compared with ship positioning method by genetic algorithm with the same input data. The result is shown in Table 2.

Table 2 Comparison of Least Square vs. genetic algorithm method

Least Square vs. Genetic Algorithm Method				
Celestial Body	**GMT**	**Ho**	**GHA**	**Dec**
Altair	18-00-00	37^{0}53’0	325^{0}06’6	08^{0}51’4
Fomalhaut	18-04-00	27^{0}54’0	279^{0}24’2	- 29^{0}39’1
Achemar	18-08-00	17^{0}46’5	240^{0}21’7	-57^{0}15’8
Rasalhague	18-12-00	41^{0}35’5	002^{0}04’8	12^{0}34’1
Dead Reckoning	-035^{0}13’0		Saint – Hilaire	-035^{0}19’0
	005^{0}20’0			005^{0}26’0
Compass errors	0^{0}01	Ship Positions	Genetic Algorithm	-035^{0}18’6
Speed log errors	0.01			005^{0}27’0
Course	220		Least Square	-35^{0}18’8
Speed	18			005^{0}26’9

3.2. Experiment on Board a Ship

The experiment to validate the program and compare with the Saint–Hilaire method and GPS position was performed. On 15 June 2020, the MV. THAI BINH 05 of Song Diem Shipping Joint Stock Company proceeded at 00h00, at speed of 9 kn, course 155005 in Gulf of Tonkin.

The ship maintained her speed and course, the ship’s position was obtained from GPS, and the estimated position was plotted with compass and tachometer errors. The time of nautical twilight on 06-16-2020, was at ship’s local time—04h15m00s (Greenwich Mean Time 21h15m00s). The observation for ship positioning by the Saint Hilaire method and the calculation of ship positioning by the Least Squares method have taken place. The ship’s position from GPS is also noted. The results of comparison between the Least Squares method, Saint Hilaire method and GPS position are shown in Table 3.

Table 3 Comparison of Least Squares method with Saint – Hilaire method and GPS position

Least Square Difference Altitude Celestial Body Method				
Celestial Body	**GMT**	**Ho**	**GHA**	**Dec**
Vega	21 - 15 - 00	36^{0}12’8	313^{0}35’1	38^{0}47’7
Mirffak	21 - 18 - 00	27^{0}29’2	181^{0}08’2	50^{0}05’0
Fomaihaut	21 - 21 - 00	35^{0}01’5	225^{0}05’5	39^{0}55’0
Dead Reckoning	20^{0}36’5 N		Saint – Hilaire	20^{0}50’5 N
	106^{0}55’5 E			107^{0}05’7 E
Compass errors	0^{0}05	Ship Positions	Least Square	20^{0}38’9 N
Speed log errors	0.2			107^{0}01’5 E
Course	155^0		GPS Position	20^{0}39’6 N
Speed	09			106^{0}59’9 E

4. Discussion

The ship's position was determined by three methods based on observation of four celestial bodies as follows:

Saint Hilaire method and Genetic Algorithm method: 0.9 nm/course 064°,

Saint Hilaire method and Least Squares method: 0.8 nm/course 255°,

Genetic Algorithm method and Least Squares method: 0.2 nm/course 202°.

The results of the first step reveal that two new methods (Genetic Algorithm and Least Squares) show a more accurate ship's position. In order to verify our proposed method effectiveness, an experiment was carried out onboard, as shown in the next section.

After approximately four hours from 0h00m00s to 04h21m00s on 06-16-2020, the difference between the estimated position and GPS position is: 5.2 nm/course 233°.

The difference between the position by the Saint Hilaire method and GPS position is: 12.2 nm/course 027°.

The difference between the position by the Least Squares method and GPS position is 1.7 nm/course 255°.

Based on the results in Tables 2 and 3, the proposed method showed its accuracy in ship positioning. Furthermore, it can reduce the variation between the estimated position and the ship's position. It is very important when using two independent ECDIS onboard a ship, to have a celestial method as a backup during offshore navigation.

The program of determining ship's position by the celestial altitude difference using the Least Squares Method is constructed using Excel with an easy-to-understand algorithm and simple user-oriented interface. It is suitable for the conditions seafarers experience whilst working onboard a ship. It can be done in a short space of time, including the process of celestial body observation and the calculation to achieve the results, making it an easy application to practice. The program also meets the requirements for a backup ship positioning method noted in the amendments of the STCW Convention 78/2010.

5. Conclusion

The paper proposed a new method in determining the ship's most probable position by calculating the least squares of the altitude variation to celestial bodies. This method satisfies the requirements for a backup method of ship positioning for ECDIS according to the amendments of the STCW Convention 78/2010. It shows the advantage that determined ship's position is independent of the estimated position accuracy. The mathematical methodology is simple and suitable for the qualifications of a marine officer. Based on the proposed methodology, the automatic ship positioning program is constructed. The result of the experiment on Motor Vessel (M/V) Thai Binh 05, has shown tremendous accuracy in the ship's position as opposed to previous methods. This program is simply designed, easy to apply and appropriate for the capabilities of officers and crew onboard a ship.

References

[1] MING-CHENG T. Genetic algorithm for solving celestial navigation fix problems. *Polish Maritime Research*, 2012, 19(3): 53-59. https://doi.org/10.2478/v10012-012-0031-5

[2] VULFOVICH B., & FOGILE V. New ideas for celestial navigation in the third millennium. *The Journal of Navigation*, 2010, 63(2): 373-378. https://doi.org/10.1017/S0373463309990348

[3] VAN-SUONG N., NAM-KYUN I. M., and QUANG-DAN D. Azimuth method for ship position in celestial navigation. *International Journal of e-Navigation and Maritime Economy*, 2017, 7: 55-62. https://doi.org/10.1016/j.enavi.2017.06.006

[4] CHIH-LI C., TIEN-PEN H., and JIANG-REN C. A novel approach to determine the astronomical vessel position. *Journal of Marine Science and Technology*, 2003, 11(4): 221-235. https://jmst.ntou.edu.tw/marine/11-4/221-235.pdf

[5] ZHANG J., YANG J., WANG S., LIU X., WANG Y., and YU X. A self-contained interactive iteration positioning and orientation coupled navigation method based on skylight polarization. *Control Engineering Practice*, 2021, 111: 104810. https://doi.org/10.1016/j.conengprac.2021.104810

[6] PIERROS F. Stand-alone celestial navigation positioning method. *The Journal of Navigation*, 2018, 71(6): 1344-1362. https://doi.org/10.1017/S0373463318000401

[7] LUSIC Z. Astronomical position without observed altitude of the celestial body. *The Journal of Navigation*, 2018, 71(2): 454-466. https://doi.org/10.1017/S037346331700073X

[8] NGUYEN V. S. A novel approach to determine the ship position with an azimuth of celestial body and factors of ship route. *International Journal of Civil Engineering and Technology*, 2019, 10(2): 1162-1167. https://iaeme.com/Home/article_id/IJCIET_10_02_113

[9] INTERNATIONAL MARITIME ORGANIZATION. *International Convention on Standards of Training, Certification and Watchkeeping for Seafarers (STCW 78/2010).* https://www.imo.org/en/OurWork/HumanElement/Pages/STCW-Convention.aspx

[10] BURCH D. *Celestial Navigation.* Starpath Publications, Seattle, Washington, 2010.

[11] NGUYEN T.-D. Evaluation of the accuracy of the ship location determined by GPS global positioning system on a given sea area. *Journal of Physics: Conference Series*, 2020, 1515: 042010.

[12] NGUYEN T.-D. Determining safety limitations and the frequency of determining vessel location when establishing marine bases on the Vietnam's sea areas. *Journal of Physics: Conference Series*, 2020, 1515: 042012.

[13] NGUYEN T.-D. Determine the effect of distortion of Mercator chart, Gnomonic chart on maritime safety and method of navigation when using Gnomonic chart. *Journal of Physics: Conference Series*, 2020, 1515: 052080.

[14] NGUYEN T.-D., & NGUYEN T.-D. Calculation of parameters for altering the ship's course at sea using Artificial Neural Network (ANN). *IOP Conference Series: Materials Science and Engineering*, 2020, 862: 032097. https://doi.org/10.1088/1757-899X/862/3/032097

[15] DMITRIEV V. I., GRIGORYAN V. L., and KATENIN V. A. *Navigation and Sailing Direction*. Morkniga, Moscow, 2009.

参考文:

[1] MING-CHENG T. 遗传算法，用于解决天文导航定位问题。波兰海事研究，2012，19（3）：53-59。https://doi.org/10.2478/v10012-012-0031-5

[2] VULFOVICH B. 和 FOGILE V。**第三个千年天体**导航的新思路。导航学报，2010，63（2）：373-378。https://doi.org/10.1017/S0373463309990348

[3] VAN-SUONG N.，NAM-KYUN I. M. 和 QUANG-DAN D. **方位角法用于天体**导航中的船舶位置。国际电子航海与海洋经济杂志，2017，7：55-62。https://doi.org/10.1016/j.enavi.2017.06.006

[4] CHIH-LI C.，TIAN-PEN H. 和 JIANG-REN C. **一种确定天文船只位置的新**颖方法。海洋科学与技术学报，2003，11（4）：221-235。https://jmst.ntou.edu.tw/marine/11-4/221-235.pdf

[5]张健，杨健，王珊，刘新，王Y，**余新。一种基于天窗偏振的自包含交互式迭代定位与定向耦合**导航方法。控制工程实践，2021，111：104810. https://doi.org/10.1016/j.conengprac.2021.104810

[6] PIERROS F. **独立的天体**导航定位方法。导航学报，2018，71（6）：1344-1362。https://doi.org/10.1017/S0373463318000401

[7] LUSIC Z. **没有**观测到天体高度的天文位置。导航学报，2018，71（2）：454-466。https://doi.org/10.1017/S037346331700073X

[8] NGUYEN V. S. **一种用天体方位角和航路因素确定船舶位置的新**颖方法。国际土木工程技术学报，2019，10（2）：1162-1167。https://iaeme.com/Home/article_id/IJCIET_10_02_113

[9]**国**际海事组织。**海**员培训，发证和值班标准国际公约（STCW 78/2010）。https://www.imo.org/cn/OurWork/HumanElement/Pages/STCW-Convention.aspx

[10] BURCH D. **天体**导航。星际之路出版物，华盛顿州西雅图，2010。

[11] NGUYEN T.-D。**通**过全球定位系统**全球定位系**统在给定海域上确定的船舶位置的准确性进行评估。物理学杂志：会议系列，2020，1515：042010。

[12] NGUYEN T.-D。**在越南海域建立海洋基地**时，确定安全限制和确定船只位置的频率。物理学杂志：会议系列，2020，1515：042012。

[13] NGUYEN T.-D。**在使用**地精图表时，确定墨卡托图表，地精图表的变形对海上安全和导航方法的影响。物理学杂志：会议系列，2020，1515：052080。

[14] NGUYEN T.-D. 和 NGUYENT.-D。**使用人工神**经网络（人工神经网络）计算用于更改船舶航向的参数。眼压**会**议系列：材料科学与工程，2020，862：032097. https://doi.org/10.1088/1757-899X/862/3/032097

[15] DMITRIEV V. I.，GRIGORYAN V. L. 和 KATENIN V. A. **航行和航行方向。莫**尔尼加（莫尔尼加），**莫斯科**，2009。